A TEXT BOOK OF
PRACTICAL PHYSICS

(for Degree Classes of Indian Universities)

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VOLUME II

MAGNETISM ELECTRICITY AND MODERN PHYSICS

THIRTEENTH REVISED EDITION

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Vol. I

(General Properties, Sound, Light and Heat)

Vol. II

(Magnetism, Electricity and Modern Physics)

The book is also available in one combined volume.

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PREFACE TO THE THIRTEENTH EDITION

This popular book has now reached its thirteenth milestone and in the process has tried to enrich the student community with illimitable resources of physics.

In the present edition many new diagrams have been placed in place of old ones, and the text has been thoroughly revised to keep the student abreast of present developments.

The author will feel grateful for any suggestions for improvement of the book.

INDU PRAKASH

PREFACE TO THE TWELFTH EDITION

We have indeed great pleasure in bringing out the twelfth edition of the book. With the introduction of Heat in B. Sc. Part I Examination by some Universities, we have reorganised volume I ; it now contains General Properties of Matter, Sound, Light and Heat. Volume II contains Magnetism, Electricity and Modern Physics.

To cope with the changes in the syllabi of some Universities, we have reorganised Volume I by incorporating the following new experiments :

1. Surface Tension by the method of Ripples,
2. Viscosity of water by rotating cylinder method,
3. Viscosity of water by variable head method,
4. Thermal conductivity of glass by Lees and Chorlton's method
5. Stefan's constant.

and by transferring the following experiments from Electricity Section to the section on Heat.

1. Temperature coefficient of Resistance of platinum,
2. J by Callendar and Barne's method.

The following new experiments have been added in Volume II.

1. Power factor of an a. c. circuit,
2. Capacitance of a condenser by Wien's series resistance bridge,
3. Self inductance of a coil by Maxwell's Inductance bridge,
4. Study of a series resonant a. c. circuit,
5. Static characteristics of a tetrode valve,
6. '*e*' by Millikan's oil drop method,
7. '*e/m*' of an electron by Thomson's method.
8. Comparison of frequencies by C. R. O.,
9. Characteristic curves of a photocell.
10. Frequency response of a photocell.

We have pleasure in extending our thanks to Shri R. N. Kapoor, Asstt. Professor of Physics, V. S. S. D. College, Kanpur, for his help in performing these new experiments and for the lively discussions that followed.

We trust and hope that this edition will prove to be more useful and will continue to serve the needs of the students.

Suggestions for the improvement of the book will be gratefully acknowledged.

PREFACE TO THE FIRST EDITION

The book is self-contained and the treatment followed in it is thoroughly modern. Every experiment is preceded by the just necessary and relevant theory which enables the student to grasp its underlying principles. Detailed instructions about each experiment are given in elaborate manner to enable the student to understand thoroughly the full experimental procedure. A brief description of the actual apparatus illustrated by diagrams has been given where necessary. The various precautions to be observed in an experiment have not only been emphasised in the method but have also been exhaustively discussed separately. The observation tables illustrate the sequence of the method and help the student to record his observations in an orderly and scientific manner. The criticism of the method given at the end of each experiment together with the exhaustive list of questions given at the close of each chapter and thoroughly dealt with in the text are the special features of the book. These are of considerable help to the student in his preparation for the *viva voce* test.

The book also includes practical details of description and working of apparatus of general use in the laboratory and a number of intelligent exercises based directly on experiments.

CONTENTS

INTRODUCTION

Fundamentals of Practical Physics

If we go deep into the history of human progress we find that, in every phase of the progress, one thing, which is common, and therefore, which has made this progress possible, is the application of human intelligence in overcoming his physical disabilities. Man began by exploiting the superior abilities of lower animals and gradually succeeded in inventing machines infinitely more able than any animal in their capacity for work.

With the success in inventing machines capable of doing miracles in the field of physical work, man soon realized that these machines will be equally useful in the field of knowledge, particularly in the field of physical knowledge. History of progress of physical sciences and the ever-increasing applications of instruments in the fields of other branches of knowledge amply justify the present age being called the Age of Machines, as they have influenced not only our capacity of doing, but also of thinking and willing.

If we carefully study the progress of Physics we find that the greatest contribution in this progress has been made by the inventions of more and more refined instruments of measurement. This had to be done because it was early recognised that the information of a phenomenon given to us by our unaided senses was so defective that it could not be relied upon for true knowledge. Since consistency is the criterion of truth and since sensual impressions are seldom consistent, it follows that knowledge based on unaided senses cannot always be true.

Our knowledge of a phenomenon is said to be true not only when the same observer agrees in his own experience of the phenomenon at all times but also when all observers agree in their experiences of that phenomenon under like circumstances. There are two main reasons why there may not be an agreement between different experiences of the same phenomenon :—

(*i*) Our senses being parts of a living or ever-changing body, it is not possible to keep a sense in a steady state or to reproduce that state at will nor is it possible to bring the senses of different observers in identical states.

(*ii*) It is not possible with the help of unaided senses alone to ensure that a given phenemenon is reproduced under identical conditions.

For true knowledge, therefore, we require some device which is more reliably consistent than our senses in its indications. Such devices are the various measuring instruments invented by modern scientists to make their knowledge of the physical world as free from defects as is humanly possible.

The following general scheme of a scientific investigation shows how experiment is the foundation of the development of a science. In this development we start with observations and proceed step by step, almost in an endless spiral, in which every new discovery, predicted sometimes by a theory, brings us nearer the truth.

Observation	(Qualitative) of the phenomenon as a whole generally with unaided senses.	
↓		
Experimentation	(Quantitative) with suitable instruments finding effects of various factors determining the phenomenon.	
↓		
Generalisation or law Formulation	Co-ordination of results of all cases of the same phenomena (Inductive)	to find the answer to the question (How ?) the various factors influence the phenomenon and express the answer in a formula or a graph.
↓		
Formulation of Theory	(Co-ordination of results of allied phenomena	to find the answer to the question (Why ?) the phenomenon obeys this discovered law and no other.
↓		
Prediction of new phenomena	Deduction of particular consequences of the more general proposition of the theory	Provides verification of the theory as well as discovery of newer facts of greater utility.
↓		
New experimentation	————————	with better instrument sometimes specially designed for the purpose.
↓		
Observation of new Facts		
↓		
Formulation of a new Law		
↓		
Formulation of a new Theory	or modification of the old one.	
↓		
and so on.		

In fact the aim of science is the formulation of a law governing the phenomena the science aims to study; because mere observation of isolated facts is not science. We have science only when we have discovered a proposition which includes all the possible infinite cases of the same class. The discovery of a law is in fact the discovery of the hidden element of *oneness* in the apparent *manyness* of the phenomenon. A theory aims at introducing a further element of oneness in our knowledge by showing a sort of oneness in apparently different types of phenomena·

From this point of view the aim of science, in fact of all knowledge is to show that the world of our common experience is not what it seems to be ; it is not so complex as it appears. On the other hand it is simple and one complete whole. It is in reality a Universe and not a Multiverse.

Foundation of a science, particularly of Physics is, therefore, experiment, and the aim of a carefully conducted experiment in Physics is the discovery of a law governing some phenomenon or the verification of a law derived from a theory which is acceptable to the physicist only when the experiment approves it.

With a view to make their investigations and lead us as close to the truth as possible, physicists have been not only trying to invent more and more perfect instruments but have also developed the theory of errors which suggests methods of eliminating from their investigations the possible errors.

The theory of errors starts with two assumptions :—

(*i*) That every observer is more or less careless and so liable to commit mistakes in reading an instrument. Therefore, every reading taken with the same measuring instrument is more or less erroneous ; the finer the instrument, the greater the chance of divergence in the several readings. Such errors are called *chance errors*, and their magnitudes depend on the quality of the instrument as well as that of the observer. If the instrument is free from defects and the observer is careful, the errors will be small.

(*ii*) That every instrument is more or less defective and so its indications are not free from errors. Such errors due to some inherent defects in the instrument are called *instrument or constant errors*.

Aim of this theory is, therefore, to devise means to get the right result from all these wrong observations. It is a sort of miracle the theory aims to achieve, because in brief the theory is :

Given :—A set of erroneous observations.

Required :—To find a result free from error.

With the help of this theory a physicist can either eliminate these errors or find out the magnitude of the probable errors which depend on the quality of the instrument at his disposal.

As the first kind of error depends on chance, its distribution follows the same law as any other chance event follows. Measurement of a quantity by an instrument is often compared to the hitting of a fixed target with a rifle, and the divergence of the shot from the exact point on the target is compared to the error of observation. Maxwell has theoretically derived a formula which gives the nature of the distribution of velocity amongst the molecules of a gas ; and this formula applies, with necessary changes, to the distribution of chance errors amongst various observations. Results of Maxwell's theory are based on the mathematical theory of probability which is truly applicable only to a group of a large number of individual

independent events. This will apply to set of a sufficiently large number of observations taken independently without any bias of the previously taken observations.

It can be easily shown that the arithmetic mean of such a large number of observations is the actual correct value of the quantity. Let the unknown true value of the quantity be x and let the various observations made be $x_1, x_2, x_3, \ldots \ldots x_n$, so that $e_1, e_2, e_3, \ldots\ldots e_n$, are the errors of the corresponding observation. Hence we have,

$$x_1 = x+e_1$$
$$x_2 = x+e_2$$
$$x_3 = x+e_3$$
$$\ldots \quad \ldots \quad \ldots$$
$$\ldots \quad \ldots \quad \ldots$$
$$x_n = x+e_n$$

$$\therefore \quad (x_1 + x_2+x_3+\ldots+x_n)=nx+(e_1+e_2+e_3+\ldots+e_n)$$

since $e_1, e_2, e_3 \ldots\ldots e_n$ are all chance errors, they should be so distributed that for any error, say $+e_1$ there should be an error $-e_1$, for there is no reason why mere chance should favour positive errors and not negative errors. Hence,

$$e_1+e_2+e_3+\ldots\ldots e_n=0$$

$$\therefore \quad (x_1+x_2+x_3+\ldots+x_n)=nx$$

or
$$x = \frac{1}{n}(x_1+x_2+x_3+\ldots\ldots+x_n)$$
$$= \text{arithmetic mean of observations.}$$

Hence arithmetic mean of a large number of observations is the correct value of a reading. If we call this x_m, then error of any observation say rth one, is defined by

$$e_r=x_m - x_r$$

A reading x_r is said to be precise or sharp if its error e_r is small or zero. Precision or freedom from these chance errors depends on the carefulness of the observer as well as the sensitivity of the instrument.

As the number of readings is usually not very large, $e_1+e_2+e_3+\ldots+e_n \neq 0$ and so the mean of errors $\frac{1}{n}(e_1+e_2+e_3+\ldots+e_n)=e_m$ is a small quantity and so the actual value $x \neq x_m$ but $=x \pm e_m$

Three kinds of average values of errors are used :

(*a*) Arithmetic mean, $e_m=\frac{1}{n}(e_1+e_2+e_3+\ldots+e_n)$

(*b*) Root mean square $e_r=\frac{1}{n}\sqrt{e_1^2+e_2^2+\ldots e_n^2}$

(*c*) Probable error $e_p=\cdot 6745\, e_m$

If a given error, e, occurs in f observations, then f is called the frequency of its occurrence and f/n is called the relative frequency in the total number of n observations. The most probable error is that for which f/n has the highest value and its relation with the mean value is shown to be $e_p=\cdot 6745\, e_m$.

To eliminate these chance errors, therefore, the experiments should take as large a number of observations as possible and should keep his mind perfectly free from any bias.

On the other hand, elimination of instrumental errors is not quite so easy as that of the chance errors. For the elimination of errors due to some likely defects in the make of the instrument, the experimenter has first to carefully study the various factors which determine the two chief qualities of the instrument, *viz.*, the sensitivity and the accuracy, and then see which part of a given instrument affects these and in what manner. *Accuracy* of an observation is freedom from these instrumental errors. *Sensitivity* of an instrument is measured either by the smallest amount of the difference in the quantity the scale of the instrument is capable of measuring or by the number of scale divisions which a given small difference in the quantity to be measured will produce. For example, sensitivity of a balance may be defined either as the number of m. gm. required to produce a shift of the pointer through one division or by the number of divisions the pointer will shift on the addition of an extra 1 m.gm. weight on one of the pans. Usually one of these gives insensitivity rather than sensitivity. In the example of the balance the first method gives insensitivity because a balance which requires a larger weight to shift the pointer through one division is less sensitive.

Sensitivity and accuracy of an instrument usually go against each other, for, if we try to increase one, the other is automatically decreased. In rare cases we may have both improved simultaneously.

The final result of the experiment is usually calculated from a set of observations taken with a number of measuring instruments and connected by means of a formula. It can be shown that each of these observed quantities does not influence the result similarly, someone influences more than the other. If, for example, a quantity, X depends on other quantities A, B and C in the manner given by the equation

$$X = A^p \ B^q \ C^r$$

Then,

$$\frac{\delta X}{X} = p\frac{\delta A}{A} + q\frac{\delta B}{B} + r\frac{\delta C}{C} \qquad \text{(Ia)}$$

$\therefore$ proportional error in X$=p$ times the proportional error in A$+q$ times the proportional error in B$+r$ times that in C.

Therefore, for given possible errors in A, B and C, the effect on the final result is greater for the factor having higher power, and that (the factor) having smaller value. For example, if A is the smallest quantity and p is the highest power, the measurement of A should be most carefully made, *i.e.*, δA should be made least.

A few examples are given below to illustrate the application of equation (Ia) for the purpose of calculating the extreme error.

I. Mean Coefficient of apparent expansion of water with a weight Thermometer;

The equation used for determining the mean coefficient of apparent expansion of a liquid, say water, is

$$\gamma_a = \frac{m_1 - m_2}{m_2(t_2 - t_1)}$$

The extreme proportionate error in γ_a is given by

$$\frac{\delta\gamma_a}{\gamma_a} = \frac{\delta(m_1 - m_2)}{(m_1 - m_2)} + \frac{\delta m_2}{m_2} + \frac{\delta(t_2 - t_1)}{(t_2 - t_1)}$$

The following are the observations in an experiment with water, giving a mean value of γa as 5.1×10^{-4} per °C.

$m_1 = 6.6958$ gm.	$\delta(m_1 - m_2) = 0.0002$ gm.
$m_2 = 6.4224$ gm.	$\delta m_2 = 0.0001$ gm.
$t_1 = 16.5°C$	$\delta(t_2 - t_1) = 1.°C$
$t_2 = 99.5°C$	

$\therefore$ $m_1 - m_2 = 0.2734$ gm. and $t_2 - t_1 = 83.0$ °C

Substituting the above values,

$$\frac{\delta\gamma_a}{\gamma_a} = \frac{0.0002}{0.2734} + \frac{0.0001}{6.4224} + \frac{1}{83}$$
$$= 0.0007 + 0.00001 + 0.01204$$
$$= 0.0128$$

Hence the percentage error in γ_a is 1.28% of which the major contribution is by the measurement of temperature difference with a thermometer reading upto $\frac{1}{2}$°C. We have taken $\delta(t_2 - t_1)$ a 1°C, assuming that the errors in the measurement of t_1 and t_2 (each being of $\frac{1}{2}$°) are such as to add up.

From the above analysis it is at once apparent that the use of an analytical balance for the different weighings reduces the errors in $(m_1 - m_2)$ and m_2 considerably and that the use of a thermometer reading to $\frac{1}{5}$°c would reduce the error in the measurement of $(t_2 - t_1)$.

II. Thermal Conductivity of Copper by Searle's apparatus.

The expression used for determining the thermal conductivity by Searle's apparatus is :

$$K = \frac{m(\theta_4 - \theta_3)d}{\pi r^2(\theta_1 - \theta_2)} = \frac{M}{t} \frac{(\theta_4 - \theta_3)d}{\pi r^2(\theta_1 - \theta_2)}$$

where m is the rate of flow of water as determined by collecting a mass M gm. in a time t secs.; the other letters have their usual significance.

The extreme proportionate error in K is given by

$$\frac{\delta K}{K} = \frac{\delta M}{M} + \frac{\delta(\theta_4 - \theta_3)}{(\theta_4 - \theta_3)} + \frac{\delta d}{d} + \frac{\delta t}{t} + \frac{2\delta r}{r} + \frac{\delta(\theta_1 - \theta_2)}{(\theta_1 - \theta_2)}$$

The following are the observations for this experiment :
Volume of water collected=10·0 cc ; Least count of graduated cylinder= 0·5 c.c.

∴ $M=10{\cdot}0$ gm.	$\delta M=0{\cdot}5$ gm.
$\theta_4=58{\cdot}0°C$	$\delta\theta_3=\delta\theta_4=0{\cdot}2°C$, the least count of the thermometers
$\theta_3=31{\cdot}2°C$	
$d=10{\cdot}50$ cm.	$\delta d=0{\cdot}01$ cm. L. C. of callipers
$r=2{\cdot}54$ cm.	$\delta r=\frac{1}{2}\times0{\cdot}001$ cm., L. C. of screw gauge being 0·001 cm.
$t=120$ sec.	$\delta t=0{\cdot}2$ sec.
$\theta_1=90{\cdot}2°C$	$\delta\theta_1=\delta\theta_2=0{\cdot}2°C$ L. C. of the thermometers.
$\theta_2=84{\cdot}8°C$	

Substituting the above values

$$\frac{\delta K}{K}=\frac{0{\cdot}5}{10{\cdot}0}+\frac{0{\cdot}04}{(58-31{\cdot}2)}+\frac{0{\cdot}01}{10{\cdot}50}+2\times\tfrac{1}{2}\times\frac{0{\cdot}001}{2{\cdot}54}+\frac{0{\cdot}2}{120}+\frac{0{\cdot}04}{(90{\cdot}2-84{\cdot}8)}$$

$$=0{\cdot}05+0{\cdot}0015+0{\cdot}0009+0{\cdot}0018+0{\cdot}0016+0{\cdot}0078$$
$$=0{\cdot}0636$$

Hence the extreme proportionate error in K is 0·0636, *i.e.*, the percentage error in K is 6·36% of which 5% is contributed by the measurement of M and 0·78% by the measurement of $(\theta_1-\theta_2)$

From the above it is evident that the student made the mistake of having too low a rate of flow of water and thus the error in its measurement amounted to 5% ; this low rate of flow also accounts for the small temperature difference between θ_1 and θ_2, which gives an error of 0·78%. Hence the rate of flow should have been more—This helps in reducing the percentage error in K. This is also rendered evident from another set of observations on the same rod.

Other factors being same, the altered observations are

$M=44$ gm. ; $\theta_1=83{\cdot}6°C$; $\theta_2=70{\cdot}4°C$
$\theta_4=45{\cdot}4°C$; $\theta_3=31{\cdot}0°C$

Making the calculation of the error in K we have

$$\frac{\delta K}{K}=\frac{0{\cdot}5}{44}+\frac{0{\cdot}04}{(45{\cdot}4-31{\cdot}0)}+\frac{0{\cdot}01}{10{\cdot}50}+2\times\tfrac{1}{2}\times\frac{0{\cdot}001}{2{\cdot}54}+\frac{0{\cdot}2}{120}+\frac{0{\cdot}04}{(83{\cdot}6-70{\cdot}4)}$$

$$=0{\cdot}0113+0{\cdot}0027+0{\cdot}0009+0{\cdot}0018+0{\cdot}0016+0{\cdot}0030$$
$$=0{\cdot}0213$$

Hence the percentage error in K is 2·13%

III. Determination of H with Vibration and Deflection magnetometers.

The equation employed for calculating H from the observations is

$$H=\frac{2\pi}{T(d+l)(d-l)}\left(\frac{2\,I\,d}{\tan\theta}\right)^{\frac{1}{2}}$$

The extreme proportionate error in H is given by

$$\frac{\delta H}{H}=\frac{\delta T}{T}+\frac{\delta(d+l)}{(d+l)}+\frac{\delta(d-l)}{(d-l)}+\frac{1}{2}\left[\frac{\delta I}{I}+\frac{\delta d}{d}+\frac{\sec^2\theta\delta\theta}{\tan\theta}\right]$$

From the following observations in an experiment, we can calculate the error.

$T=3\cdot14$ sec. $\delta T=\frac{0\cdot2}{50}$ sec. (as 50 vibrations were timed)

$d=18\cdot5$ cm. $\delta d=0\cdot1$ cm.

$l=2\cdot505$ cm. $\delta l=0\cdot2$ cm., assuming this to be the uncertainly in locating the poles.

Hence, $\delta(d\pm l)=0\cdot3$ cm.

$\theta=40°$ $\delta\theta=1°=0\cdot0175$ radian

$m=55\cdot750$ gm. $\delta m=0\cdot001$ gm.

$a=5\cdot10$ cm. $\delta a=\delta b=0\cdot01$ cm., least count of

$b=1\cdot60$ cm. vernier callipers.

Since $I=m\frac{a^2+b^2}{12}$

$$\therefore\ \frac{\delta I}{I}=\frac{\delta m}{m}+\frac{2\delta a}{a}+\frac{2\delta b}{b}$$

$$=\frac{0\cdot001}{55\cdot750}+\frac{2\times0\cdot01}{5\cdot10}+\frac{2\times0\cdot01}{1\cdot66}$$

$$=0\cdot0269$$

Hence,

$$\frac{\delta H}{H}=\frac{0\cdot2}{50\times3\cdot14}+\frac{0\cdot3}{21}+\frac{0\cdot3}{16}+1\left[0\cdot0269+\frac{0\cdot1}{18\cdot5}+\frac{0\cdot0175}{(0\cdot7760)^2\times(0\cdot8391)}\right]$$

$$=0\cdot0013+0\cdot0143+0\cdot0187+0\cdot0134+0\cdot0029+0\cdot0173$$

$$=0\cdot0679$$

Thus the percentage error in the measurement of H is 6·79%. The major contribu ns to this error are due to the uncertainty in locating the poles, and 1·73% in the measurement of θ.

If θ were 25°.

$$\frac{\delta(\tan\theta)}{\tan\theta}=\frac{\sec^2\theta\delta\theta}{\tan\theta}=\frac{0\cdot0175}{(0\cdot9063)^2\times(0\cdot4663)}=0\cdot04569$$

Hence the error contribution due to this factor would have been $\frac{1}{2}\times0\cdot04569=0\cdot0228$, *i.e.*, 2·28% as against 1·73% when θ is 40°. It is, therefore, necessary to have θ in the neighbourhood of 45° consistent with the fact that d is taken large. Taking d large reduces the errors in the factors involving $(d+l)$ and $(d-l)$.

IV. Determination of difference in two nearly equal resistances with a Carey Foster's Bridge.

In a Carey Foster's Bridge

$$X-Y=(l_2'-l_1')\rho$$

where

$$\rho=\frac{R}{(l_2-l_1)}$$

Hence,

$$X-Y=\frac{R}{(l_2-l_1)}\times(l_2'-l_1')$$

The proportionate error in (X—Y) is given by

$$\frac{\delta(X-Y)}{X-Y}=\frac{\delta R}{R}+\frac{\delta(l_2-l_1)}{l_2-l_1}+\frac{\delta(l_2'-l_1')}{l_2'-l_1'}$$

In an experiment the observations were

$$R=1{\cdot}00\ \Omega \qquad \delta R=0{\cdot}001\ \Omega$$

$$\left.\begin{array}{ll}(l_2-l_1)=49{\cdot}8\text{ cm.} & \delta(l_2-l_1)=0{\cdot}2\text{ cm.}\\ (l_2'-l_1')=4{\cdot}5\text{ cm.} & \delta(l_2'-l_1')=0{\cdot}2\text{ cm.}\end{array}\right\}\delta l_1=\delta l_2=0{\cdot}1\text{ cm.}$$

Substituting the values above

$$\frac{\delta(X-Y)}{X-Y}=\frac{0{\cdot}001}{1{\cdot}00}+\frac{0{\cdot}2}{49{\cdot}8}+\frac{0{\cdot}2}{4{\cdot}5}$$

$$=0{\cdot}001+0{\cdot}004+0{\cdot}0444$$

$$=0{\cdot}0494$$

Hence the proportionate error in (X—Y) is 0·0494, *i.e.*, the percentage error in the determination (X—Y) is 4·94%.

Three points are worth noting here.

(1) In order to determine ρ, choose a large value of R to have (l_2-l_1) large; then alone the proportionate error in the determination of (l_2-l_1) would be less.

In the above experiment, with R=0·1 ohm, (l_2-l_1) would be 5cm., and the proportionate error in its measurement would be 0·2/5= 0·04, *i.e.*, the percentage error in its determination would be 4%. It is ten times the value of 0·4% with R=1 ohm.

(2) From the observations above, it is apparent that the *difference* between X and Y is of the order of 0·09 ohm. The total extreme error is 4·94% of which that contributed by the measurement of $(l_2'-l_1')$ is 4·4%. Hence the error in the difference of the resistances X and Y is (0·09×4·94)/100 =0·0044 ohm. Thus this error contribution of 4·4% in the measurement of $(l_2'-l_1')$ need not be alarming, as this error is 4·4% of the *difference*.

This point is further illustrated by taking a value of $(l_2'-l_1')$ as 1·5 cm., when the difference between the two resistances is of the order of 0·03 ohm. In this case

$$\frac{\delta(X-Y)}{X-Y}=0{\cdot}001+0{\cdot}004+\frac{0{\cdot}2}{1{\cdot}5}=0{\cdot}1383$$

i.e., the percentage error in the measurement of (X—Y) is as high as 13·83%. But the difference is of the order 0·03 ohm, and hence the error in the difference (X—Y) is $0{\cdot}03 \times \frac{13{\cdot}83}{100} = 0{\cdot}0041$ ohm.

(3) The method is suitable for finding the difference in the value of two very nearly equal resistances, for then the error contribution in the determination of ρ becomes negligible.

V. Comparison of two resistances with a potentiometer.

We know that

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

In an experiment, $l_1 = 918{\cdot}1$ cm., $l_2 = 900{\cdot}5$ cm. $\delta l_1 = \delta l_2 = 0{\cdot}1$ cm. Hence the proportionate error in the determination of the ratio R_1/R_2 is

$$\frac{\delta(R_1/R_2)}{R_1/R_2} = \frac{\delta l_1}{l_1} + \frac{\delta l_2}{l_2}$$

$$= \frac{0{\cdot}1}{918{\cdot}1} + \frac{0{\cdot}1}{900{\cdot}5}$$

$$= 0{\cdot}000108 + 0{\cdot}000111$$

$$= 0{\cdot}00019 = 0{\cdot}00022$$

Thus the percentage error is 0·022% and it is almost equally distributed between the two factors. The method should be applied to the comparison of two nearly equal resistances.

VI. Determination of E. C. E., of Cpper using a Tangent Galvanometer.

The electro-chemical equivalent of copper is given by

$$z = \frac{m}{K.\ t.\ \tan\theta}$$

The proportionate error in z is given by

$$\frac{\delta z}{z} = \frac{\delta m}{M} + \frac{\delta K}{K} + \frac{\delta t}{t} + \frac{\delta(\tan\theta)}{\tan\theta}$$

In an experiment the observations gave

$m = W_2 - W_1 = 99{\cdot}9752 - 99{\cdot}6102$ gm. $= 0{\cdot}3650$ gm. | $\delta m = 0{\cdot}0002$ gm., m being the diff., of two weighings.

K is known to a high degree of accuracy as H, n and r are known fairly accurately ; r is usually supplied by the manufacturer. Hence the factor $\delta K/K$ can be left out.

$t = 1200$ sec. $\delta t = 1$ sec.

$\theta = 42^\circ$ $\delta\theta = 1^\circ = 0{\cdot}0175$ radian

Substituting the values in the equation above

$$\frac{\delta z}{z} = \frac{0{\cdot}0002}{0{\cdot}3650} + \frac{1}{1200} + \frac{\text{Sec}^2 42 \times 0{\cdot}0175}{\tan 42}$$

$=0{\cdot}00054+0{\cdot}00083+0{\cdot}03416$
$=0{\cdot}0356$

Hence the extreme percentage error in z is $3{\cdot}56\%$,the major contribution being due to an error in the observation θ. As the weighings are done on a chemical balance, $\delta m/m$ factor contributes $0{\cdot}054\%$ error only. Using a stop watch of least count $0{\cdot}2$ sec., would not in any way improve the result as the error contribution by a stop-clock of least count 1 sec. is $0{\cdot}083\%$ when the duration for which the current is passed is 20 minute. If the duration be raised to 30 minutes, it would be less. We therefore conclude that great care must be exercised in the observation of θ, the angle of deflection. It is assumed, however, in the discussion that the setting of the galvanometer is correct.

GRAPHICAL REPRESENTATION OF DATA

I. Advantages of Graphs. The presenting of data by a graphical method is an adaptation of the principles of Descarte's analytical geometry, whereby numerical values are represented in geometrical form by the length of a line, the area of a surface etc.

It may conveniently be said in favour of graphs that

(*i*) They appeal to the attention of the reader,

(*ii*) They permit an easy reference to data,

(*iii*) They facilitate comparisons of values,

(*iv*) They reveal some significant features in a set of data —*e.g.*— presence of maxima or minima or points of inflection— which may not be easily noticed in a survey of the data.

II. Choosing the Co-ordinate scales. For the under-graduate students the purpose of plotting a graph is limited. His job is not to investigate whether or not a relationship exists between the variables considered, and if such a relationship exists, to determine its mathematical form. His job is to show that the relationship, as embodied in the mathematical equation which he is using in his experimental work, does exist ; he may further use the graph as a tool for making some determinations.

Further the under-graduates have not to make any choice about the type of graph paper to be used in the graphical representation of the experimental data. He has usually to use the rectangular graph paper which has either mm. divisions or 1/10″ divisions. The main problem before him is the *choice of the co-ordinate scales.*

"A poor choice of scales for the co-ordinates, more than any other single factor, will make an otherwise acceptable graph unsatisfactory as a tool." This being the case, the need of suitability rules is evident. Certain general rules may be stated as follows :

1. *It is customary to choose the scale for the independent variable along the X=axis.*

This is an established custom. Which of the two variables is independent is usually greatly influenced by the experimental procedure adopted. While taking observations in an experiment, the values of one quantity are arbitrarily fixed,and the corresponding values of the other quantity are observed; in such a case it is evident that the former quantity is to be regarded as an independent variable.

In the experiment for the determination of difference in two nearly equal resistances by C. F. Bridge, the formula used is

$$R_1 - R = \rho(l_1 - l_2)$$

where l_1 is the length of the bridge wire at the balance with R_1 in left gap and R in the right, l_2 the corresponding length when R_1 and R are interchanged, ρ is the resistance per unit length of the bridge wire.

In the experiment under reference R is varied arbitrarily by using suitable resistance in the decimal ohm box and the corresponding difference $(l_1 - l_2)$ is computed by observing l_1 and l_2. R is therefore an independent variable while the corresponding value of $(l_1 - l_2)$ is a dependent variable. The graph (G. A.) has been plotted with R along X-axis and $(l_1 - l_2)$ along Y-axis. The data for the graph has been taken from the table below.

TABLE—*For Measurement of* R_1

R	l_1	l_2	$(l_1 - l_2)$
2·4	56·0	47·9	8·1
2·5	54·4	49·5	4·9
2·6	52·9	51·2	1·7
2·7	51·2	52·8	—1·6
2·8	49·2	54·4	4·5
2·9	48·2	55·9	7·7

2. *The choice of the scale should be such that the co-ordinates of any point on the graph may be ascertained easily and quickly.*

For rectangular graph paper with successive main lines divided into ten parts (each part equal to 1 mm. in a cm. graph sheet or 1/10" in an inch graph sheet), the scales convenient to use are those in which the distance between two consecutive main lines represents a difference in value of 1, 2, 4 or 5 units or these values multiplied by 10^n where n is an integer. It would certainly be very inconvenient to plot the points and also troublesome to read the co-ordinates of a point on the plot, if the distance between the consecutive main lines represented a difference of 3, 6, 7, 9...........units.

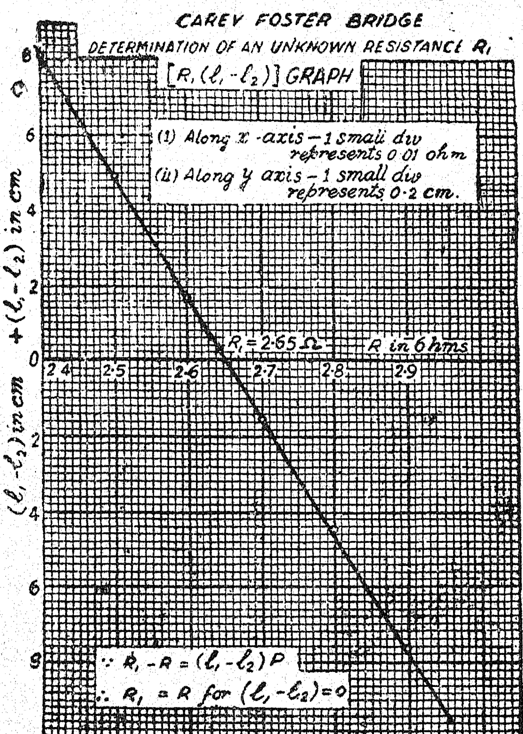
Thus in the graph (G.A.) between R and $(l_1 - l_2)$ the choice of scales along the two axes is

(*i*) 0·1 ohm., difference between two consecutive *main* lines along the X-axis ;

(*ii*) 2 cm., difference between two consecutive *main* lines along the Y-axis.

3. *The choice of the scale should be such that the resultant curve is as extensive as the sheet permits but subject to the condition that "uncertainties of measurements are not made thereby to correspond to more than one or two of the smallest divisions."*

The co-ordinate units need not usually start from zero values. It is customary to begin the scale for each variable from the lowest rounded value in the data and end it at the highest value or just greater than the highest value.



Graph (G. A.) between R, (l_1-l_2)

The above graph has been drawn by taking the above rule into consideration with the exception that (l_1-l_2) has a starting point zero It is necessary in this case because (l_1-l_2) has both +ve and —ve values for different values of R.

Supposing there were observations only for +ve values of (l_1-l_2), even then the starting point for (l_1-l_2) would have to be taken as

zero, because the calculation of R_1, the unknown resistance, is based on finding the value of R for $(l_1—l_2)=0$. For, from the equation

$$R_1—R=(l_1—l_2)\ \rho$$

it is evident that as R is altered, $(l_1—l_2)$ is correspondingly altered, R and ρ being constants. A plot of R and $(l_1—l_2)$ is expected to be a straight line, as shown in graph GA.

When $(l_1—l_2)=0$,

$$R_1=R$$

i. e., the intercept of the plot line on the R axis for a value $(l_1—l_2)=0$, gives the value of R_1. Hence the starting point for $(l_1—l_2)$ has to be taken zero in this case of plotting the graph.

We conclude therefore that except where necessary we begin the scale of each variable from the lowest rounded value. For R we begin the scale with 2·4 ohm but for $(l_1—l_2)$ we start with zero at the origin because it is necessary to do so.

Secondly, where the sheet is large enough we could have spread the $(l_1—l_2)$ scale but it would have been incorrect to spread the scale such that each small division in the graph reads, say, 0·05 cm., for the least count of the metre scale used to read l_1 or l_2 is 0·1 cm.

4. *Scales are to be marked properly by labelling the main co-ordinate lines with values which they represent.*

It should be noted that,

(*a*) The numbers used in marking the scale on the main lines should contain as many significant figures as the data justify.

(*b*) For a satisfactory scale designation it is necessary to have along each axis the name of the quantity represented and the units in which it is measured.

As shown in graph GA, along X-axis it is labelled as, R in ohm along Y-axis, $(l_2\text{-}l_2)$ in cm.

5. *Plotting of the points on the graph sheet should be done very carefully and each point should be indicated by a cross* (×) *or a circle* (O).

Where two plots on the same sheet intersect or are very close together, the points of one may be marked with a cross (×) and of the other with a circle (O).

6. *Fitting a curve to the plotted points.*

The curve should be smooth and should pass as close as reasonably possible to all the plotted points.

When the number of observations is fairly large, of the points which do not lie on the curve but close to it, about one half should fall on one side of the curve and the other half on the other side of the curve.

A point lying too remote from the best fit curve suggests obviously a mistake in the taking of the observation corresponding to this.

7. *Appropriate caption*

An appropriate and descriptive caption accompanying a graph should clearly indicate what the graph is intended to show.

As the primary aim of a scientific investigator is the discovery of truth, he should take all possible care to eliminate all those factors that tend to distort truth. One of the most important factors that is responsible for this distortion is the mental prejudice and habit of carelessness of the investigator. Defects of instruments are only of secondary importance. Most of the pioneer workers in the fields of scientific investigations did not use very perfect instruments, for which like a bad carpenter our students are seen quarrelling. Every serious student and teacher of Physics knows well that a student learns most when he works with an imperfect instrument, for it develops resourcefulness in the worker.

For these and other considerations a student of practical physics will do well to begin his work after carefully going through these few pages and thus putting himself in the proper scientific attitude of mind of an original investigator. It is for want of this attitude of mind that a student sometimes does not feel interested in this work. A student will, therefore, do well if he carefully reads and tries to follow throughout his work in the laboratory, the rules laid down below :

1. The aim of experimental science being the discovering of truth, it should be carefully borne in mind by every student of Physics that a perfectly open and unbiased state of mind should be maintained during experimental work.

2. Before beginning any of the experimental work, the student should make a self-study of the work and thus have a clear idea of the aim of his work and of the way how he is going to realize that aim.

3. Forethought should be brought to bear upon every phase of work in hand. It counts more than anything else in the saving of time, in the prevention of trouble, and in the assurance it gives of satisfactory work.

4. If no plan or diagram of the arrangement of the apparatus is provided, one should be carefully made and verified before actually setting up the apparatus.

5. A sufficiently complete description of every piece of apparatus as also the order of taking observations, should be recorded in complete details in a carefully pre-arranged form.

6. In all cases as many observations as possible should be taken and entered in a suitable tabular form to be so planned by the student that it gives all the details of his observations and in certain cases also the results of his calculation.

7. These observations and results are to be directly entered in the book in which the final and fair report of the work has to be written.

8. In most of his experiments, the student has to determine in the value of a physical quantity which is not directly observed but is calculated from the observations of other quantities that are directly measured. It often happens that several of these directly measurable quantities are capable of more accurate determination than others, so it becomes desirable to previously determine the degree of accuracy to which each quantity should be measured. For this the student should refer to equ. (Ia) and the following examples which illustrates its use.

9. In the calculation of results also, no advantage will be found to be gained by using more than just the necessary number of figures. Thus the use of contracted and approximate methods of calculations, and also the use of log. tables up to a limited number of figures is desirable.

10. The student should make a free and frequent use of graph in the representation of his observations and also in the interpretation of their result. A graph gives a deeper insight into the phenomenon than even the tabular form.

The record of the experiment should be entered in the following form :

Date *Experiment No.* *Page*

1. A clear and concise statement of the object of the experiment.

2. A discussion of the theory involved in the experiment resulting in the formula from which the quantity is to be calculated from the direct observation of other quantities. In certain cases it may suffice to give the formula and the meaning of each symbol used in it.

3. A list of the various pieces of apparatus actually used.

4. A clear description of the assemblage of the apparatus with a linear, self-explanatory diagram neatly and carefully drawn.

5. All original observations in a neatly tabulated form.

6. Calculations.

7. Method of taking observations.

8. Precautions that were actually taken to be stated with reasons.

9. Result and percentage Error. Never omit unit.

10. Discussion of the result and criticism of the apparatus.

SECTION V

MAGNETISM AND ELECTRICITY

CHAPTER XXIV

MAGNETIC MEASUREMENTS

24·1. Field Strength. The space around a magnet within in which magnetic material will experience magnetic effect is termed a *magnetic field.* Theoretically this field is of infinite extent but since its effect dies away very quickly as the distance from the magnet increases, the field is appreciable only in a limited region around the magnet. At every point in the field the magnetic force has a definite strength depending upon the distance from the poles. This *strength or* **intensity of the field** *at a point is equal to the force in dynes experienced by a unit north pole if placed at that particular point, it being assumed that the introduction of the unit pole does not affect the configuration of the field.* Thus, if a pole of strength m webers be placed at a point in a magnetic field where the intensity of the field is H units, the force in dynes experienced by the pole will be given by

$$F = mH$$

The direction of the field at a point is the direction in which an *isolated* north pole would tend to move if placed at that point. The unit of field strength is *oersted.* *A magnetic field is said to be of one* **oersted** *when the force experienced by a unit north pole in it is one dyne.*

24·2. Magnetic moment and intensity of magnetisation. Referringto fig. 24·1, let a magnet of pole strength m be placed in a *uniform* field of strength H so as to be free to rotate about a vertical axis through its C. G. The force on *each* pole of the magnet is mH. If the magnet makes an angle θ with the direction of the field, these two *equal opposite* and *parallel* forces mH constitute a couple whose moment is equal to $m\text{H} \times \text{SA}$, where SA is the perpendicular distance between the lines of the two forces. But $\text{SA} = 2l \sin\theta$, where $2l$ is the length of the magnet. Hence the moment of the *restoring couple* is $m\text{H}\ 2l \sin\theta = 2ml.\text{H}\sin\theta = \text{M}\sin\theta$, where $\text{M} = 2ml$. Thus for any position of the magnet in a uniform field the couple depends upon the quantity M. This characteristic quantity M is called the *magnetic moment of the magnet.* The couple for all magnets having different pole strengths and lengths is the same provided their magnetic moments are the same.

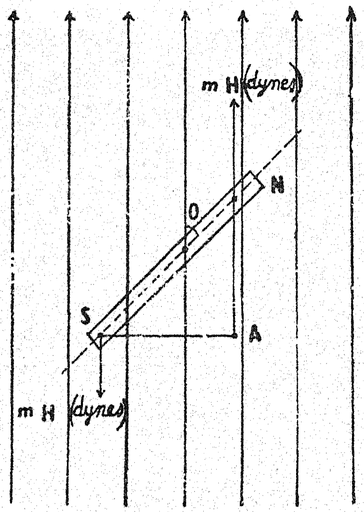


Fig. 24·1

The effect of this couple is to set the magnet *parallel* to the direction of the magnetic field in which position the couple vanishes, for then $\theta=0$. There is, however, no translatory force acting on the magnet to make it move bodily. The couple is *maximum* when $\theta=90°$ and has the value MH. If the field is of unit strength, *i.e.*, $H=1$ and also $\theta=90°$, the couple is numerically equal to M. Thus *the* **magnetic moment** *of a magnet is defined as the product of its pole strength and the distance between the poles and it is numerically equal to the moment of the couple acting on the magnet when it is placed at right angles to a uniform field of unit intensity.* Magnetic moment is a *vector* quantity and can be resolved into components. Its C. G. S. unit is weber × centimetre.

The **intensity of magnetisation** *of an element of a magnet is the magnetic moment per unit volume.* If the magnet is *uniformly* magnetised, the intensity of magnetisation will be the *same* throughout its volume. If M be the magnetic moment of a *uniformly* magnetised bar magnet, V its volume, L its length and A the area of its pole face, the *intensity of magnetisation* of the magnet is given by

$$I=M/V=mL/LA=m/A$$

where m is the strength of the magnet. Thus *the* **intensity of magnetisation** *is also defined as the pole strength per unit area of the pole face.*

24·3. Magnetic field due to a bar magnet. Let m be the pole strength and $2l$ the distance between the poles of a bar magnet NS. Let P be a point distance d from the centre of the magnet. The intensity at the point P due to the north pole is m/NP^2 directed along NP and that due to south pole is m/SP^2 directed along PS. The strength of the field at P due to the magnet is the resultant of these two component intensities. We shall calculate the resultant intensity at the point P in the following two simple cases :—

(a) **When the point P lies on the axial line of the magnet.** Let the point P lie on the axis of the magnet produced as depicted in fig. 24·2. Then $NP=(d-l)$ and $SP=(d+l)$. Since the two component intensities due to the two poles are in the *same* straight line, the resultant field at P is given by

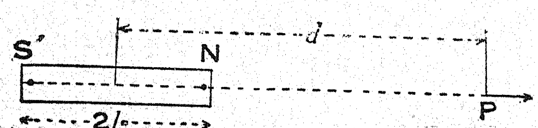


Fig. 24·2

$$F = \frac{m}{NP^2} - \frac{m}{SP^2}$$

$$= m\left(\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right)$$

$$= \frac{4mdl}{(d^2-l^2)^2}$$

But since $2\ ml = M$, this reduces to

$$F = \frac{2Md}{(d^2-l^2)^2} \quad \text{(along NP)} \quad (24{\cdot}1)$$

In this case *the magnet is said to be* **end-on** *to the point* P.

Expanding the right-hand side of the above equation, we get

$$F = \frac{2\ M}{d^3}\left(1 + \frac{2\ l^2}{d^2} + \dots\dots\right)$$

or if $d > l$,

$$F = \frac{2M}{d^3}\left(1 + \frac{2\ l^2}{d^2}\right)$$

If $d >> l$ so that l^2 is negligible when compared to d^2 we have

$$F = \frac{2\ M}{d^3} \quad \dots \quad \dots \quad (24{\cdot}2)$$

(b) When the point P lies on the equatorial line of the magnet. Let the point P lie on the line *perpendicular* to the axis of the magnet and passing through its centre as shown in fig. 24·3. The point P will be *equidistant* from the two poles so that NP=SP=r (say). The component intensities at P due to the two poles will, therefore, be *each* equal to m/r^2. Let them be represented by PA and PB in the figure. Then completing the parallelogram PACB, the resultant is represented by PC. From similar $\triangle$ PNS and APC, we have

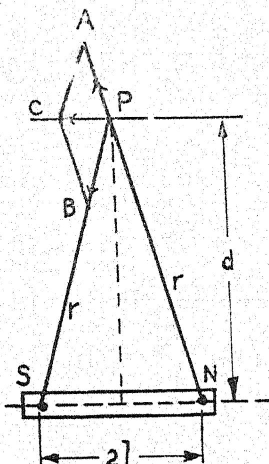


Fig. 24·3

$$\frac{PC}{PA} = \frac{NS}{PN} = \frac{2l}{r}$$

$$\therefore \text{ Field at P} = PC = \frac{2\ l}{r} \cdot PA = \frac{2\ m\ l}{r^3}$$

But $2ml = M$ and $r = (d^2+l^2)^{1/2}$

(24·3)

$$\therefore \quad F = \frac{M}{(d^2+l^2)^{3/2}}$$

In this case *the magnet is said to be* **broad-side-on** *to the point* P. Note that the field at P is *parallel* to the magnet. Expanding the right-hand side of the above equation, we get

$$F = \frac{M}{d^3}\left(1 - \frac{3}{2}\frac{l^2}{d^2} + \dots\dots\right)$$

or if $d > l$,
$$F = \frac{M}{d^3}\left(1 - \frac{3}{2}\frac{l^2}{d^2}\right)$$

If $d >> l$ so that l^2 can be neglected when compared to d^2, we have

$$F = \frac{M}{d^3} \tag{24·4}$$

Comparing equations (24·2) and (24·4) we see that *the field at a point due to magnet in the* **end-on** *position is* **twice** *that due to the same magnet in the* **broad-side-on** *position, the distance of the point from the centre of magnet remaining the same.* Further, the field at a sufficiently great distance depends only on the magnetic moment M and not on the values of the factors m and $2l$ separately and the same is true for the field at a great distance in any direction from the centre of the magnet.

24·4. Two mutually perpendicular uniform magnetic fields. When a *small* compass needle, freely suspended or pivoted, is placed in a *uniform* magnetic field of intensity H, it comes to rest with its axis *parallel* to the direction of the field. If now a second *uniform* field of intensity F be superposed over the first field in a direction *perpendicular* to that of the first, the needle will be deflected from its rest position and will set itself in the direction of the *resultant* where the couples due to the fields balance each other.

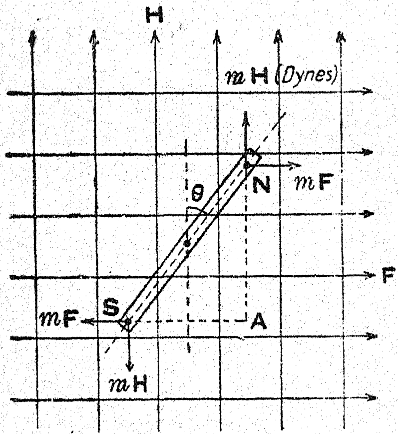


Fig. 24·4

Referring to fig. 24·4, let the angle which the needle makes with the direction of the *first* field in the *equilibrium position* be θ. The couple on the needle due to the *first* field of intensity H is mH × SA and that due to the *second* field of intensity F is mF × AN, where m is the pole-strength of the needle. Since in the *equilibrium position* of the needle, the two couples are *equal*, we have

$$mF \times AN = mH \times SA.$$

or
$$F = H \times \frac{SA}{AN}$$

But
$$\frac{SA}{AN} = \tan\theta$$

$\therefore$
$$F = H \tan\theta \quad \ldots \quad (24{\cdot}5)$$

The resultant field is *uniform* and is of magnitude $\sqrt{F^2+H^2}$ It makes an angle $\tan^{-1}(F/H)$ with the direction of the *first* field of intensity H.

Note that the position occupied by the compass needle is *independent* of its magnetic moment, pole-strength or length.

(a) Tangent A Position of Gauss. Let a magnet NS be placed with its axis *perpendicular* to the magnetic meridian, and let a *small* compass needle be placed at a point P on the *axial line* of the magnet produced as shown in fig. 24·5. The field F at P due to the magnet will be *perpendicular* to the *horizontal component* of the earth's magnetic field. Hence, if θ be the deflection of the needle from the *magnetic meridian*, then from equ. 24·5).

$$F = H \tan\theta$$

Since the magnet is *end-on* to the point P, we have from equ. (24·1)

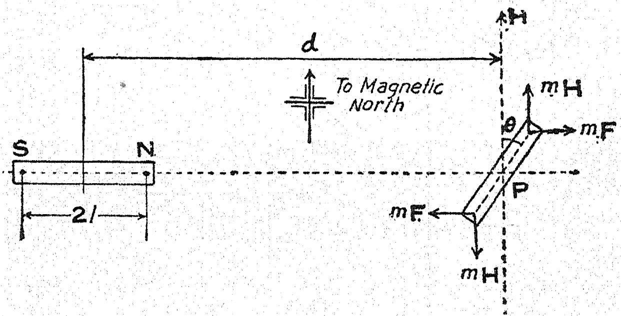


Fig. 24·5

$$F = \frac{2Md}{(d^2-l^2)^2}$$

Equating these two values of F, we have

$$\frac{2Md}{(d^2-l^2)^2} = H \tan\theta$$

or
$$\frac{M}{H} = \frac{(d^2-l^2)^2}{2d}\tan\theta \quad (24{\cdot}6)$$

If $d >> l$ so that l^2 can be neglected when compared to d^2, we have,

$$\frac{M}{H} = \frac{d^3}{2}\tan\theta \quad (24.7)$$

The above arrangement of the magnet and the needle in earth's field is called **A tangent position of Gauss.**

(b) Tangent B Position of Gauss. Let a *small* compass needle be placed at a point P on the *equatorial line* of a magnet NS placed east and west, *i.e.*, with its axis *at right angles to* the magnetic meridian as depicted in fig. 24·6. The field F at the point P is *perpendicular* to the *horizontal component* of the earth's magnetic field H so that

$$F = H \tan \theta$$

where θ is the deflection of the needle from the *magnetic meridian.* Since the magnet is *broadside-on* to the point P, we have from equ. (24·3).

$$F = \frac{M}{(d^2+l^2)^{3/2}}$$

Substituting this value of F in the previous equation, we get

$$\frac{M}{(d^2+l^2)^{3/2}} = H \tan \theta$$

or $$\frac{M}{H} = (d^2+l^2)^{3/2} \tan \theta \quad (24{\cdot}8)$$

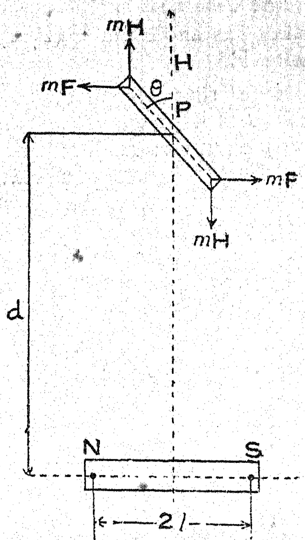


Fig. 24·6

If $d >> l$ so that l^2 is negligible when compared to d^2, we have

$$\frac{M}{H} = d^3 \tan \theta \quad (24{\cdot}9)$$

The above arrangement of the magnet and the needle in the earth's field is called **B tangent position of Gauss.**

Note that the field F due to the magnet is *perpendicular* to H both in A and B tangent positions of Gauss. The compass needle used in both the positions of Gauss should be *small* as the field F due to the magnet is uniform only in a small region round the needle. The magnets used should be strong in order that the effects of friction at the pivot may be insignificant.

If the same magnet be used in A and B tangent positions of Gauss and, if the deflections of the needle from the magnetic meridian in the two cases be θ_1 and θ_2 respectively, then for the same value of d, from equations (24·7) and (24·9) we have, for a *short* magnet.

$$\frac{\tan \theta_1}{\tan \theta_2} = 2 \quad (24{\cdot}10)$$

This formula was used by Gauss to prove *inverse square law* experimentally.

24·5. Deflection Magnetometer. As shown in fig. 24·7, this consists essentially of a *small* magnetic needle pivoted at the centre of a circular scale of degrees, so as to move *freely* in a *horizontal* plane. At *right angles* to the needle is attached a *light* aluminium pointer which is used to read off the deflections of the needle on the circular scale. To avoid *error due to parallax*, a plane mirror is placed un-

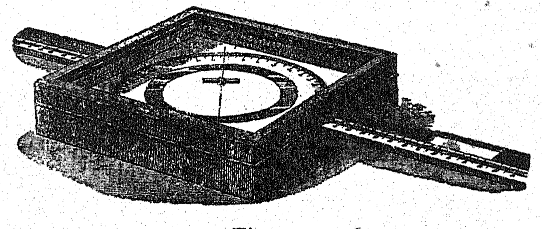


Fig. 24·7

derneath the needle. The whole is enclosed in a box with a glass lid which shields the needle from air draughts. This box is placed at the centre of a long wooden board, the two sides of which on the *opposite* sides of the needle constitute the two arms of the magnetometer. Each arm of the magnetometer is provided with a half metre scale, the zeros of the two scales being coincident with the centre of the needle.

The instrument is used for the determination or comparison of *strengths of magnetic fields* or the *magnetic moments* of bar magnets and the verification of *inverse square law.* In all these experiments either *A tangent* or *B tangent position of Gauss* is used. For A tangent position of Gauss, the arms of the magnetometer are set in the magnetic *east and west* direction and the bar magnet placed on the arms of the magnetometer with its axis *parallel* to them. For B tangent position of Gauss, the arms of the magnetometer are adjusted to be in the magnetic *north and south* direction and the bar magnet placed on the arms of the magnetometer with its axis *perpendicular* to them.

24·6. Vibration of a bar magnet in a magnetic field. When a bar magnet, *freely* suspended in a magnetic field, is deflected through a *small* angle from its position of rest, it begins to execute *simple harmonic* oscillations about the axis of rotation as soon as the deflecting influence is withdrawn. For, if M be the *magnetic moment* of the magnet, H the *intensity* of the magnetic field and θ the *small* deflection of the magnet, the moment of the *restoring reaction* on the magnet due to the magnetic field is MH $\sin\theta$ which, as θ is *small*, reduces to $MH\theta$. If I be the moment of inertia of the magnet about the axis of rotation, *i.e.*, the *vertical* axis through its C. G., the moment of the *inertial reaction* is $I\dfrac{d^2\theta}{dt^2}$. From *Newton's third law of motion*, the sum of these two *reactions*, in the absence of the deflecting influences, must be equal to *zero*. Hence

$$I\frac{d^2\theta}{dt^2} + MH\,\theta = 0$$

or
$$\frac{d^2\theta}{dt^2} + \frac{MH}{I}\,\theta = 0$$

or
$$\frac{d^2\theta}{dt^2} + p^2\,\theta = 0$$

where $p^2 = MH/I$

The above equation represents a *simple harmonic motion* of period

$$T = 2\pi\sqrt{\frac{I}{MH}} \qquad (24{\cdot}11)$$

In the derivation of the above equation, we have neglected the moment of the torsional reaction due to suspension fibre and also the change in the magnetic moment of the magnet due to inductive action of the field. The effect on the motion of the magnet due to these two factors, however, is very small. If torsional reaction is not negligible when compared to $MH\theta$, the formula for period will become

$$T = 2\pi\sqrt{\frac{I}{MH+C}}$$

where C is the moment of *torsional reaction* for *unit* radian twist.

24·7. Vibration magnetometer. It consists of a wooden box having a vertical long glass tube projecting at the middle of the top of the box as shown in fig. 24·8. Inside the box is suspended a very *light* stirrup of aluminium wire for holding the magnet. The suspension is either an *unspun* silk thread or a camel hair and is tied at the top to the *torsion head* resting on the top of the glass tube. On the base of the box below the magnet, is fixed a strip of plane mirror with a straight line marked *parallel* to the *longer* side of the box. There are two slits in the wooden top of the box vertically above the line on the plane mirror and are used to observe the oscillations of the magnet through them. In each of the two long sides of the box there is a glass strip that can be slid in and out.

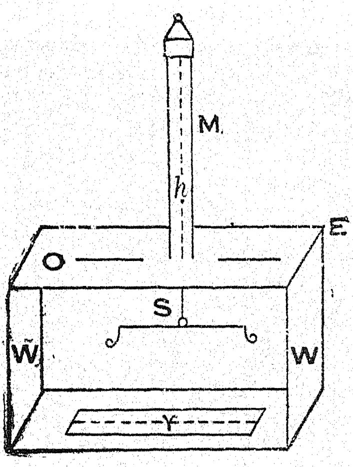


Fig. 24·8

To adjust the instrument it is placed on a table with the line on the plane mirror lying in the *magnetic meridian*. Then a brass bar is placed in the stirrup and allowed to come to rest. When the brass bar is at rest the suspension fibre will be free from twist. The *torsion head*

is then turned until the bar lies *parallel* to the line on the plane mirror. The bar is then replaced with the magnet whose oscillations are to be observed. When the magnet is at rest its axis lies in the magnetic meridian and the suspension fibre will be free from any twist.

The instrument is used for the comparison of *magnetic fields* or *magnetic moments* and the verification of *inverse square law*.

Experiment 24.1

Object. To determine the *magnetic moment* M of a bar magnet and to find the absolute value H of *horizontal component of earth's magnetic field* in the laboratory at......, by using deflection and vibration magnetometers.

Apparatus. A bar magnet whose M is to be determined, a deflection magnetometer, a vibration magnetometer, a brass bar of about the *same* mass and size as the magnet, a compass needle, a stop-watch, a vernier callipers, a metre scale and a balance.

Theory. To determine the absolute values of M and H, two experiments are performed:

(*a*) A *deflection experiment* in which the bar magnet is used to deflect a magnetic needle in A *tangent position of Gauss.* If θ be the deflection of the needle from the *magnetic meridian* when the *centre* of the bar magnet is at a distance d from it, we have from equ. (24·6)

$$\frac{M}{H} = \frac{(d^2-l^2)^2}{2d} \tan\theta \qquad (24{\cdot}12)$$

where M = Magnetic moment of the magnet

l = Half the distance between its poles

and H = Horizontal component of earth's field.

(*b*) An *oscillation experiment* in which the *same* bar magnet is allowed to oscillate in earth's field *alone*. If T be the *period of oscillation* of the magnet, then from equ. (24·11) we have

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

or

$$MH = \frac{4\pi^2 I}{T^2} \qquad (24{\cdot}13)$$

where I = *Moment of inertia* of the magnet about the *vertical* axis through its C.G.

For a *rectangular* magnet

$$I = m\left(\frac{a^2+b^2}{12}\right)$$

where m = mass of the magnet

a = length of the magnet

and b = breadth (*horizontal*) of the magnet.

Multiplying equations (24·12) and (24·13) and then taking the square root, we have

$$M = \frac{\pi(d^2-l^2)}{T}\sqrt{\frac{2\,l}{d}\tan\theta} \quad \cdots \quad (24{\cdot}14)$$

which gives the value of M.

Dividing equ. (24·13) by equ. (24·12) and then taking the square root, we have

$$H = \frac{2\pi}{T(d^2-l^2)}\sqrt{\frac{2\,ld}{\tan\,\theta}} \quad \cdots \quad (24{\cdot}15)$$

which gives the value of H.

Method.—(a) **Deflection experiment.** Place the deflection magnetometer on a *rigid horizontal* table and turn it until its arms are *parallel* to the aluminium pointer. If the compass box is fixed and the instrument is perfect, the aluminium pointer will point o—o on the circular scale. If, however, the compass box is not fixed, rotate it until the aluminium pointer reads o—o on the scale, keeping the arms of the magnetometer parallel to the pointer. *This sets the magnetometer for A tangent position of Gauss.*

Now place the magnet whose magnetic moment is to be determined on one of the arms of the magnetometer and arrange its position on it in such a manner that *its magnetic axis is parallel to the arms of the magnetometer and when produced passes through the centre of the magnetometer needle.* Adjust the distance of the *centre* of the magnet from the *centre* of the needle so as to get a deflection in the neighbourhood of 45° or failing which between 25° and 65° *at the same time* keeping the value of *d* quite *large.* Then after gently tapping the compass box read the positions of *both* ends of the pointer on the circular scale. Note down the distance *d* of the centre of the magnet from the centre of the needle.

Then turn the magnet *end for end* so that its N and S-poles are *interchanged* keeping its distance from the centre of the needle the *same* and again note down the deflections at the two ends of the pointer. Next transfer the magnet to the *second* arm of the magnetometer, and keeping its distance from the centre of the needle the *same*, repeat the above determinations. Take down the mean of the above *eight* values of deflection of the needle which gives the *mean* value of θ. Repeat the above observations for θ twice with the *same* value of *d*, and take the mean.

(b) **Oscillation experiment.** Represent the *magnetic meridian* on a *rigid horizontal* table by a light chalk line, drawn with the help of a compass needle and a metre scale. Place the vibration magnetometer on the table with its *longer* side *parallel* to the magnetic north and south line.

Next place the compass needle inside upon the line marked on the plane mirror fixed to the base of the box and adjust the magneto-

meter, if at all necessary, *to bring the index line on the plane mirror exactly in the magnetic meridian.*

Now place in the stirrup a brass bar of about the *same* mass and size as the magnet. If there is any twist in the suspension fibre the brass bar will turn round until the fibre is without twist. *Take care to check the motionof the bar after every few vibrations otherwise when the fibre is untwisted the inertia of thebar may cause it to twist in the opposite direction.* When the bar at is rest observe the angle between the bar and the index line on the plane mirror beneath it and turn the *torsion head* through *this* angle so that *the brass bar may rest parallel to the index line.* Now holding the stirrup tight in position so that no further twisting of the fibre takes place, remove the brass bar and put in the magnet used in the deflection experiment *with its north pole pointing towards magnetic north.* The magnet should be laid *flat* on the stirrup so as to lie perfectly *horizontal.* When the magnet comes to rest it must be *parallel* with the *index line.* Then close the sliding glass window of the box taking care not to disturb the magnet when the sliding cover is put on.

Now rotate the suspended magnet through a *small* angle about its axis of suspension by bringing *slowly* and *cautiously* towards it another magnet *presented end on with its north pole towards the north pole of the suspended magnet.* Then take away the second magnet *immediately* to a *sufficient* distance from the suspended magnet when the latter will start oscillating about its position of rest. Determine *twice* with an *accurate* stop-watch the time for a *large* number of oscillations of the magnet say, 30 and calculate its *period of oscillation.* Repeat the observations for period of the magnet with *altered* number of oscillations and find out the *mean* value of T.

Next locate the position of the poles of the magnet by tracing a few lines of force in the *neighbourhood* of *each* pole of the magnet. *The two points one near each end of the magnet at which the lines of force seem to start or terminate correspond to the positions of the two poles.* Measure the distance between these two points which gives the *effective* length $2l$ of the magnet. Then weigh the magnet, determine its length and breadth (horizontal) and calculate the moment of inertia I of the magnet. Finally calculate the value of M and H from the equations (24·14) and and (24·15).

Sources of error and precautions. (1) The magnetometers should be placed on a *rigid* table preferably of stone. The bases of the magnetometers must be in perfect *horizontal* level and once the instruments have been adjusted, they should not be disturbed throughout the whole experiment.

(2) All pieces of magnetic materials and current-bearing conductors should be removed to a considerable distance from the magnetometers.

(3) The dimensions of the magnet should not be measured between the deflection and oscillation experiments as contact with steel measuring instruments, *e.g.*, vernier callipers, may affect the magnetic moment of the magnet.

For deflection experiment only

(4) The magnet should be so placed on the arms of the magnetometer that *its magnetic axis when produced passes through the centre of the magnetometer needle*, otherwise the eight values of deflection of the needle obtained as explained under method, will differ *appreciably* from the *mean* value of deflection.

(5) When the deflection of the needle from the magnetic meridian is 45°, the readings of θ will be *least* liable to error, for if $d\theta$ be the *small* change in the value of θ corresponding to a *small* change dF in the value of the field F at the centre of the needle due to the bar magnet, then since

$$F = H \tan \theta$$

we have

$$dF = H \sec^2 \theta . \; d\theta$$

and

$$\frac{dF}{F} = \frac{\sec^2 \theta}{\tan \theta} . d\theta = \frac{2}{\sin 2\theta} . \; d\theta$$

For greatest accuracy of reading dF/F must be the *least*. dF/F is *minimum* when $\sin 2\theta = 1$ or $2\theta = 90°$, *i.e.*, $\theta = 45°$. When $\theta = 15°$ or 75°, the relative accuracy of reading is *half* of that at $\theta = 45°$, and with $\theta < 15$ or $\theta > 75$ it is even less than this.

The *effective* length $2l$ of the magnet is a quantity which cannot be determined accurately. Hence in order that the percentage error in the value of the term $(d^2 - l^2)$ may be small d should be kept *fairly large* compared with l. Further the larger the value of d, the greater is the *uniformity* of the field at the needle. Hence *for greater accuracy of result the deflection should be in the neighbourhood of* 45°, *and at the same time* d *should be quite large.*

(6) While reading the position of the pointer on the circular scale the eye should be placed in such a position that the pointer covers its image in the plane mirror otherwise a considerable *error due to parallax* may be made in reading the position of the pointer.

(7) As the pivot on which the needle rotates may not be exactly at the centre of the circular scale, *both* ends of the pointer should be read thus avoiding *error due to eccentricity* of the pivot with respect to the circular scale.

(8) If the deflecting magnet is *not* magnetised *uniformly*, its *magnetic centre* will not coincide with its *geometric centre*, and hence d will not represent the correct distance between the centre of the needle. Observations for deflection of the needle should, therefore, be taken first with one pole of the magnet pointing towards the needle and then, by *reversing* the magnet *end for end*, with the second pole pointing towards the needle.

(9) As the pivot of the needle may not be *exactly coincident* with the zeros of the linear scales fixed along the arms of the magnetometer, the observations for deflection of the needle should be taken with the magnet placed in a *similar* position on *both* the rms of the magnetometer.

For oscillation experiment only

(10) The stirrup for holding the magnet should be *very light* so that its moment of inertia about the axis of rotation may be neglected when compared to that of the magnet. It may be made of thin aluminium wire or the stirrup may be dispensed with altogether and the magnet suspended in a double loop made at the end of the suspension fibre.

(11) In the derivation of the formula $T=2\pi\sqrt{\frac{I}{MH}}$, the *torsional reaction* due to suspension was supposed to be negligibly *small.* The stirrup should, therefore, be suspended by a silk fibre. But, if the fibre is twisted initially, the torsional reaction may be quite appreciable even with silk fibre. Hence the suspension must be let untwist itself slowly under the weight of a *non-magnetic* bar of the *same* mass and size as the magnet and the *torsion head* should be rotated until the bar rests *parallel* to the magnetic meridian. If horse hair is used for suspension, there is no *initial* twist.

(12) The oscillations of the suspended magnet should be started by means of another magnet presented *end on* with its north pole towards the north pole of the suspended magnet so that *the motion of the magnet is rotational in a horizontal plane.*

(13) The amplitude of oscillation of the magnet should be *very small,* say about 5° to satisfy the condition of theory that $\sin\theta=\theta$.

(14) The oscillations of the magnet should be observed with the eye placed *vertically* above the rest position of the magnet and should be timed with reference to the *index line* on the plane mirror with an *accurate* stop-watch correct up to 1/5 of a second.

Observations. [*A*] *Determination of* θ

S. No.	Distance of the *centre* of magnet from the needle	Deflection of the pointer with the magnet on the								mean θ	tan θ
		East arm				West arm					
		N. pole towards needle		S. pole towards needle		N. pole towards needle		S. pole towards needle			
	cm.	Rt. end	Lft. end	Rt. end	Lft. end	Rt. end	Lft. end	Rt. end	Lft. end		
1.											
2.											
3.											

[B[*Determination of T.*

Least count of stop-watch = sec.

S. No.	No. of oscillations	Time taken min.	Time taken sec.	Period T sec.
1	30			
2	"			
3	25			
4	"			
		Mean		

[C] *Distance between the poles of the magnet* $= 2\,l =$ cm.

[D] *Constants of the magnet.*

(*a*) Mass of the magnet = gm.

(*b*) Length of the magnet = cm.

(*c*) Breadth of the magnet

vernier constant = cm.

Zero error = cm.

S. No.	Breadth of the magnet cm.
Mean	

Calculations

$$I = m\left(\frac{a^2+b^2}{12}\right)$$

$$=$$

$$= \quad \text{gm.} \times \text{cm.}^2$$

$$M = \frac{\pi(d^2 - l^2)}{T} \sqrt{\frac{2\ I}{d} \tan\theta}$$

$$=$$

$$=$$

and $$H = \frac{2\pi}{T(d^2 - l^2)} \sqrt{\frac{2\ I\ d}{\tan\theta}}$$

$$=$$

$$= \quad \text{oersteds}$$

Result. (a) The value of *magnetic moment* of the given magnet = webers × cms.

(b) The value of the *horizontal component* of earth's magnetic field in the laboratory at......... = oersteds.

Standard value of H at .... = oersteds.

∴ Percentage error =

Criticism of the method. The method gives a fairly satisfactory result. For greater accuracy the value of T should be corrected for :

(i) *finite* amplitude of oscillation of the magnet,

(ii) the moment of inertia of the stirrup in the vibration magnetometer,

(iii) the *torsional reaction* of the suspension fibre and

(iv) the change in the magnetic moment of the magnet when oscillating due to *inductive action* of the field.

The principal defects of the method are the difficulty in determining accurately the distance $2l$ between the poles of the magnet and the impracticability of getting suitable deflections in all cases. The pointer-over-scale system of measuring deflections has a very limited accuracy, but this can be increased considerably by fixing a plane mirror vertically at the centre of the needle and measuring deflection by lamd and scale or telescope and scale arrangements. Further as the magnetometer needle is not sufficiently short, the field at the needle is not absolutely uniform ; and as the needle is pivoted and saddled with a long pointer, the frictional resistance to its movement is also not negligible.

To get better results for the value of H more accurate instrument, *e.g.*, Kew magnetometer or those methods which depend upon the magnetic effects of current and which are far more accurate should be used.

24·8. Earth's Field and its Magnetic Elements. The directional properties of a *freely* suspended bar magnet show the presence of a magnetic field in the *whole* region on and around the earth's surface. To account for the shape of this earth's field, *a powerful small hypothetical magnet may be supposed to be situated at the centre of the earth with its south-seeking pole pointing approximately towards geographical north.* The magnetic axis of the

hypothetical magnet does not coincide with the earth's geographical axis. The lines of force in earth's field do not lie parallel to the earth's surface but are inclined to it, and hence a compass needle suspended so as to be able to take up any direction sets itself in the direction of the earth's magnetic force and is found to be inclined to the horizontal. A magnetic needle capable of totation in a horizontal plane takes up a direction corresponding to rhe projection of the lines of force upon this plane for then it is acted upon only by the hcrizontal component of the earth's field.

The intensity and direction of earth's field vary from place to place but within regions of not too great extent, the earth's magnetic field can be regarded as *uniform*, for the lines of force at a place are *parallel* straight lines and even the most sensitive methods fail to detect any force tending to produce translation in a freely suspended magnet.

To determine earth's magnetic field at a place completely, the following three quantities, known as the *magnetic elements* at that place, have to be determined :—

(*a*) *Declination* or variation,

(*b*) *Dip* or inclination and

(*c*) *Horizontal component* H of the earth's total magnetic intensity.

Declination *or variation at any place is the angle between the magnetic meridian and the geographical meridian at that place.* In the Fig. 24·9, PGKO represent a part of the *geographical meridian* at a place where QP is the geographical north and south line and PMNO represents a part of the *magnetic meridian* where RP is the magnetic north and south line. ∠GPM is the *declination* or variation.

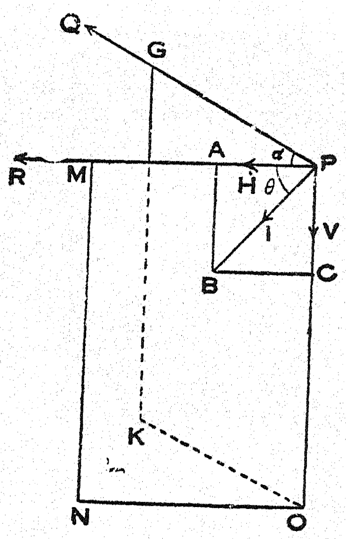


Fig. 24·9

Dip *or inclination at any place is the angle which the direction of earth's total magnetic intensity makes with a horizontal line in the magnetic meridian at that place.* In the fig. 24·9, PB represents the direction of *total* magnetic force of the earth's field. ∠MPB is the *dip* or inclination. The dip is measured with a **dip needle,** *i.e., a magnetic needle capable of free rotation in a vertical plane about a horizontal axis.* The angle which the magnetic axis of the dip needle makes with a *horizontal line* in the *magnetic meridian* gives the value of dip.

If H and V be the *horizontal* and *vertical* components respectively of earth's *total* magnetic force I, and ψ the *angle of dip*.

$$H = I\cos\psi \quad \text{and} \quad V = I\sin\psi$$

$$\text{or} \quad \tan\psi = V/H \quad \text{and} \quad I = \sqrt{H^2+V^2}$$

The values of ψ and H are usually determined experimentally and those of I and V calculated from them with the help of above equations.

The earth's magnetic elements vary from place to place and also from time to time at the same place. The cyclic variations called *secular changes* have a period of about 1000 years. The daily and yearly changes are very small. The daily variations are very irregular but are distinct from the effects due to magnetic storms which cause large and irregular fluctuations and which are usually unpredictable.

Oral questions

Define the terms: pole-strength and magnetic moment of a magnet, equivalent or effective length of a magnet and intensity or strength of a magnetic field at a point. What are C. G. S. units of pole strength, magnetic moment and intensity of magnetic field ? What is oersted ? How do you determine the direction of a magnetic field ? What are lines of force and what are their properties ? What are unit tubes of force and how do you define field strength in terms of them ? What is a uniform field ? Give an example of it. What is intensity of magnetisation and how is it related to pole-strength or magnetic moment of a bar magnet ? What do you understand by "end on" and "broadside on" position of a bar magnet ? What are the expressions for field at a point in the two cases and what is the relation between them, if any, for a very short magnet ? What are tangent A and B position of Gauss ? How do you use these two positions of Gauss to compare the magnetic moments of two magnets or to compare two magnetic fields ? Is it the null or the deflection method in A or B tan-position of Gauss which gives the maximum accuracy ? What are other methods of comparison of magnetic moments of bar magnets ? What are relative merits and demerits of deflection and oscillation methods for the comparison of magnetic moments ?

DETERMINATION OF M AND H

What do you understand by H and what is its value at your place ? How do you measure H with the help of vibration and deflection magnetometer ? Describe the deflection magnetometer. Why is the needle enclosed in a box ? Which is the needle, the shorter one or the longer one ? Why is the needle made very small ? Why must there be two, a needle and a pointer ? Could not the circular scale be made short or the needle made sufficiently long so that the latter may serve for the pointer as well ? What are the requirements of an ideal pointer ? Can it be made of iron in this case ? Why is the pointer attached at right angles to the needle and not parallel to it ? How can you measure very small deflections ?

How do you adjust the deflection magnetometer for A tangent position of Gauss ? Why do you perform the deflection experiment in A tangent position and not in B tangent position of Gauss ? How do you place the magnet on the arms of the magnetometer and why ? Why is the needle deflected when the magnet is placed on the arms of the magnetometer ? Why is there no deflection when comparing magnetic moments of bar magnets by null method ? What is the "tangent law" of two magnetic fields ? What are the two essential conditions for its application ? How do you produce a uniform field at the centre of the needle e Why do you adjust the deflection in the neighbourhood of 45° ? If you fail to get

deflection in the neighbourhood of 45° for fairly large values of d, what should be the limits between which the deflection must lie? Why do you read both ends of the pointer? Explain with a diagram how the error due to eccentricity of the pointer with respect to the circular scale is eliminated by reading both ends of the pointer? Why is the plane mirror fixed below the needle? Why do you reverse the magnet end for end? Why do you place the magnet on both the arms of the magnetometer?

Describe the vibration magnetometer. Why is the stirrup for holding the magnet made very light? Can you not dispense with it altogether? Why is the suspension made of silk fibre or horse hair and not of cotton thread? Why do you prefer horse hair to silk fibre? On what factors does the period of a magnet depend when oscillating in a uniform magnetic field? Is it essential that the field should be uniform? To keep the field uniform a short needle is used in the deflection magnetometer while in vibration experiment the oscillating magnet is quite big: why do you not object to this? If you are given two magnets of the same size and mass, how can you determine which has got greater pole-strength? Which is stronger: a magnet oscillating quickly or the one swinging slowly in a uniform magnetic field? Why do you level the vibration magnetometer? Why do you place it on a rigid table? How do you remove the twist from the suspension fibre? What is the harm if this twist is not removed? Can you use an iron bar in place of brass bar? Is it necessary that its mass and size should be nearly the same as that of the magnet? When the twist has been removed why do you turn the torsion head to set the bar parallel to the magnetic meridian? Why is a plane mirror fixed at the base of the oscillation box and what is the use of the line marked on it? While replacing the bar by the magnet, is there any harm if the stirrup is not held tight? Why should the magnet be placed perfectly horizontal? How do you start the magnet oscillating? Why should the amplitude of oscillation be small? Why do you place your eye vertically above the rest position of the magnet when timing the oscillations of the magnet? What are the defects of this method of measuring H? Which is the most accurate method of determining H and why?

What are Earth's magnetic elements at a place? Are their values constant at every place and remain constant with time? What is dip and how is it measured? Do the north and south poles of the Earth exist individually or connected together like those of a magnet? How can the latter be possible at all when the Earth is known to be not entirely made of iron.

CHAPTER XXV

MAGNETIC EFFECTS OF CURRENTS

25·1. Magnetic field due to current in a conductor. When an electric current is started in a conductor, immediately a magnetic field is established in the space around the conductor. The conductor itself does not become a magnet for it does not possess any magnetic poles neither can it attract iron filings. The nature or form of the field depends upon the shape of the conductor and can be explored with iron filings or a compass needle in the usual manner. The intensity F of the *magnetic field* at a point P due to a *short* element of the conductor as shown in fig 25·1, is (*i*) directly proportional to the apparent length of the element as seen from the point P, (*ii*) directly proportional to the strength *i* of the current and (*iii*) inversely proportional to the square of the distance *r* of the element from the point P. Thus

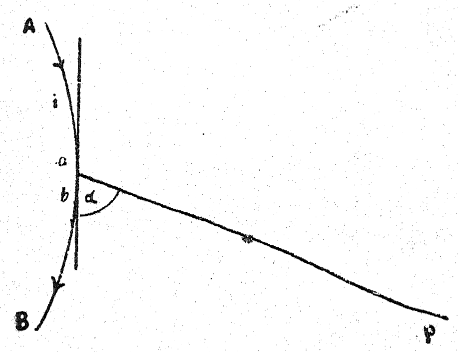


Fig. 25·1

$$F = k\frac{i\ ab \sin \alpha}{r^2}$$

The unit of current is so chosen that $k=1$. Hence

$$F = i\ ab \sin \alpha / r^2$$

The above rule for calculating the intensity of the magnetic field at a point due to a small element of a conductor carrying current is known as *Laplace's rule.* The rule is applicable to each of the small elements of a current circuit and hence can be extended to the whole circuit. The direction of the field is dependent upon the direction of the current and can be obtained from any of the following rules :—

(*a*) Look along the conductor in the direction of the current, the magnetic lines of force will go round in the direction in which the hands of a clock move, *i.e.*, the north pole of the magnetic needle will be deflected in the clockwise direction.

(*b*) *Ampere's Rule.* Imagine a man swimming in the direction of the current with his face towards the needle, then the north pole of the needle will be deflected towards his left hand.

(*c*) *Maxwell's Corkscrew Rule.* Imagine an ordinary right-handed screw to be twisted along the wire so as to move in the direction of the current, then the direction in which the thumb rotates is the direction in which north pole tends to move round the wire.

25·2. Magnetic Field due to Linear Current. Referring to fig. 25·2, let AB be a straight wire carrying i e. m. units of electric current and P be a point at a *perpendicular* distance DP$=d$ from the axis of the wire. Let ab be a *small* element of the wire at a distance aP$=r$ from P. Let the $\angle$DP$a=\theta$, and the *small* angular increment aP$b=d\theta$. The magnetic field dF at P due to the *small* element ab is by *Laplace's law* given by

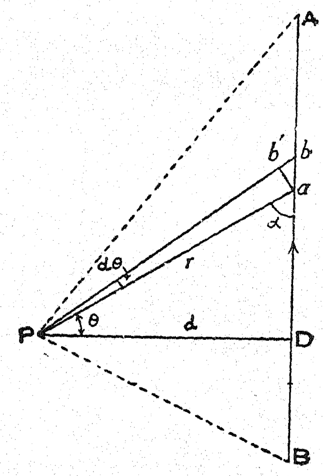


Fig. 25·2

$$dF = i\ ab\ \sin\alpha/r^2 = \frac{i\ ab'}{r^2}$$

But $\quad ab' = rd\theta$ and $r = d/\cos\theta$

$$\therefore \qquad dF = \frac{i\cos\theta}{d}\,d\theta$$

If the angles APD and BPD be θ_1 and θ_2 respectively, then the total field at P due to the whole conductor AB is given by

$$F = i\int_{\theta_1}^{\theta_2}\frac{\cos\theta}{d}\,d\theta$$

$$= \frac{i}{d}(\sin\theta_1 + \sin\theta_2)$$

For a conductor of *infinite* length $\theta_1=\theta_2=\pi/2$ and hence

$$F = \frac{2i}{d} \text{ oersteds}$$

If the current i be in *amperes*

$$F = \frac{2i}{10d} \text{ oersteds} \qquad \ldots \qquad (25\cdot1)$$

The magnetic field at a point due to a *linear current*, therefore, varies *inversely* as the perpendicular distance of the point from the conductor and is the same at all points at the same distance from it. The direction of field at P is *at right angles* to the plane through P and the axis of the wire and hence the field has no component in that plane. The magnetic lines of force are, therefore, circles in planes perpendicular to the axis of the conductor with their centres on its axis. In practice, however, the field in the neighbourhood of the conductor will be the resultant of the field due to the straight conductor and that due to the earth.

Experiment 25.1

Object. (i) To plot the resultant magnetic field of a vertical *straight* current and the earth in a horizontal plane,

(ii) to show that the field due to the straight current varies *inversely* as the distance, and

(*iii*) to calculate the value of the horizontal component of earth's magnetic field from the position of the neutral point.

Apparatus. A large rectangular framework covered with several turns of copper wire and mounted vertically as shown in fig. 25·3, with one of its vertical sides passing through a drawing board centrally, a bulb resistance of about 140 ohms, a suitable

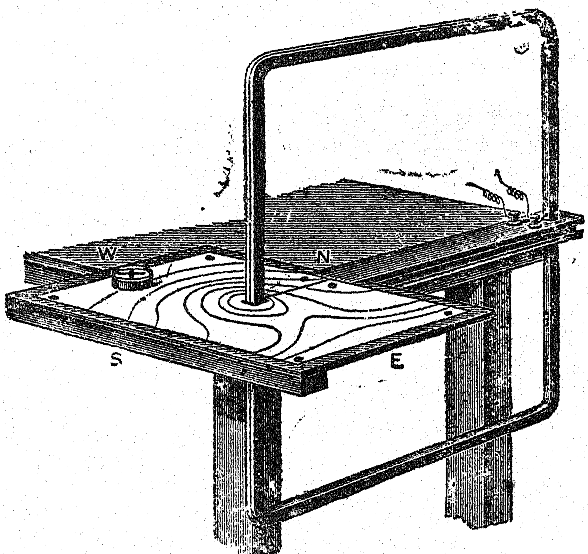


Fig. 25·3

rheostat, an ammeter, a compass needle, a Searle's oscillating needle, a stop-watch, a sheet of drawing paper and wax.

Theory. Referring to fig. 25·4, let AB be the *magnetic east-west* line passing through the axis of the straight conductor which cuts the *lines of force* in the *resultant* field of the straight current and the earth at points x_1, x_2, x_3, etc., on the *opposite* side of the *neutral point*, If prent tangent is drwn at any of these points, say the the corresponding any line force, the tangent will lie in the *magnetic north-south* direction. This is

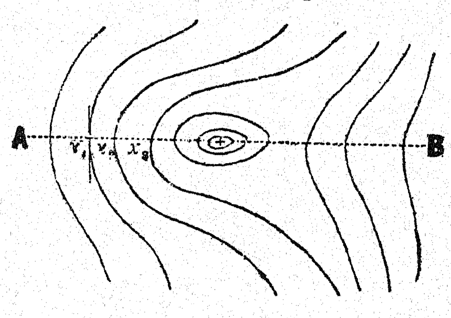


Fig. 25·4

because the lines of force in the field of the straight current *alone* are concentric circles. Evidently the resultant magnetic field at x_1 is

(F+H), where F is the field due to the straight current and H the horizontal component of the earth's field.

Now, if a Searle's needle be allowed to oscillate at x_1, its *period* of oscillation is given by

$$T = 2\pi\sqrt{\frac{I}{M(F+H)}}$$

This gives

$$F+H = \frac{4\pi^2 I}{MT^2} = \frac{C}{T^2} \qquad (25{\cdot}2)$$

where $C=4\pi^2 I/M$ is a constant.

If the *same* needle were allowed to oscillate in earth's field *alone* its period would be

$$T_o = 2\pi\sqrt{\frac{I}{MH}}$$

whence $$H = \frac{4\pi^2 I}{MT_o^2} = \frac{C}{T_o^2} \qquad 25{\cdot}3)$$

Subtracting equ. (25·3) from equ. (25·2), we have

$$F = C\left(\frac{1}{T^2} - \frac{1}{T_o^2}\right)$$

If the field due to the straight current varies *inversely* as the distance

$$F = \frac{k}{r}$$

where r is the distance of the point x_1 from the conductor and k is a constant of proportionality, hence we have

$$\frac{k}{r} = C\left(\frac{1}{T^2} - \frac{1}{T_o^2}\right)$$

or $$r\left(\frac{1}{T^2} - \frac{1}{T_o^2}\right) = \frac{k}{C} = K, \text{ another constant} \qquad (25{\cdot}4)$$

Equ. (25·4) can also be written as

$$\frac{1}{T^2} = \frac{K}{r} + \frac{1}{T_o^2}$$

If a graph is plotted taking $1/r$ as abscissae and $1/T^2$ as ordinates, it should come out to be a *straight line* with an intercept on the Y-axis equal to $1/To^2$.

At the *neutral point* the field due to the straight current is $2ni/10d$, where d is the distance of the neutral point from the conductor, i the current in *amperes* flowing through it and n is the number of turns of wire in the conductor. Since at the neutral point *resultant* field intensity in the horizontal plane is *zero*, the field due to the straight current is exactly *equal* and *opposite* to the horizontal component H of the earth's field. Hence

$$H = \frac{2ni}{10d} \text{ oersteds} \quad \ldots \qquad (25{\cdot}5)$$

Method. Fix with wax a sheet of drawing paper on the drawing board in such a manner that the vertical side of the rectangular framework carrying the straight conductor passes through the paper *centrally*. With the help of the compass needle draw the magnetic north and south line, on the drawing paper by a fine arrow. Connect the two open ends of the straight conductor to the mains through a bulb resistance, an ammeter and a suitable rheostat all *in series*, the value of the bulb resistance being so chosen as to produce a current of strength between one and two amperes.

After the electric connections have been completed and thoroughly checked, start the current in the conductor when a magnetic field due to the current in the conductor will be produced in the space around it. With the help of the compass needle, trace on the drawing paper in the usual manner, the magnetic lines of force in the combined field of the straight current and the earth and locate the *neutral point*. The neutral point will lie due magnetic east or west of the conductor according as the current is flowing downwards or upwards through the conductor. The neutral point will be found to be enclosed by four sets of lines of force forming a curvi-linear quadrilateral. Diminish the area of this quadrilateral by tracing lines of force within it. When the area of the quadrilateral has been considerably reduced, its center will give the position of the neutral point. Around the neutral point mark a circle of needle's size and measure the distance d of the neutral point from the straight conductor. Then note down the reading of the ammeter and calculate the value of H from equation (25·5).

Now with the help of the compass needle draw the magnetic east-west line through the axis of the conductor, and on the *opposite* side of the neutral point mark on this line a number of poins x_1, x_2, x_3, etc., at known distances r_1, r_2, r_3, etc., from the coductor. Allow a *Searle's needle* to oscillate at any one of these pots, say x_1 in the *resultant* field (F+H) and note down the time taken byitto complete 20 oscillations and thus find out the period of oscillation of the needlc at x_1. Then allow the needle to oscillate at other points and determing as before the period of oscillation T at the various points. Now switch off the current and determine the period of oscillation T_0 of the needle at any point in the earth's field *alone*. For each point x_1, x_2, etc., calculate the value of the expression $r\left(\frac{1}{T^2}-\frac{1}{T_0^{\,2}}\right)$. This will be the *same* for *each* point. Plot a graph taking $1/r$ along the X-axis and $1/T^2$ along the Y-axis. This will come out to be a straight line with an intercept on the Y-axis equal to $1/T_0^2$.

Sources of error and precautions. (1) All magnetic materials, current-bearing conductors and the mains should be at a considerable distance from the straight conductor otherwise the field produced in the space around the conductor will be more complex in character than that which is intended to be produced.

Connections of the straight conductor with the rest of the circut should be made by twin *flexible wires* twisted together so that the passage of current through them may not affect the field of the straight conductor.

(2) The magnetic meridian should be represented on the drawing paper by a fine arrow and once this magnetic north and south line has been drawn, the paper should never be disturbed throughout the experiment.

(3) Unless it is a hot-wire instrument, the ammeter should be kept at a *sufficient* distance from the conductor as most ammeters contain a strong permanent magnet.

(4) The current flowing through the straight conductor should be of one to two amperes so that a *strong* magnetic field is produced in the space around the conductor and hence the *neutral point* may be at a considerable *distance* from it. Further, the magnetic field produced should be *steady* and hence the reading of the ammeter should remain constant during the experiment, the rheostat being adjusted from time to time, if necessary, to keep the current constant.

(5) After tracing a magnetic line of force, its direction should be clearly marked by an arrow head.

(6) For determination of T *Searle's needle* should be allowed to oscillate only at points lying on the magnetic *east-west* line passing through the conductor and on *opposite* side of the *neutral point.*

(7) The amplitude of oscillation of the needle should be small, say 3 or 4.

(8) The graph between $1/r$ and $1/T^2$ should be a *straight line* and should be *smoothly* drawn.

Observations. [*A*] *Determination of H*

(*i*) Distance of the *neutral point* from the conductor $d =$ cm.
(*ii*) Current through the circuit $i =$ amp.
(*iii*) Number of turns in the conductor $n =$

[*B*] *Determination of* T

Least count of stop-watch = sec.

Set No.	Distance r of point x from conductor cm.	No. of oscillations	Time taken min.	Time taken sec.	Period T sec.	*Mean* period T sec.
A						

(C) Determination of T_0

S. No.	No. of oscillations	Time taken		Period T_o
		min.	sec.	
1.				
2.				
3.				
			Mean	

Calculations

Point on graph	$\frac{1}{r}$	$\frac{1}{T^2}$	$r\left[\frac{1}{T^2}-\frac{1}{T_o^2}\right]$

For point x_1, $r =$ cm., T= sec.

$$r\left[\frac{1}{T^2}-\frac{1}{T_o^2}\right]= \qquad =$$

(*N.B.*—Similar calculations may be done for other points)

$$H = \frac{2ni}{10d}$$
$$=$$
$$= \quad \text{oersteds}$$

Result. (*i*) The graph between $1/r$ and $1/T^2$ is a *straight line* showing that the field due to a straight current varies *inversely* as the distance.

(*ii*) The value of horizontal component of earth's magnetic field H in the laboratory at...... = oersteds

Standard value of H at...... = oersteds

∴ Percentage error =

Criticism of the method. It is not an accurate method of determining the value of the horizontal component of earth's field for the resultant field due to the linear conductor and the earth is liable to be affected by the presence of neighbouring apparatus carrying current. The magnetic field in the neighbour-

hood of the neutral point is very *weak* and the behaviour of the needle in this region becomes almost *uncertain*. The position of the neutral point cannot, therefore, be *correctly* located and hence the accuracy of result cannot be expected to be great.

25·3. Magnetic field at centre of a circular current. Let a conductor AB be looped in a circle of a radius r as shown in fig. 25·5 and let it carry a current of *i.e.* m. units. The field at the *centre* C of the circle due to a *small* element ab is given by

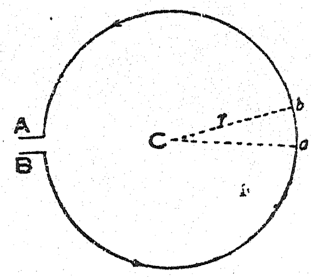


Fig. 25·5

$$dF = iab/r^2$$

and the *total* field at C due to the *whole* conductor is given by

$$F = \Sigma \frac{iab}{r^2}$$

Now since each *small* element of the conductor is *similarly* situated relatively to C, the distance r of the element from C is the *same* for all elements and hence

$$F = \frac{i}{r^2} \Sigma ab$$

But $\Sigma ab = 2\pi r$

$\therefore$ $F = 2\pi i/r$ oersteds ... (25·6)

If the conductor is a circular coil consisting of n turns and the *mean* radius of the coil is r, the field at the centre of the coil is given by

$$F = 2\pi n i/r \text{ oersteds} \qquad (25·7)$$

If the current flowing through the conductor is i *amperes*

$$F = \frac{2\pi n i}{10\, r} \text{ oersteds}$$

The direction of the field at C is *at right angles* to the plane of the coil and may be determined by any one of the rules given in § 25·1. But the following rule is very convenient:—*Looking at the face of the coil, if the current is clockwise, the direction of the field inside the coil is away from the observer; if the current is counter-clockwise the direction of the field inside the coil is towards the observer.* Within a small region at the centre of the coil, the magnetic lines of force are approximately parallel equidistant straight lines, *i.e.*, the field is practically *uniform*.

25·4. Units of current strength and quantity of electricity. Equation (25·6) gives us a definition of unit current which is called absolute or C. G. S. electromagnetic of unit current

strength. The *absolute e. m. unit* of current is that current which when flowing in a wire 1 cm. long bent into an arc of 1 cm. radius produces a magnetic field of intensity 1 oersted at the centre of the circle or exerts a force of 1 dyne on a unit pole placed at the centre. This absolute e. m. unit of current is rather large and for practical work another unit called *ampere* (after Ampere) which is 1/10 of the absolute unit is used. Thus *ampere* is the *practical* unit of current and *is that current which when flowing through a wire 1 cm. long bent into an arc of a circle of one cm. radius exerts a force of* 0·1 *dyne on a unit pole placed at the centre.* The ampere thus defined is called "true ampere."

One true ampere$=1/10$ e.m. unit$=3\times10^9$ e. s. units.

Since quantity of electricity conveyed$=$current strength$\times$time in seconds, the C. G. S. electromagnetic unit of quantity of electricity is the quantity conveyed by the electromagnetic unit current in one second. The *practical* unit of quantity is the *coulomb* (after Coulomb) and is 1/10 of the absolute or C. G. S. e. m. unit of quantity.

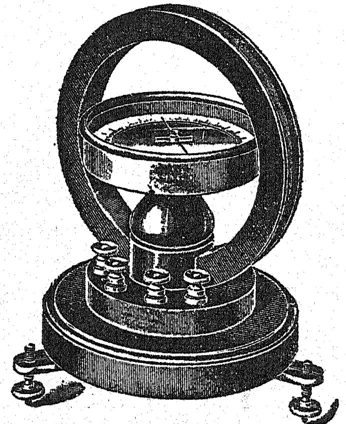


Fig. 25·6

Thus (true) **coulomb** *is the quantity of electricity conveyed by a current of one (true) ampere in one second.* One true coulomb$=1/10$ e. m. unit $=3\times10^9$ e. s. units.

Another practical unit of quantity is termed the **ampere-hour** and is *the quantity of electricity conveyed by a steady current of one ampere flowing for one hour.*

One ampere-hour$=60\times60=$ 3600 coulombs.

25·5. Tangent Galvanometer. As depicted in fig. 25·6, the tangent galvanometer consists essentially of a magnetic needle pivoted or suspended at the centre of a circular coil consisting of many turns of insulated copper wire, the plane of the coil being *vertical.* The magnetic needle is *small* and the radius of the coil *large* compared with the needle so that the field due to the current in the coil in entire space in which the needle moves may be *uniform* and equal to the field at the centre of the coil. At right angles to the needle is attached a light aluminium pointer which reads off the deflection of the needle on the horizontal circular scale graduated in degrees. The needle, the pointer and the scale are enclosed in a metal box with glass cover to shield them from air draughts. Below the needle at the base of the box is fixed a plane mirror with the help of which *errors* due *to parallax* in reading deflections are avoided. To level the instrument the base is provided with three levelling screws.

The Pye tangent galvanometer usually employed in labora-

tories has 500 turns of copper wire wound on the circular frame which terminate in four binding screws. Between the first and second terminals there are only two turns of wire which have a resistance of about 0·02 ohms. This is known as *ammeter coil* and is used with *strong* currents. Between the first and the third terminals there are 50 turns of wire which have a resistance of about 1·1 ohms and are used with *moderate* currents. Between the first and the fourth terminals there are 500 turns of wire having a resistance of about 226 ohms. This is called *voltmeter coil* and is used with *weak* currents.

To use the instrument the plane of the coil is set in the *magnetic meridian* when the needle and the coil will lie in the *same* vertical plane. Then the current to be measured is passed through the coil and the deflection θ produced in the needle read off on the scale. Since the field F at the centre of the coil is perpendicular to earth's horizontal field H, we have from equ. (24·5)

$$F = H \tan \theta$$

If the current i be in e.m. units, the value of F from § 25·3 is also given by

$$F = 2\pi ni/r \tag{25·7}$$

Equating the two above expressions for F, we get

$$2\pi ni/r = H \tan \theta$$

whence

$$i = \frac{rH}{2\pi n} \tan \theta \tag{25·8}$$

or

$$i = \frac{H}{G} \tan \theta \tag{25·8a}$$

where $G = 2\pi n/r$ is called the *galvanometer constant* or coil constant. It depends only on the radius and the number of turns in the coil and hence is a *constant* for a galvanometer.

Putting $2\pi n/r = G$ in equation (25·7) above, we get

$$F = G\,i$$

If $i = 1$, G = F, *i.e.*, the **galvanometer constant** *is numerically equal to the strength of the magnet field at the centre of the coil due to unit current in the coil.*

Now putting H/G = K in equ. (25·8a) above, we get

$$i = K \tan \theta \tag{25·9}$$

where K denotes the *reduction factor* of the tangent galvanometer. *This is a constant which when multiplied by the tangent of the angle of deflection gives the current in the galvanometer.* Since its value is $rH/2\pi n$, it depends upon the radius of the coil and the number of turns in it and also on the value of horizontal component of the earth magnetic field. Hence it is a constant for a galvanometer at a

particular place but varies for the same galvanometer from place to place.

When $\theta=45°$, tan $\theta=1$, and hence from equ. (25·9)

$$K=i$$

i. e., the **reduction factor** *is numerically equal to the current required to produce a deflection of* 45° *in the galvanometer.*

Experiment 25.2

Object. To determine the *reduction factor* of the two-turns coil of a tangent galvanometer using a copper voltameter and then to calculate the value of H, the *horizontal component of the earth's magnetic field.*

Apparatus. The tangent galvanometer, a battery, a rheostat, a commutator, a key, a copper voltameter, a test plate of copper, a weight box, a chemical balance and a spirit level.

Theory. Let the plane of the coil of the tangent galvanometer be placed in the *magnetic meridian* and let a current of strength *i amperes* be allowed to pass through the coil. If the needle is deflected through an angle θ from the *magnetic meridian*

$$i = K \tan\theta \qquad (25·9)$$

where K is the *reduction factor* of the tangent galvanometer. If the *same* current passes through a copper voltameter placed *in series* with the tangent galvanometer and if the mass of copper *deposited* on the cathode plate in t seconds be m gms., then from equ. (27·1)

$$m = Z\,i\,t \qquad (25·10)$$

where Z gms. per coulomb is the *electro-chemical equivalent* of (cupric) copper.

Eliminating i between (25·9) and (25·10), we get

$$K = \frac{m}{Zt.\tan\theta} \qquad (25·11)$$

This equation gives the value of K, the value of Z being already known from table of constants.

Now, if n be the number of turns in the galvanometer coil and r be its *mean* radius

$$K=\frac{10\,r\,H}{2\,\pi\,n}$$

whence

$$H=\frac{2\,\pi\,nK}{10r} \text{ oersteds} \qquad (25·12)$$

Method. Clean the copper *cathode plate* of the copper voltameter *thoroughly* with sand paper and a rug or emery cloth until it is quite bright and weigh it in an accurate *chemical balance* correct up to 1/10 of a milligramme. Then level the compass box

$R = m/2t$.

and also the base of the tangent galvanometer by means of levelling screws testing the levelling with a spirit level. Now rotate the coil if it is movable, or the instrument as a whole if the coil is fixed, about a verical axis till *the coil, the needle and its image in the plane mirror, all lie in the same vertical plane.* In the sposition of the coil, if the instrument is perfect, the aluminium pointer in the compass box, if the latter is fixed, will point o—o. If the box is, however, not fixed, rotate it *without disturbing the position of the coil*, till at least one of the ends of the pointer stands at zero. Now using the *test plate* of copper as the cathode in the copper voltameter connect the tangent galvanometer (coil of 2 turns) through a commutator K to a battery *in series* with a suitable rheostat, key and the copper voltameter as shown in fig. 25·7. Start the current and after gently tapping the compass box with figur ; read the deflections at *both* ends of the pointer. Then *reverse* the current through the tangent galvanometer by means of the commutator and again read the deflection at the two ends of the pointer. If the *mean* values of readings for deflection before and after rever. sing the current in the tangent galvanometer differ, if at all, by more than 1°, slightly turn the coil until they agree as closely as possible. *This sets the plane of the coil of the tangent galvanometer parallel to the magnetic meridian.* Again level the galvanometer if necessary and then mark the positions of the levelling screws of the tangent galvanometer on the table. Adjust the rheostat so that a deflection in the neighbourhood 45° or failing which between 25° and 65° is produced in the tangent galvanometer.

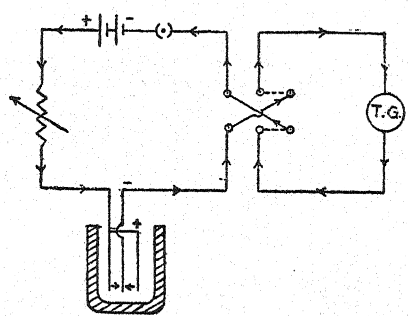


Fig. 25.7.

Now stop the current and take out the *test plate* of copper from the copper voltameter. If the connections of the voltameter are correct the plate should be covered with a smooth salmon coloured deposit of copper. Now replace this plate in the voltameter by the *clean* copper plate over which the depositof copper is to be made, taking care not to touch it with fingers but holding it from the top screw or between a double strip of paper. Then start the current and *immediately* start stop-watch. Note down the deflection in the tangent galvanometer at the *two* ends of the pointer after *every* 5 minutes. Keep the deflection constant by adjusting the rheostat, if necessary. When the current has passed for ten minutes, quickly reverse the direction of the current in the tangent galvanometer by means of the commutator, and again note down the deflection at the two ends of the pointer after every 5 minutes. At the end of another ten minutes stop the cur-

rent as well as the stop-watch. Find out the exact time t for which the deposition of copper has been made. Take the *mean* of the above readings for deflection which gives the value of θ.

Next remove the cathode plate from the voltameter and *immediately* immerse it in a jar of *tap* water already placed near the voltameter. This will remove the copper sulphate solution left on the plate. Then transfer the plate to another jar containing distilled water to which two or three drops of sulphuric acid per litre have been added. Next press the plate *without rubbing* between sheets of filter paper or clean blotting paper to remove the moisture as far as possible. Then dry the plate finally in warm air coming out of a hot-air blower. Weigh the *cool dry* plate in the chemical balance to the nearest tenth of a milligramme with the help of a rider and thus find out the mass m of the copper deposited. Then taking $Z=0{\cdot}0003295$ *gm. per coulomb*, calculate the value of K from the equation (25·11). Finally putting $n=2$ and using the value of r for two turns as specified by the maker, calculate the value of H from the equation (25·12).

Sources of error and precautions. (1) The experiment should be performed on a *rigid* table preferably of stone. The base of the tangent galvanometer and the compass box should be carefully levelled so that the plane of the coil is *vertical* and the needle is *free* to swing in a *horizontal* plane.

(2) The plane of the galvanometer coil should be set in the *magnetic meridian* otherwise the *tangent law* will not hold good.

(3) When the tangent galvanometer has been carefully levelled and its coil placed in the magnetic meridian and the setting tested, the position of the levelling screws on the table should be lightly marked as a precaution against accidental displacement of the instrument.

(4) All magnetic materials, current-bearing conductors and the mains should be at a considerable distance from the tangent galvanometer. The connections between the galvanometer and the commutator should be made by two *flexible wires* twisted together and they should lead away from the galvanometer *so that the magnetic field produced by the passage of current through them has no appreciable effect on the needle.* As there are only two turns of copper wire in the galvanometer coil, the effect of current in any wire near the galvanometer on the needle is easily *comparable* with the effect due to the current in the galvanometer coil itself.

(5) The copper sulphate solution in the copper voltameter should have a density of about 1·18 gms./c.c. and should be made slightly more acid than the aqueous solution of the salt by the addition of 0·1% by volume of concentrated sulphuric acid which increases the conductivity of the solution. The total area of the cathode surface immersed in the solution should be about 50 sq. cm. per ampere passing otherwise the deposit of copper on the

plate will not be smooth and firm and hence will be liable to scale off. The area for deposition of copper can be doubled by depositing it on both sides of the cathode plate for which two anode plates connected together should be immersed in the solution one on *each* side of the cathode plate.

(6) The cathode plate over which deposit of copper is made should be clean and free from any trace of brown oxide or grease, otherwise the deposits will not adhere properly and evenly. The plate should be thoroughly cleaned with sand and moist rug, or with emery cloth or with sand paper or pumice until the surface on either side becomes quite bright. It should never be touched with fingers otherwise the parts touched will be rendered greasy.

(7) The *cathode plate* of the voltameter, *i.e.*, the plate over which the deposit of copper is to be made should be connected to the —ve pole of the battery or to a lower potential point in the circuit. The plate should always be weighed before and after deposit in an accurate *chemical balance* upto 1/10 of a milligram with the help of a rider.

(8) The strength of the current should be between 1·5 and 2 amperes so that in about 20 minutes a deposit of about $\frac{1}{2}$ gm. of copper may be obtained. With weaker current a very long time is required to obtain a deposit sufficient to be weighed accurately. On the other hand, if the current is too strong a very large area of the plate has to be immersed in the solution (see prec. 5), otherwise the copper will adhere very poorly.

(9) The strength of the current should be kept *constant* when the deposition of copper on the plate is being made. The deflection in the tangent galvanometer should, therefore, be kept constant by adjusting the rheostat from time to time, if necessary.

(10) The deflection in the tangent galvanometer should be about 45° for then the readings of deflection would be *least liable to error*. In no case the deflection should be less than 25° and greater than 65°, (see prec. 5 expt. 58).

(11) While reading the deflection in the tangent galvanometer, avoid *error due to parallax*.

(12) Both the ends of the pointer in the compass box should be read to avoid *error due to eccentricity* of the pivot on which the needle rotates with respect to the circular scale.

(13) Readings of deflection in the galvanometer should be taken first with current passing in one direction and then by *reversing* it to avoid *errors due to thermo-electric effects* and due to any *want of accurate setting of the galvanometer coil in the magnetic meridian*. A commutator should be used to reverse the current in the galvanometer without changing its direction in the rest of the circuit.

(14) When the cathode plate is taken out of the voltameter after the deposition of copper has been made, it should be *immediately* immersed in a beaker or jar of *tap* water otherwise the delay will help in the oxidation of fine deposit of copper to copper oxide.

Observations. [*A*] *Determination of m and t.*

Mass of cathode plate *before* deposition of copper = gm

Mass of cathode plate *after* deposition of copper = gm.

Time for which current was passed = min. sec.

[*B*] *Determination of θ*

Time	Deflection of pointer with current				Mean	Tan θ
	Direct		Reverse			
min.	Rt. end	Left end	Rt. end	Left end	θ	
0	45°	45°	...	...		
5	45°	45°	...	...		
10	45°	45°	45°	45°	45°	
15	...	...	45°	45°		
20	...	...	45°	45°		

[*C*] *Constants of the coil.*

Number of turns used =

Circumference of the coil = cm.

Calculations. Mass of copper deposited on the cathode plate = gm.

Electro-chemical equivalent of copper from tables

= gm. per coulomb.

$$K=\frac{m}{Zt.\tan\theta}$$

= amperes

Mean radius of the coil = cm.

$$H=\frac{2\,\pi n K}{10\,r}$$

= oersteds.

Results. (*i*) The *reduction factor* of the tangent galvanometer for two-turns coil = amperes

(*ii*) The value of *horizontal component of earth's magnetic field* in the laboratory at...... = oersteds.

Standard value of H = oersteds

∴ Percentage error =

Criticism of the method. The method suffers from a number of defects. (1) The plane of the coil cannot be set exactly in

the magnetic meridian and there is always an uncertainty in this adjustment.

(*ii*) The needle is not always exactly at the cent.. of the coil.

(*iii*) Unless the coil has a single and exactly circular layer, the value of r is somewhat uncertain. The magnetic field is uniform only in a *small* space at the centre of the needle and since the needle has a *finite* size its movement can never be in an absolutely uniform field.

The last two defects have been removed in Helmholtz modification of tangent galvanometer in which the needle is placed between two coils in the *uniform* field produced by them.

The friction resistance to the movement of the needle is not negligible for it is pivoted and saddled with a long pointer. The 'pointer over scale method' of measuring deflections has a very limited accuracy. This can, however, be considerably increased by fixing a plan- mirror vertically at the centre of the needle and measuring the deflection by lamp and scale or telescope and scale method.

25·6. Magnetic Field at a point on the axis of a Circular Current. Referring to fig. 25·8, let AB be a circular coil of radius r carrying an electric current i e.m. units and let P be a point

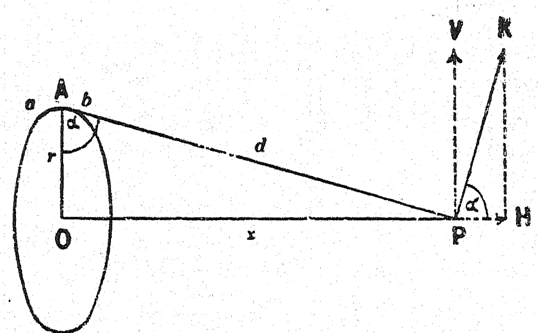


Fig. 25·8

on the axis of the coil at a distance x from its centre. The magnetic field at P due to a *small* element ab of the coil is by *Laplace's law* equal to $i.ab/d^2$ *at right angles to* AP. This can be resolved into two components, one $(iab/d^2)\cos\alpha$ *along* the axis of the coil and another $(iab/d^2)\sin\alpha$ *perpendicular* to it. Considering the field at P due to the whole coil, the components *perpendicular* to the axis of the coil due to various elements will *cancel* each other and hence the total field at P due to the whole coil is given by

$$F = \Sigma \frac{iab}{d^2} \cos a$$

Now $\cos\alpha = PH/PK$. But from *similar* Δ's HPK and OAP we have

$$\frac{PH}{PK} = \frac{OA}{AP} = \frac{r}{d} \quad \therefore \cos\alpha = \frac{r}{d}$$

Hence

$$F = \Sigma \frac{riab}{d^3}$$

$$= \frac{r\,i}{d^3}\,\Sigma\,ab$$

the distance d of each element of the coil from P being the *same*.

Now $\sum ab = 2\,\pi\,r$, and from Δ OAP, $d^2 = (x^2+r^2)$

$$\therefore \qquad F = \frac{2\,\pi r^2\,i}{(x^2+r^2)^{3/2}}$$

Thus for a coil consisting of n turns of wire and having a *mean* radius r, the magnetic field at a point P on its axis is given by

$$F = \frac{2\pi n\,r^2\,i}{(x^2+r^2)^{3/2}} \qquad \text{— (25·13)}$$

If the current i be in amperes

$$F = \frac{2\pi n r^2 i}{10(x^2+r^2)^{3/2}} \qquad \text{... (25·14)}$$

When $x=0$, equ. (25·13) reduces to $2\pi n\, i/r$, the same expression as that obtained previously for the field at the centre of the coil.

If the values of field F and the corresponding values of x for various points on the axis of a circular coil carrying current be plotted on a graph, a curve as shown in fig. 25·9 is obtained. The curve is at first concave towards O, the point corresponding to the centre of the coil, but the curvature goes on decreasing as x increases and soon *changes* its sign, the curve becoming convex towards O. At the *point of inflection*, *i.e.*, where the curvature changes its sign $d^2F/dx^2=0$

Now

$$F = \frac{2\pi n r^2 i}{(x^2+r^2)^{3/2}} = \frac{a}{(x^2+r^2)^{3/2}}$$

$$\therefore \qquad \frac{dF}{dx} = a\,\frac{d}{dx}(x^2+r^2)^{-3/2} = -3a\,x\,(x^2+r^2)^{-5/2}$$

and

$$\frac{d^2F}{dx^2} = -3\,a\,[(x^2+r^2)^{-5/2} - 5x^2(x^2+r^2)^{-7/2}]$$

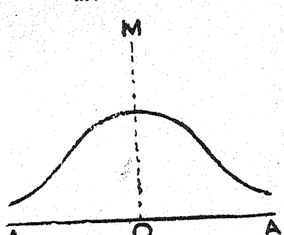


Fig. 25·9

Now if $\dfrac{d^2F}{dx^2} = 0$

$$-3a[x^2+r^2)^{-5/2} - 5x^2(x+r^2)^{-7/2}] = 0$$

or $\qquad 5x^2\,(x^2+r^2)_{-1} = 1$

or $\qquad 5x^2 = x^2 + r^2$

whence $\qquad x = \frac{1}{2}\,r \qquad$ (25·15)

Thus at the *point of inflection* $x = \frac{1}{2}r$.

At this point the rate of change of field dF/dx is *constant*. Hence, if two similar coils be placed with their axis co-incident and separated by a distance equal to the radius of *either*, and if the *same* current is passed through them in the *same* direction, *the rate of increase of field due one coil at mid-point between the coils is equal to the rate of decrease of field due to the other at the same point*, and as we move along the axis from the mid-point any *diminution* in the intensity of the field due to one coil is *exactly*

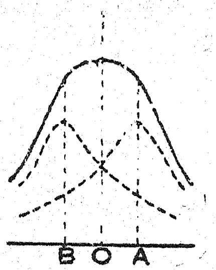


Fig. 25·10

compensated by the *increase* in the field due to the other so that the field between the coils is practically *uniform*. This fact is depicted in fig. 25·10 and utilised in the *Helmholtz tangent galvanometer*. The variation of field with distance for the two coils is shown graphically in fig. 25·11.

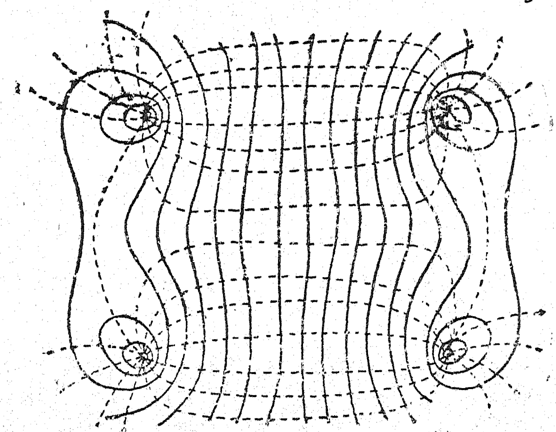


Fig 25·11

As depicted in fig. 25·12, the *Helmholtz tangent galvanometer* consists of two *equal coaxial* coils separated by a distance equal to the radius of the coils.

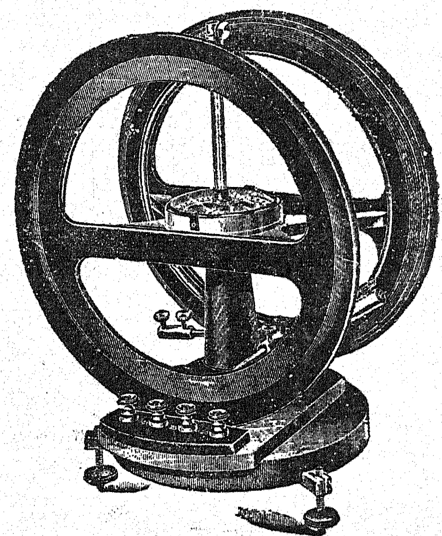


Fig. 25·12

The needle is pivoted *mid-way* between them ; and the *same* current is passed through the coils in the *same* direction so that fields at the mid-point due to the two coils *assist* each other. Putting $x=r/2$ in equation (25·13) and remembering that there are two coils, the total field at the mid-point between the coils is given by

$$F=2\left\{\frac{2\pi n r^2 i}{\left\{(r/2)^2+r^2\right\}^{\frac{3}{2}}}\right\}$$

or $$F = \frac{32\pi n i}{5\sqrt{5}r}$$

When the planes of the coils are set *parallel* to the *magnetic meridian*, the field F is also given by

$$F = H \tan \theta$$

where θ is the deflection of the needle from the *magnetic meridian* and H the horizontal component of earth's field.

Thus from the above two equations, we get

$$\frac{32\pi n i}{5\sqrt{5}r} = H \tan \theta$$

whence $$i = \frac{5\sqrt{5}r}{32\pi n} H \tan \theta \qquad (25{\cdot}16)$$

If the current i be in *amperes*

$$i = \frac{50\sqrt{5}r}{32\pi n} H \tan \theta$$

Experiment 25.3

Object. To plot graph showing the variation of magnetic field with distance along the axis of a circular coil carrying current and to estimate from it the radius of the coil.

Apparatus. Tangent galvanometer of the Stewart and Gee type (Fig. 25·13) a storage battery, a suitable rheostat, a commutator and a plug key.

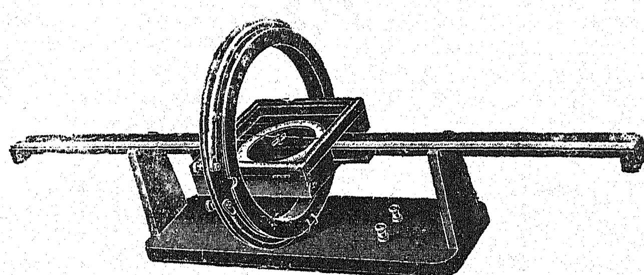


Fig. 25·13

Description of Apparatus. The apparatus called *Stewart and Gee tangent galvanometer* consists of a circular coil of thin copper wire fixed with its plane vertical on a suitable horizontal bench, and a magnetometer compass box which can be slid on the bench so that the centre of the needle always lies on the axis of the coil. Along the bench on both sides of the coil is fixed a scale graduated in millimeters on which can be read off the distance of the centre of the needle from the centre of coil.

Theory. Let the plane of the coil be placed *parallel* to the *magnetic meridian* and let a current of i amperes be allowed

to pass through the coil. The magnetic field F at a point situated on the axis of the coil and at a distance x from its centre, due to the current in the coil is given by

$$F = \frac{2\pi n r^2 i}{10\,(x^2+r^2)^{3/2}}$$

where r is the radius of the coil and n the number of turns in it. The field F is *perpendicular* to the direction of the horizontal component H of the earth's field and hence, if θ be the deflection from the *magnetic meridian* of a compass needle placed at that point, then from § 24·4,

$$F = H \tan\theta$$

Equating the above two expressions for F, we get

$$\frac{2\pi n r^2 i}{10\,(x+r^2)^{3/2}} = H \tan\theta$$

If a graph be plotted taking the distances of various points along the axis of the coil from *one end* of the instrument as abscissae and the corresponding values of tan θ as ordinates, the curve obtained will be as in fig. 25·15. At the *point of inflection, i.e.*, where the curvature changes sign, $x=\frac{1}{2}r$ (see § 25·6); and hence *the distance between the points of inflection* P and Q *on the two branches of the curve situated on the two sides of the line* XC *will be equal to r, the radius of the coil.* The line XC which is equidistant from the points of inflexion will give the position of centre of the coil.

Method. Place the magnetometer compass box on the sliding bench so that its magnetic needle is at the centre of the coil. Rotate the bench in a horizontal plane until *the coil, the needle and its image all lie in the same vertical plane.* This sets the coil roughly in the *magnetic meridian* and hence its axis in the magnetic east and west direction. In this position of the bench the pointer in the compass box will read o—o on the circular scale.

Now without disturbing the above adjustment, connect the thin-wire coil through a commutator to a storage battery B *in series* with a suitable rheostat R and a plug key K_1. Then start the current and adjust its value by means of the rheostat so as to produce a *large* deflection, say of about 75° in the compass box when the needle is at the *centre* of the coil. If the current is too weak to produce the above deflection the number of cells in the storage battery may be increased. Now gently tapping the compass box with finger read off the deflection at the *two* ends of the pointer. Then *reverse* the current in the coil by means of the commutator and again read off the

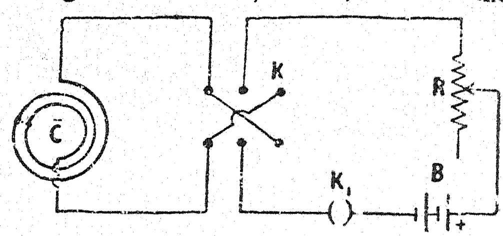


Fig. 25·14

deflection at the two ends of the pointer. If the *mean* value of readings for deflection before and after reversing the current in the coil differ by more than 1°, slightly turn the coil with the help of the bench until *they agree as closely as possible.* This sets the coil in the *magnetic meridian.*

Now place the compass box on east or west arm at its *end* and note down the readings of deflection before and after reversing the current in the coil, and from the four readings of deflection thus obtained, calculate the *mean* value of θ. Measure the distance of the needle from the *and* of the arm. Then shift the compass box along the bench by 2 cm. and determine the value of θ for another point measuring its distance again from the end of the arm. Continue to move the compass box on the bench in steps of 2 cm. and determine the *mean* value of θ for each position of the needle on the axis of the coil till the compass box passing through the centre of the coil reaches the *other* end of the bench or the deflection is reduced to about 5°, *taking special care in determining deflection when x is nearly equal to r/2.*

Next plot a graph taking the various values of distances of the centre of the needle from the *end* of the bench as abscissae and the corresponding values of tan θ, the tangent of the *mean* value of deflection, as ordinates. This will consist of two symmetrical branches, as shown in fig. 25·15. Find out the two points of *inflection* P and Q on the curve and determine the difference between their X-co-ordinates A and B which gives the value of r, the radius of the coil.

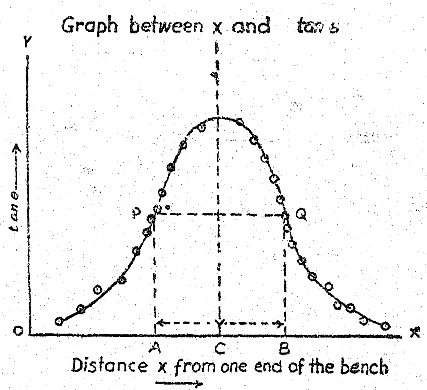


Fig. 25·15

Sources of error and precautions. (1) All magnetic materials and current-bearing conductors should be at a considerable distance from the apparatus.

(2) The plane of the coil should be *parallel* to the *magnetic meridian*, otherwise the *tangent law* will not hold good.

(3) The current through the coil should be sufficient to give a *large* deflection, say of about 75° when the needle is at the centre of the coil and its strength should be *constant.* The current through the circuit should, therefore, be sent from a *steady* supply battery, *e.g.*, a storage battery.

(4) The *error due to parallax* in reading deflection of the needle should always be avoided.

(5) *Both* ends of the pointer in the compass box should be read to avoid *error due to eccentricity* of the pivot on which the needle rotates with respect to the circular scale.

(6) Readings of deflection in the compass box should be taken first with current passing in one direction and then by *reversing* it.

(7) Since the *exact* position of the centre of the coil is not accurately known, the distances of the centre of the needle in all positions of the compass box should be measured from one of the ends of the bench.

(8) Since the curve becomes almost *vertical* in the neighbourhood of $x = r/2$, observations for deflection when the distance of the needle from the centre of the coil is approximately equal to *half the radius* of the coil should be very carefully made by shifting the compass box in steps of about a cm.

(9) The curve showing the variation of tan θ with distance should be *smooth*.

Observations.

S No.	Distance of the needle from *one end* of bench	Deflection of pointer with current				Mean θ	Tan θ
		direct		reverse			
		Rt. end	Lft. end	Rt. end	Lft. end		
1.							
2.							
...							
...							

Result. The radius of the coil as estimated from the *distance—tan θ* curve = cm.

Radius of coil by *actual* measurement = cm.

∴ Percentage error =

Criticism of the method. The theory underlying the method assumes (*a*) that the plane of the coil lies exactly in the *magnetic meridian* and is exactly *vertical* (*b*) that the needle is *infinitely small*, its axis

is exactly *horizontal* and its centre always lies exactly on the axis of the coil, and (*c*) that the coil is exactly circular. These assumptions are obviously not all justifiable in practice and hence the result obtained by this method cannot be very accurate. Further the friction resistance to the motion of the needle is not negligible for it is pivoted and saddled with a long pointer. The accuracy of the pointer-over-scale method of measuring deflections is very limited and consequently the position of inflection points on the curve cannot be determined very accurately.

Oral questions

STRAIGHT CURRENT

What is Laplace's rule for calculating the intensity of magnetic field at a point due to small current-element? How can you determine the direction of the magnetic field at a point due to a current in a conductor? State the rules. How does the magnetic field at a point due to a straight current vary with the distance of the point from the conductor? What is the direction of magnetic field at a point due to straight current? What type of lines of force do you get in the field of a straight current? What will be the character of the resultant field of straight current and the earth? How many neutral points do you get in this case? Where will it lie? How can you determine the value of H by plotting the resultant field of the straight current and the earth's horizontal field? What precautions do you observe in this experiment? Why should the straight conductor be connected with the rest of the apparatus by flexible wires? Why should the ammeter be kept at a distance from the straight conductor? Why do you take current from mains? Can an accumulator of E. M. F. 2 volts not do? How can you accurately locate the position of the neutral point? Why does the behaviour of the needle in the neighbourhood of the neutral point become uncertain? What is the accuracy of the result obtainable by this method? Do you know any other method of determining H?

TANGENT GALVANOMETER

Describe the construction of tangent galvanometer. Why is it so called? What is the tangent law? Why is only a short needle used in the compass box? What is the relation between current and the angle of deflection? How is the needle of the T. G. deflected? What are the two fields? Point out their directions. How is the direction of magnetic field at the centre of the coil determined? What are the factors on which the field at the centre of the coil depends? Define e. m. unit of current and ampere and state their relation. What is meant by reduction factor of tangent galvanometer? Explain its physical significance. Why is it so called? What is galvanometer constant or coil constant of a T. G.? Is it the same as R. F. of a T. G.? Do their values remain constant at all places? How does R. F. vary with (*a*) the number of turns (*b*) the radius of the coil and (*c*) the place of working? How will you determine R. F. of a T. G.? How can your result be verified? What is the use of the pointer? Why is it made of aluminium? Can you replace it by an iron pointer or by a pointer made of straw? Why is the pointer attached at right angles to the needle? Is it at all necessary to attach it at right angles to the needle? Can you not do without the pointer? What is the purpose of the mirror fixed at the base of the box? What is the harm if this error is not avoided? Why do you place the plane of the coil in the magnetic meridian? How will the deflection change if the coil be displaced from the magnetic meridian? How do you set the coil in the magnetic meridian and how is the accuracy of the adjustments tested? What will be the deflection if the plane of the coil be placed at right angles to the magnetic meridian? Why are both ends of the pointer read? Why do you take readings of deflection with current direct and reverse? What is the accuracy of observations with a T. G.? How can it be increased? Why should the deflection be in the neighbourhood of $45°$? What is the harm if the deflections are very small or very large? What is the

range in which deflections should lie ? What is the approximate resistance of your instrument ? Of which wire is the coil of the tangent galvanometer made ? What is meant by sensitivity of a galvanometer ? How can a T. G. be made more sensitive ? Is tangent galvanometer a dead-bead galvanometer ? If not, how can you make it so ? Why are three levelling screws provided at the base of the instrument ? How are the readings affected if the base of the galvanometer is not perfectly levelled ? How do you level a T. G. ? Having levelled the instrumene why is it necessary to mark the positions of the levelling screws ? What are the uses of a T. G. ? Why in the Pye T. G. there are three coils of turns 2, 50 and 500 ? What is a sine galvanometer ?

How do you measure current in this experiment ? Why is copper voltameter preferred to an ammeter to measure current ? Describe a copper voltameter and explain the reactions which take place when the current is passed through it. What should be the strength of the copper sulphate solution ? Why is some H_2SO_4 added to it ? Why do you connect the plate on which deposition of copper is made to the —ve of the battery or to a lower potential point in the circuit ? From where does copper come to the cathode ? When the copper liberated at the cathode comes from the anode why don't you weigh the anode plate before and after deposition of copper to get the mass of copper liberated ? Why are two plates used for the anode ? Is it necessary to clean the anode plates also ? What will be the harm if they remained dirty ? How do you clean the cathode plate? How should a clean plate be handled and why ? Why do you weigh the cathode plate to such a high degree of accuracy ? While removing the cathode plate from the voltameter after deposition of copper, why do you immediately immerse it into water ? How is the plate dried ?

How do you measure H with a tangent galvanometer ? Discuss the accuracy in the value of H obtainable by this method.

CIRCULAR COIL

How does the field due to a circular current vary along its axis ? How does it vary with *(a)* the distance and *(b)* the number of turns in the coil ? Where is field maximum ? How do you find out the radius of the coil from the x—tan θ graph ? What sort of graph do you get in this case ? Why is the needle in the compass box very small ? Do you get uniform field around any point on the axis of the coil ? If not, how can you modify the apparatus to get uniform field ? What is an Helmholtz tangent galvanometer ? What advantages does it possess over an ordinary tangent galvanometer ? What precautions do you take in this experiment ? Why do you use a storage cell for this experiment ? Can't you use a Leclanche cell ? If not, why ? Discuss the accuracy of the results obtained in this experiment.

CHAPTER XXVI

MEASUREMENT OF RESISTANCE AND POTENTIAL DIFFERENCE

26·1. Potential Difference and its Unit. When a current flows through a conductor, it gets heated. This is due to the fact that when electricity moves from one end of the conductor to the other, it does so at the expense of a part of its electrical energy which is transformed into heat energy in the conductor. This *energy transformed into heat or work done when unit quantity of electricity passes between any two points equals the potential difference (P. D.) between those two points.* If the work done or the energy transformed into heat between the two points is unity when unit quantity of electricity passes between them, then P. D. between the two points is also unity. This gives definition of unit P. D. which is called the absolute or C. G. S. electro-magnetic unit of P. D. The C. G. S. electro-magnetic unit of P. D. is too small for practical purposes and hence for such work a bigger unit called the *volt* (named after Volta) which is equal to 10^8 e.m. units is chosen. Thus **volt** *is the practical unit of P. D. and is equal to the P. D. between the two points which are such that the amount of work done or energy transformed into heat is one joule or* 10^7 *ergs when one true coulomb of electricity passes between them.* The volt so defined is called the *true* volt.

1 true volt$=10^8$ absolute e.m. unit$=1/300$ e. s. unit

26·2. Ohm's Law and unit of Resistance. We know that when the flow of liquid in a tube is *streamline*, the quantity of liquid flowing through the tube per second is directly proportional to the pressure difference between the ends of the tube. In exact analogy with this *when a current is allowed to pass through a conductor, the strength of the current through it is directly proportional to the P. D. between the ends of the conductor, provided the physical state of the conductor remains the same.* Thus mathematically

$$I=k\,V$$

where V and I are the P.D. and current respectively and k is a quantity characteristic of the particular conductor, which measures the capacity of the conductor to allow electricity to pass through it and is called its *electrical conductivity.* This important relation between I and V is called *Ohm's law.*

Now $$V/I=1/k=R$$

where $R=1/k$ is called the *electrical resistance* of the conductor for the greater the value of R, greater must be the P. D. established for the same current through the conductor.

Now, if V and I each be equal to unity, R is also unity. Hence a conductor has a resistance of one absolute or C. G. S. electromagnetic unit if a P. D. of one e.m. unit applied to its ends causes a current of one e.m. unit to flow through it. The C. G. S. electromagnetic unit of resistance is too small for practical purposes and for such work a bigger unit called the ohm (named after Ohm) which is 10^9 e.m. units is used. *The resistance of a conductor is one* **ohm** *when a P. D. of one true volt applied at its ends causes a current of one true ampere to flow through it.* The ohm so defined is called the *true* ohm.

$$1 \text{ true ohm} = 10^9 \text{ absolute e. m. units} = \frac{1}{9\times 10^{11}} \text{ e. s. unit}$$

Since conductivity is the reciprocal of resistance, the practical unit of conductivity is reciprocal ohm or *mho.*

Since the resistance of a conductor remains constant so long as its physical state remains the same, attempts have been made to construct a standard ohm. The metal chosen for the construction of standard ohm is mercury, for it being a liquid is free from mechanical strains, and can be obtained in a chemically *pure* state by distillation and further its temperature can be kept constant more easily than that of any other metal. The standard mercury ohm called *the legal or* **international ohm** *is the resistance of a column of mercury of uniform cross section weighing* 14·4521 *gm. and measuring* 106·300 *cm. at* 0°C. This corresponds to an area of cross-section of about 1 sq. mm., but on account of difficulty of obtaining a glass tube of uniform bore the mercury standard is preferably specified in terms of mass of mercury present. The mercury standard is reliable to about 1 part in 50000. The international ohm is slightly bigger than the true ohm.

$$1 \text{ international ohm} = 1{\cdot}00052 \text{ true ohm}$$

26·3. Electromotive Force (E. M. F.) and Potential Difference (P. D.) We have seen above that whenever electricity passes between two points of a conductor there is an expenditure of energy which is subtracted from the circuit and appears as heat between those two points. This implies that in order to maintain the current in the circuit, energy must be continuously supplied to it by some source situated somewhere in the circuit. This source may be a cell in which case the necessary energy is supplied by the chemical action inside it or it may be a dynamo in which case the energy is provided by some kind of heat engine. But whichever may be the source of supply of energy in the circuit we say, from analogy of flow of water in a closed circuit where the flow is maintained by a mechanical effect of a pump which may be called the water-motive-force, that the current in an electrical circuit is maintained by an electrical effect called the *electromotive force* (E. M. F.). This **E. M. F.** *of a source of supply of energy to an electrical circuit is measured by the rate of working or the energy supplied per second when it is maintaining unit current in the circuit* and is thus clearly equal to the work done or energy transformed when

unit quantity of electricity passes completely round the circuit containing the source. This work is equal to the sum of the work done to drive unit quantity of electricity in the *external* circuit, *i.e.*, the potential difference V across the external resistance and the work done in driving unit quantity of electricity inside the source, *i.e.*, the potential difference v across the *internal* resistance of the source ; and hence if E be the E. M. F. of the source of electrical energy in the circuit.

$$E=V+v$$

But from *ohm's law* $V=I\,R$ (26·1)

and $v=I\,r$

where I is the current flowing through the circuit and R and r are the external resistance of the circuit and the internal resistance of the source respectively.

Thus $E=IR+Ir$ (26·2)

Dividing equ. (26·1) by equ. (26·2) we get

$$\frac{V}{E}=\frac{R}{R+r}=1-\frac{r}{R+r}$$

whence $$V=E\left(1-\frac{r}{R+r}\right) \qquad (26{\cdot}3)$$

Thus the P. D. between the terminals of a battery or a dynamo is always *less* than its E. M. F. when it is supplying a current. If, however, no current is being supplied by the source, *i.e.*, it is *on open circuit, the P. D. between its terminals equals the E. M. F. of the source.* The E. M. F. is measured in the same units as the P. D.

An E. M. F. like a mechanical force is a *directed* quantity and determines the direction of the current in the circuit. If there are more than one E. M. F. in the circuit the direction of current depends upon the *resultant* or effective E. M. F. in the circuit. An E. M. F. corresponds to energy transformation in a *reversible* process for, if an E. M. F. opposes the flow of current which is maintained by a bigger E. M. F. in the circuit then energy will be absorbed from the circuit at the smaller E. M. F. and not supplied to it and hence the energy of the source of E. M. F. opposing the flow of current will increase at the expense of the energy of the source of E. M. F. maintaining the current in the circuit. On the other hand, P. D. has a direction which depends upon the direction of the current in the circuit and hence upon that of the resultant or effective E. M. F. in the circuit. P. D. corresponds to energy transformations in an *irreversible* process for whichever may be the direction of current in the circuit, P. D. will always correspond to the dissipation of electrical energy from the circuit in the form of heat.

26·4. Combination of Conductors. When several conductors are connected in a circuit, the following general relationships hold :—

(1) **Law of Resistors in Series.** Referring to fig. 26·1, when several conductors are connected in *series*, *i. e.*, one after another so that the *same current* passes through them, *the total resistance is equal to the sum of the individual resistances*, *i.e.*,

$$R=r_1+r_2+r_3+\ldots=\Sigma r_n$$

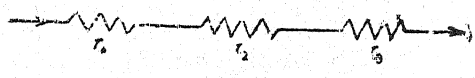


Fig. 26·1

(2) **Law of Resistors in Parallel.** Referring to fig. 26·2, when several conductors are connected in *parallel*, *i.e.*, one end of all conductors is joined to one point of the circuit and the other end of all conductors is joined to a second point of the circuit so that the *same* P. D. exists across each conductor, *the equivalent resistance is such that its reciprocal is equal to the sum of reciprocals of the individual resistances*, *i.e.*,

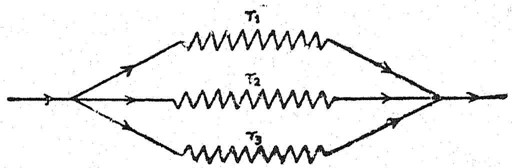


Fig. 26·2

$$\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\ldots=\Sigma\frac{1}{r_n}$$

Since $1/R=k$, the *electrical conductivity*, it follows from above that *when several conductors are connected in parallel, their total conductivity is equal to the sum of the individual conductivities*, *i.e.*,

$$k=k_1+k_2+k_3+\ldots=\Sigma k_n$$

Note that *when the resistors are connected in parallel, the total resistance is always less than the least of the component resistances.* If $r_1=r_2=\ldots=r$, $R=r/n$, *i.e.*, when n equal resistances are connected in parallel, the total resistance is equal to nth part of one of the resistances.

26·5. Current in divided circuits and use of shunt. Let fig. 26·3 represent a portion of a circuit in which the main curent I divides itself among a number of resistors connected in parallel. Let V be the P. D. across each resistor and let R be the equivalent resitance of these resistors connected in paralel, then from Ohm's Law

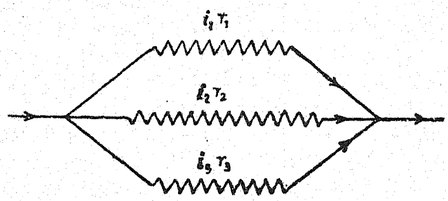


Fig. 26·3

$$V=IR=i_1\,r_1=i_2r_2=i_3r_3 \quad\ldots\quad (26\cdot4)$$

and since the *total* current is unchanged

$$I = i_1 + i_2 + i_3 + \ldots$$

From equ. (26·4), we get

$i_1 = IR/r_1$; $i_2 = IR/r_2$ and so on, where R is given by

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

If there are only two conductors r_1 and r_2 in parallel, then

$$R = \frac{r_1 r_2}{r_1 + r_2}$$

and hence

$$i_1 = \frac{Ir_2}{r_1 + r_2} \quad \text{and} \quad i_2 = \frac{Ir_1}{r_1 + r_2} \qquad \ldots \quad (26·5)$$

or

$$\frac{i_1}{i_2} = \frac{r_2}{r_1}$$

i.e., the current divides inversely as the resistances.

The above principle is used in sending only a *small* portion of the main current through an instrument, *e.g.*, a galvanometer by connecting a low resistance wire in *parallel* with it. This is illustrated in fig. 26·4. The wire connected across the strument is called a 'shunt' and provides a by-pass for the current and hence reduces the sensitiveness of the instr ent as well as prevents it from being damaged when the otal current is large.

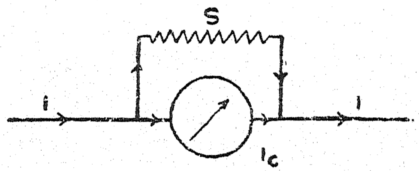


Fig. 26.4

Let I be the total current and I_g the current through the galvanometer after it has been shunted with a resistance S. Then, if G be the resistance of the galvanometer, we have from equation (26·5),

$$I_g = \frac{S}{S+G} I$$

or

$$I = \frac{S+G}{S} I_g$$

The factor (S+G)/S by which the galvanometer current has to be multiplied to get the total current is called the *multiplying power* of the shunt. Now, if only 1/10 of the total current is to be passed through the galvanometer, then clearly the multiplying power of the shunt must be 10, *i.e.*,

$$\frac{S+G}{S} = 10$$

whence $S = G/9$

Similarly, if 1/100 or 1/1000 of the total current is to be passed through the galvanometer, the resistance of the shunt must be 1/99 or 1/999 respectively of the galvanometer resistance.

Sometimes galvanometers are provided with special shunt boxes containing shunts of different multiplying powers. These are known as *range multipliers*. This system has the disadvantage that a shunt box can be used only with the instrument for which it has been made and hence each galvanometer requires its own special shunt box. This difficulty is overcome in Ayrton and Mather Universal shunt which will serve for any galvanometer.

26·6. Kirchhoff's Laws. The total resistance and the current in various branches of a divided circuit can be readily calculated by the application of laws of resistances in series and in parallel and *Ohm's law* when the circuit is a *simple* one, *i.e.*, when there are no cross connections in the network. But when complicated circuits are to be dealt with where there are cross connections as is depicted in Fig. 26·5, the above method fails and the problem is then solved by using *Kirchhoff's* following two *laws*:

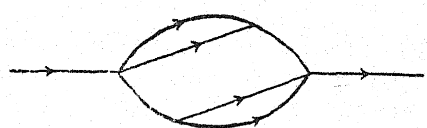


Fig. 26·5

I Law. The sum of currents arriving at any point in a circuit equals the sum of the currents leaving that point, *i.e.*, *the algebraic sum of the currents at any point in a circuit is zero* or mathematically

$$\Sigma i = 0$$

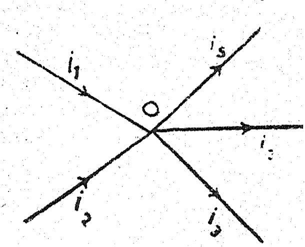


Fig. 26·6

Thus referring to fig. 26·6, if two currents i_1 and i_2 flow *towards* a point, O in a circuit and the currents i_3, i_4 and i_5 flow *away* from it, then applying the law to the point O, we have

$$i_1 + i_2 = i_3 + i_4 + i_5$$

or

$$i_1 + i_2 - i_3 - i_4 - i_5 = 0$$

This law is simply an expression of the fact that for a steady current in a circuit, *there can be no accumulation of charge at any point in the circuit*, and corresponds exactly to the *equation of continuity* of a perfectly incompressible fluid.

II Law. *The algebraic sum of the electromotive forces acting in a closed circuit (or mesh) in a network of conductors is equal to the algebraic sum of the products of current and resistance of each part of the circuit* or mathematically

$$\Sigma E = \Sigma (ir)$$

The currents flowing in the clockwise direction and the E. M. F.'s tending to cause the current to flow in the clockwise direction are reckoned as *positive* while currents flowing in the

anti-clockwise direction and the E. M. F.s tending to cause the current to flow in the anti-clockwise direction are taken as *negative*.

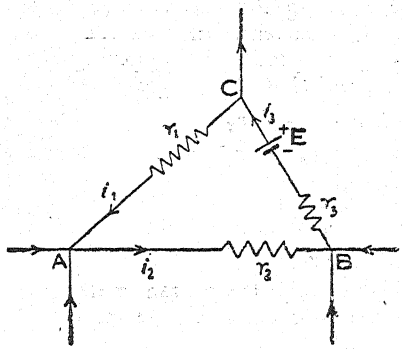


Fig. 26·7

Applying the law to the circuit ABC in the network of fig. 26·7. we have

$$i_1\,r_1+i_2\,r_2+i_3\,r_3=E$$

This law is merely a generalisation of Ohm's law.

26·7. Specific Resistance or Resistivity. The resistance of a conductor is directly proportional to its length and inversely proportional to its area of cross-section. Thus, if R be the resistance of a conductor of length l and of area of cross section S assumed to be uniform, then $R \propto l/S$, or

$$R=\rho l/S$$

where ρ is a constant which depends upon the nature of the material of the conductor, and is called the *specific resistance* or *resistivity* of the material of the conductor. If $l=1$ and $S=1$, then $R=\rho$. Thus *the* **specific resistance** *of a material is numerically equal to the resistance of a piece of it of unit length and uniform unit cross-sectional area.* It is also *numerically equal to the resistance offered by a cubical block of the material with edge of unit length when the flow of current is normal to one of its faces.* The unit of resistivity is *ohm × cm.* The reciprocal of resistivity is called *specific conductance* and is expressed in *mho per cm.*

26·8. Wheatstone Bridge. Wheatstone bridge or net

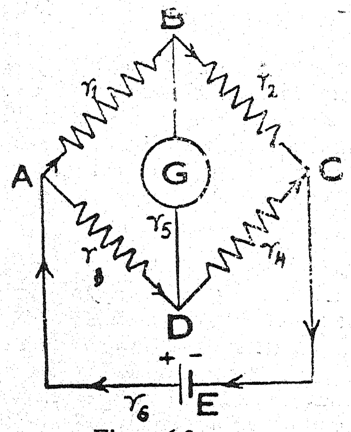


Fig. 26·8

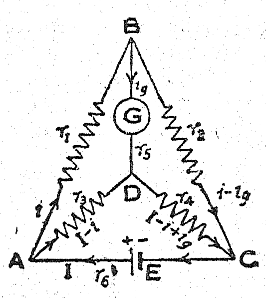


Fig. 26·9

consists of four resistances r_1, r_2, r_3 and r_4 arranged as shown in fig. 26·8, the one pair of *opposite* junctions of the four resisters, such as A and C, being connected by a battery of E. M. F., e and

internal resistance r_6 and the other pair of opposite junctions B and D through a galvanometer of resistance r_5. Let I be the current in the battery circuit, i that along AB and i_g that along BD. Then by *Kirchhoff's first law*, the currents in BC, AD, and DC will be $(i-i_g)$, $(I-i)$, $(I-i+i_g)$ respectively. Applying *Kirchhoff's second law* to the meshes ABDA and DBCD, we get

$$ir_1+i_g r_5-(I-i)\, r_3 = 0$$

and

$$(i-i_g)\, r_2-(I-i+i_g)\, r_4-i_g\, r_5 = 0$$

Rearranging these equations, we get

$$i\,(r_1+r_3)+i_g\, r_5-Ir_3 = 0 \qquad (26\cdot6)$$

$$i\,(r_2+r_4)-i_g\,(r_2+r_4+r_5)-Ir_4 = 0 \qquad (26\cdot7)$$

When there is no current flowing through the galvanometer, *i.e.*, $i_g=0$, the bridge is said to be *balanced* and in that case equations (26·6) and (26·7), reduce to

$$i\,(r_1+r_3)=Ir_3 \quad =0 \qquad (26\cdot8)$$

$$i\,(r_2+r_4)=Ir_4 \quad =0 \qquad (26\cdot9)$$

Dividing equ. (26·8) by equ. (26·9), we have, since i and I are not equal to zero.

$$\frac{r_1+r_3}{r_2+r_4}=\frac{r_3}{r_4}$$

or

$$\frac{r_1+r_3}{r_3}=\frac{r_2+r_4}{r_4}$$

whence

$$\frac{r_1}{r_3}=\frac{r_2}{r_4}$$

or

$$\frac{r_1}{r_2}=\frac{r_3}{r_4} \quad \ldots \qquad (26\cdot10)$$

Thus knowing the values of any three resistances, say r_1, r_2, and r_3, when the bridge is balanced, the value of the unknown resistance r_4 can be calculated from the above relation. The relation is independent of the current in the battery circuit. An examination of fig. 10·10 will show that when the positions of the galvanometer and battery are interchanged, the above condition for no deflection of the galvanometer will still hold.

26·9. Conjugate conductors. *If two conductors of any network of conductors are so related that an E. M. F. in one produces no current in the other, they are said to be* **conjugate** *to each other.* The 'battery arm' and the 'galvanometer arm' in a wheatstone net are *conjugate* to each other when the bridge is balanced *i.e.*, when the resistance of other arms are so adjusted as to satisfy the usual wheatstone bridge relationship of equation (26·10). It can be shown that *the current in one of the two conjugate conductors due to an E.M.F. in any part of the network, is independent of the resistance of the conjugate conductor.*

26·10. Sensitiveness of Wheatstone Bridge. In the practical use of Wheatstone bridge for the measurement of a wide range of resistances, the accuracy in the determination of the bridge depends upon the *sensitiveness* of the bridge. The sensitiveness of the bridge depends upon the current in the galvanometer, for the greater the current in the galvanometer for a given *small* want of balance of the bridge, greater is the sensitiveness of the bridge. Hence to make the bridge most sensitive, the positions of the galvanometer and the battery and the values of the resistances in the various arms of the bridge should be so adjusted as to produce *maximum* current in the galvanometer for a given *small* want of balance.

(a) Positions of galvanometer and battery. The galvanometer can be connected in BD and the battery in AC as shown in fig. 26·10 or their positions can be interchanged, but the two arrangements are, in general, not equally sensitive. It can be shown that if, *of the two resistances, that of battery and that of galvanometer, the greater connects the junction of the two greatest with that of the two least of the four arms of the bridge*, the current in the galvanometer and hence the deflection in it is greatest for a given small want of balance and, therefore, the bridge most sensitive for the given resistances.

(b) Galvanometer resistance. The sensitiveness of the galvanometer should be maximum, for the greater the sensitiveness of the galvanometer, greater will be the accuracy in the determination of the proper balance of the bridge. The sensitiveness of a galvanometer is proportional to the deflection for a given small current, and this deflection is proportional to the number of turns in the coil, if the current is the same. Now, as the channel of the bobbin on which the coil is wound is fixed with respect to its cross-section and the mean radius, the number of turns can be increased only by decreasing the cross-section of the wire. Let the cross-section of the wire be decreased to $1/n$ of its previous value, then the number of turns can be increased to n times their original value. But this also increases the length of the wire n times and hence the total resistance of the coil is increased to n^2 times its original value, *i.e.*, $r_5 \propto n^2$, where r_5 is the resistance of the galvanometer and n is the number of turns in the coil. Hence for a given value of current the deflection in the galvanometer is proportional to $\sqrt{r_5}$. The deflection also varies with the current. Hence the sensitiveness of the galvanometer is proportional to the quantity $i_g\sqrt{r_5}$. It can be shown that this quantity is *maximum* when

$$r_5 = \frac{r_4(r_1+r_3)}{r_3+r_4} = \frac{r_1(r_2+r_4)}{r_1+r_2}$$

This gives the value of the resistance of the galvanometer in order to make the bridge most sensitive for the given values of r_1, r_2, r_3, o_4 and r_5. The tangent galvanometer is very unsuitable for wheat-

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stone bridge for its sensitivity is very small, *i.e.*, it requires comparatively large current to produce an appreciable deflection.

(c) **Battery resistance.** When the resistance of the battery is at the disposal of the experimenter, the bridge is most sensitive for the given values of r_1, r_2, r_3, r_4, and r_5 when the current given by the battery is maximum, and this is the case *when the external resistance equals the internal resistance of the battery.* Thus for nearly complete balance

$$r_6=\frac{(r_1+r_2)\,(r_3+r_4)}{r_1+r_2+r_3+r_4}=\frac{r_4(r_1+r_2)}{r_2+r_4}$$

This gives the battery resistance to make the bridge most sensitive for the given values of r_1, r_2, r_3, r_4, and r_5.

(d) **Galvanometer and battery resistances fixed and others variable.** Usually the resistance r_5 of the galvanometer and the resistance r_6 of the battery are not at the disposal of the experimenter and hence, if r_4 be the unknown resistance, the only resistances which can be varied to make the bridge most sensitive are r_1, r_2 and r_3. For a particular value of r_3 there will be a pair of values of r_1 and r_2 which will constitute the best arrangement for that value of r_3; and there will be a *particular* value of r_3 which with the corresponding values of r_1 and r_2, will be the best arrangement for the given values of r_4, r_5 and r_6. It can be shown that in this case the values of r_1, r_2 and r_3 which will make the bridge most sensitive should be as follows :—

$$r_1=\sqrt{r_5r_6}\,,\quad r_2=\sqrt{r_4r_6\cdot\frac{r_4+r_5}{r_4+r_6}}\quad\text{and}\quad r_3=\sqrt{r_4r_5\cdot\frac{r_4+r_6}{r_4+r_5}}$$

The resistance r_1 is independent of the value of r_4 and has a definite value $\sqrt{r_5r_6}$. Having given this value to r_1, the value of r_4 is roughly determined and then the values of r_2 and r_3 calculated.

(e) **All resistances variable.** When all resistances except r_4, the unknown resistance, can be varied, *i.e.*, the resistances of the battery, the galvanometer and other arms of bridge except the unknown resistance is at the disposal of the experimenter, the most sensitive arrangement is that in which *each of the resistances is equal to the unknown resistance, i. e.*, $r_1=r_2=r_3=r_5=r_6=r_4$.

In this case it is immaterial whether the galvanometer is connected in BD and the battery in AC or *vi.e-versa*. The heating effects will also be the same in all branches. This arrangement gives the most accurate value of the unknown resistance but such an arrangement is practically impossible for every unknown resistance requires the setting up of a special bridge and the employment of special galvanometer and battery.

26·11. Metre bridge. It is one of the practical applications of the Wheatstone bridge and, as shown in fig. 26·10, consists of a straight

and uniform wire of manganin, constantan or some other suitable alloy of *high* specific resistance, stretched along a metre scale, with its ends soldered to two copper strips of negligible resistance. The wire is one metre long and has a resistance of about 2 to 3 ohms. Between these two copper strips and a third one fixed along the panel there are two gaps, G_1 and G_2, in which are connected with the help of binding screws, the unknown resistance X and a suitable resistance box. A jockey serves to make contact with the stretched wire and a mark on it or a pointer attached to it indicates the position of the point of contact on the measuring scale. The battery is joined between terminals on the two end strips, a key being included in the circuit so that the current may be passed when required. The galvanometer is connected between a terminal on the middle copper strip and the jockey.

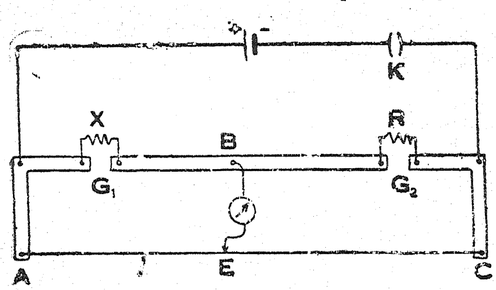


Fig. 26·10

To determine the *unknown* resistance X, the connections are made as shown. A suitable resistance R is introduced in the gap G_2 by means of the resistance box and a point of contact E is found on the wire by sliding the jockey at which there is no deflection in the galvanometer. If the two resistances into which the bridge wire is then divided be R_1, (AE), and R_2 (CE), we have by equ. (26·10) since the bridge is *balanced.*

$$\frac{X}{R} = \frac{R}{R_2}$$

Since the wire is assumed to be of *uniform* cross-section, the resistance of any portion of it is *proportional* to the length. Hence, if AE$=l$, CE$=(100-l)$ and $R_1 \propto l$ and $R_2 \propto (100-l)$. Thus

$$\frac{X}{R} = \frac{l}{100-l} \quad \ldots \quad (26\cdot11)$$

whence
$$X = \frac{l}{100-l} R$$

This gives the value of the resistance to be measured.

26·12. End corrections for metre bridge. We have assumed above that in a metre bridge the resistance of the copper pieces and the soldering at the ends of the bridge wire are negligible, but *actually they are not so small as to be neglected.* Whenever the soldering is not done carefully, the solder spreads over the

bridge wire and the cross-section of the wire no longer remains uniform. The solder forms an alloy with the material of the wire, the specific resistance of which is not equal to that of the material of the wire. Further, the bridge wire may not be exactly one metre long and the scale may not be accurately placed with respect to the wire, either because the wire is slightly longer or shorter or because of the difficulty of making the connections between the ends of the wire and the strips exactly at the ends of the scale. The effect of all these factors is to introduce some ***unknown resistance*** at the two ends of the wire. Let these unknown end resistances at the left and right ends of the wire be respectively equivalent to α and β cm. of length of the bridge wire, then the *effective* ends of the wire will be at distances α and β respectively from the 0 and 100 cm. readings of the scale and hence the equation (26·11) will be modified to

$$\frac{X}{R}=\frac{l+\alpha}{100-l+\beta}$$

where α and β are *end corrections* for the bridge wire. If the end of the bridge wire is beyond the *extreme* graduation at that end, the end correction is *positive*; while if the extreme graduation is beyond the end of the wire, end correction may be positive or negative according as the total end resistance at that end is greater or less than the resistance of the length of the wire by which it falls short of the extreme graduation of the scale.

26·13. Interchanging Commutator. This commutator is very helpful in interchanging the resistances P and Q connected in the two gaps of a metre bridge. As shown in fig. 26·11, it consists of a circular ebonite disc 4 to 5 in. diameter, drilled and fitted with 8 terminals marked 1, 2, 3,......up to 8, *equally* spaced near its edge and

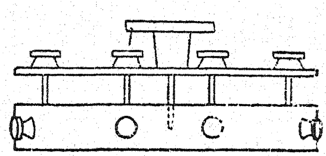


Fig. 26·11.

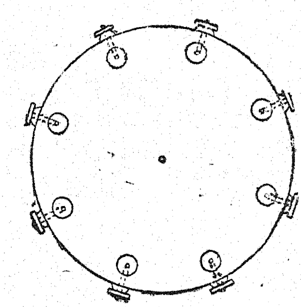


Fig. 26·12

projecting about 1/2 in. beneath the disc. Below the ebonite disc is a circular block of ebonite (Fig. 26·12) about 1 in. thick and of equal diameter. The ebonite block is drilled with 8 holes, about 3/4 in. deep corresponding in position with the terminals of the disc, and eight terminals screwed into the side of the block and marked 1, 2, 3...... up to 8, pass into the holes. The holes are filled with mercury which

secures contact between the block terminals and the upper disc terminals. The ebonite disc can be rotated about a screw passing through the centre of the disc into a hole drilled *centrally* into the ebonite block. By lifting the disc and turning it round different terminals can be made to fit into the holes in the ebonite block. The terminals on the ebonite disc marked 2 and 3, 1 and 4, 5 and 8, and 6 and 7 are connected together by means of thick copper wires (Fig. 26·13). One of the resistances to be interchanged is connected to the terminals of the ebonite block marked 1 and 2 and the other resistance between those marked 5 and 6. The terminals marked 3 and 4 are connected to one gap of the bridge and the terminals marked 7 and 8 to the other. A rotation of the ebonite disc through a *right angle* will affect the interchange of the resistances from one gap to the other.

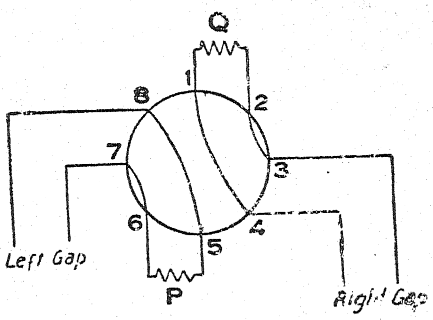


Fig. 26·13

Experiment 26·1

Object. To find *end corrections* for a metre bridge and to determine the *specific resistance* of the material of a given wire taking into account the end corrections for the bridge.

Apparatus. Metre bridge, two ordinary resistance boxes, one decimal-ohm box preferably *dial* pattern, a Leclanche cell, a weston galvanometer, wire the resistivity of whose material is to be determined, a shunt wire, connecting wires, a screw gauge and a metre scale.

Theory. Let l_1 cm. be the reading on the scale of the position of the *null point* on the bridge wire when resistances P and Q are connected in the left and right gaps of the metre bridge respectively. Then, if the *end corrections* for the left and right ends of the bridge wire are α and β cm. of length of the wire, we have

$$\frac{P}{Q}=\frac{l_1+\alpha}{100-l_1+\beta}$$

or $$Q\alpha-P\beta+Ql_1-P(100-l_1)=0 \qquad \ldots \qquad (26{\cdot}12)$$

Now let the positions of the resistances P and Q be *interchanged.* Then, if l_2 cm. be the reading on the scale of the position of the null point on the bridge wire, we have

$$\frac{Q}{P}=\frac{l_2+\alpha}{100-l_2+\beta}$$

or
$$P\alpha - Q\beta + Pl_2 - Q(100 - l_2) = 0 \quad \ldots \quad (26\cdot13)$$

Multiplying equ. (26·12) by Q and equ. (26·13) by P and then subtracting equ. (26·12) from equ. (26·13), we get

$$(P^2 - Q^2)\alpha + P^2 l_2 - Q^2 l_1 + PQ(l_2 - l_1) = 0$$

or
$$(P^2 - Q^2)\alpha + Pl_2(P+Q) - Ql_1(P+Q) = 0$$

whence
$$\alpha = \frac{Ql_1 - Pl_2}{P - Q} \quad \ldots \quad (26\cdot14)$$

Similarly
$$\beta = \frac{Pl_1 - Ql_2}{P - Q} - 100 \quad \ldots \quad (26\cdot15)$$

Interchanging Commutator—Plug type. This is depicted in fig. 26.13(a). It consists of brass blocks arranged as shown in the figure

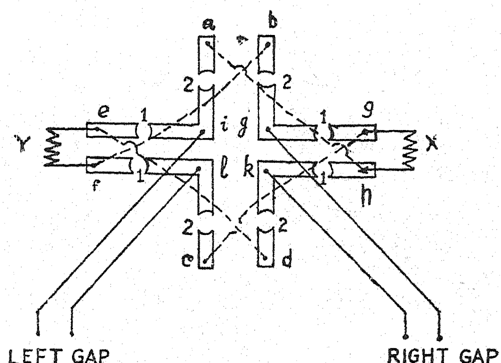


Fig. 26.14

with tapering holes for inserting the plugs; *a*, *b*, *c*,....*l* are the terminals for connexions. When in use, only eight of these will be connected. A resistance *X*, as shown, is connected across *h* and *g* while the other resistance *Y* across *e* and *f*. The terminals *i* and *l* are connected to the left gap of a carey Foster Bridge while *g* and *k* to the right gap.

With the plugs inserted in the blocks as shown by positions 1,1, 1, 1, the resistance *X* lies in the right gap while the resistance *Y* lies in the left gap. It can be seen that when the plugs are removed from the positions 1, 1, 1, 1, and inserted at the positions 2, 2, 2, 2, *X* is transferred to the left gap and *Y* to the right gap.

From these equations the values of end corrections α and β can be calculated, having determined l_1 and l_2 for known values of P and Q.

Now, if X be the resistance of the wire the specific resistance of whose material is to be determined

$$X=\rho\frac{L}{\pi r^2}$$

or
$$\rho=\frac{\pi r^2 X}{L} \quad \ldots \quad \ldots \quad (26{\cdot}16)$$

where L is the length of the wire and r its radius.

If the wire be connected in the left gap of the bridge and a known resistance R in the right gap and the reading on the scale of the position of null point on the bridge wire is l cm., we have

$$\frac{X}{R}=\frac{l+\alpha}{100-l+\beta}$$

or
$$X=\frac{l+\alpha}{100-l+\beta}R \quad \ldots \quad (26{\cdot}17)$$

The value of specific resistance ρ can be calculated from equation (26·16), having determined the value of X from equation (26·17).

Method. Make the connections as shown in fig. 26·14. Shunt the galvanometer with a low resistance wire. Introduce a resistance P=10 ohms, say in the left gap of the metre bridge and a resistance Q=1

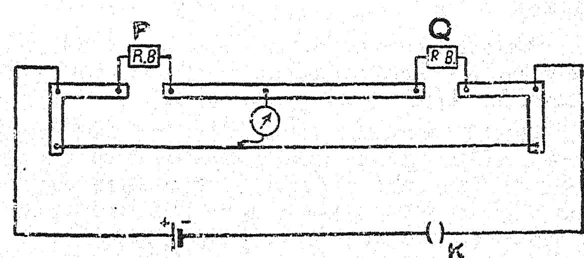


Fig. 26·15

ohm in the right gap by adjusting the respective resistance boxes. Close the cell circuit and press the jockey over the bridge wire at near its one end and note down the direction of deflection in the galvanometer. Move the jockey to near the other end of the wire and again note down the direction of deflection in the galvanometer. The directions of deflection in the two cases must be *opposite* otherwise there is a faulty contact or wrong connection in some part of the circuit.

Having tested the connections of the bridge, find by trial the approximate position of the null point. This will be near one end of the bridge wire. Then remove the shunt from the galvanometer so that it may have its full sensitivity possible and determine the accurate position l_1 of the null point.

Now interchange P and Q, and repeat the above determinations to find the value of l_2. This time the null point will be near the other end of the wire. Calculate the values of α and β from equations (26·14) and (26·15).

Keeping Q=1 ohm, alter the value of P to 15 ohms and 20 ohms. Calculate the values of α and β for each set of observations separately and then take their mean.

Now replace the resistance box in the left gap of the bridge by the wire the specific resistance of whose material is to be determined and that connected in the right gap by a decimal-ohm box.

Adjust the decimal-ohm box to a suitable resistance, say R=2 ohms. Close the battery circuit and with the galvanometer shunted with a low resistance wire, find out the position of the null point on the bridge wire and from it calculate roughly the value of the unknown resistance. Adjust the decimal-ohm box to this value and then find out the approximate position of the null point which will be in the 'middle third' of the bridge wire. Remove the shunt from the galvanometer and determine the exact position of the null point as accurately as possible with the full galvanometer sensitivity. Calculate the value of X, the unknown resistance, from equation (26·17). Take at least two more sets of observations by slightly decreasing and increasing the value of known resistance R in the right gap and calculate the value of X separately for each set of observation.

Next interchange the positions of X and R and again determine the value of unknown resistance, with the same values of known resistance R. Find the mean value of X.

Now measure the length L of the wire outside the binding screws. Then with a screw gauge measure the diameter of the *bare* wire at a number of points along its length and find out the mean radius. Finally calculate the value of specific resistance ρ of the material of the given wire from equation (26·16).

Sources of error and precautions. (1) A plug key should be included in the cell circuit which should be closed only when observations are being made. This prevents prolonged flow of current and hence consequent alteration in the values of the various resistances due to appreciable heating.

(2) The galvanometer should be shunted by a low resistance wire to avoid excessive deflection in it when the bridge is out of balance. The shunt must be removed when the null point has been almost obtained.

(3) The battery circuit should be closed before depressing the jockey on the bridge wire, but when breaking reverse order should be followed. Otherwise, if the resistance to be measured is inductive, *e.g.*, that of a coil of wire, a momentary kick will be produced in the galvanometer due to self-induced E. M. F. This momentary kick is obtained even if the exact balance exists and may sometimes be excessive enough to damage the galvanometer.

(4) The jockey should always be pressed gently so that good contact may be made with the bridge wire without flattening the jockey. The contact between the jockey and the bridge wire should not be made while the former is being moved along, otherwise the wire will be worn unevenly in various parts and hence will no longer remain uniform in cross-section.

For determination of end corrections only

(5) The resistances P and Q should differ as much as possible. If $P=Q$ and so $l_1=l_2$, the terms $\frac{Ql_1-Pl_2}{P-Q}$ and $\frac{Pl_1-Ql_2}{P-Q}$ of equations (26·14) and (26·15) will reduce to 0/0, *i.e.*, become *indeterminate* and so become the values of α and β. Hence the difference between P and Q should be as *large* as possible. Convenient values of P and Q are $Q=1$ ohm and $P=10$ to 20 ohms. With higher values of P/Q, the balance point may lie on the copper strip.

For determination of specific resistance only

(6) The null point should be as near the *middle* of the bridge wire as possible and hence the known resistance R should be of the same order of magnitude as unknown. *When the null point is in the middle of the wire, a small error in determining its position produces the least error in the value of the unknown resistance, i.e., the accuracy in the result is greatest.* For, if dl be a *small* error introduced in the determination of l and dX be the corresponding error produced in the result, we have, since

$$X=\frac{l}{L-l}R$$

$$dX=\frac{L.\,dl}{(L-l)^2}R$$

$$\therefore \quad \frac{dX}{X}=\frac{Ldl}{l(L-l)}$$

Now dX/X is minimum when $l(L-l)$ is maximum. But $l+(L-l)=L$, a constant. Hence the product $l(L-l)$ is maximum when $l=L-l$ or $l=L/2$.

(7) To eliminate error due to non-coincidence of the pointer of the jockey with the metallic edge on underside of it which comes in contact with the bridge wire, two separate measurements of the unknown resistance must be made, one by introducing the unknown resistance in one gap and known resistance in the other and the second by interchanging them.

(8) The length of only that portion of the wire which is *outside* the binding screws should be measured for specific resistance calculations. It is advisable to make sharp marks on the wire at the points where it *leaves* the binding screws *before it is unclamped.*

(9) The radius of the wire is a *small* quantity and occurs raised to the *second* power in the expression for ρ. Hence the diameter of the *bare* wire should be measured at a number of points along its length and at each point readings along two *mutually perpendicular* diameters should be taken.

Observations. [*A*] *Determination of* α *and* β.

S. No.	P ohms	Q ohms	Position of balance point on the scale with P in the Left gap l_1 cm.	Position of balance point on the scale with P in the Right gap l_2 cm.	α cm.	β cm.
1.						
2.						
3.						
				Mean		

[*B*] *Determination of unknown resistance X of the wire.*

S. No.	Known resistance R ohms	X in the left gap: Position of balance point l cm.	(100-l) cm.	X ohms.	X in the right gap: Position of balance point l' cm.	(100-l') cm.	X ohms
			Mean			Mean	

[*C*] *Constants of the wire.*

(*i*) Length of the wire = cm.

(*ii*) Measurement of diameter of the wire

Least count of the screw gauge = cm.

Zero error = cm.

S. No.	Reading along any diameter a cm.	Reading along *perpendicular* diameter b cm.	$\frac{a+b}{2}$ cm.
1.			
2.			
3.			
4.			
5.			
		Mean	

Corrected diameter of the wire = cm.

Calculations.

$$\alpha = \frac{Ql_1 - Pl_2}{P-Q}$$
$$= \quad \text{cm.}$$
$$\beta = \frac{Pl_1 - Ql_2}{P-Q} - 100$$
$$= \quad \text{cm.}$$

When X in left gap

$$X = \frac{l+\alpha}{100-l+\beta} . R$$
$$= \quad \text{ohms.}$$

When X in right gap

$$X = \frac{100-l'+\beta}{l'+\alpha} . R$$
$$= \quad \text{ohms.}$$

Mean value of resistance of wire = ohms.

Mean radius of the wire = cm.

$$\rho = \frac{\pi r^2 X}{l}$$
$$= \quad \text{ohms} \times \text{cm.}$$

Result. The *end corrections* for bridge wire are

left end α = cm.

right end β = cm.

and the *specific resistance* of the material (......) of the given wire = micro-ohms × cm.

Standard value = micro-ohms × cm.

Error = %

Criticism of the method. The method yields satisfactory results, but the accuracy is limited due to the length of

the bridge wire being one metre only, and can be increased by increasing the length of the bridge wire. While determining the end corrections for the bridge wire the balance points are obtained near the ends of the bridge wire and so the accuracy in the measurement of lengths of the ratio arms is not great and hence the values of α and β obtained are not very accurate. Further the bridge wire may not be of uniform cross-section throughout and as such it would be incorrect to take the resistances of the ratio arms proportional to their lengths. Lastly, unless the resistances of the various arms of the bridge are equal and made of the same material, they will be heated up differently and the consequent alteration in their values will not be the same.

26·14. Post Office Box. It is a compact form of Wheat stone bridge and was originally designed to measure the resistances of electric cables and telegraph wires. As shown in fig. 26·15, it consists of three separate sets of coils of known resistances arranged after fashion of a resistance box to form the three arms of a Wheatstone bridge, the fourth arm being the unknown resistance whose value is to be determined. The two sets of known resistance coils which are exactly *alike* and *similarly* placed form the two *ratio arms* P and Q of the bridge. Each ratio arm contains three coils of 10, 100 and 1000 ohms resistance. One advantage of three coils in each of the ratio arms is that we can choose the pair nearest in value to the resistance to be measured and another advantage is the extension of range. The third set of known resistance coils forms the *rheostat arm*, *i.e.*, the arm of variable known resistance R. In plug type arrangement, the rheostat arm consists of a series of coils of values from 1 to 5000 ohms to that R may be varied in steps of 1 ohm from 1 to 11,110ohms. In dial type arrangement shown in fig. 26·16, the rheostat arm consists of one dial of ten 1-ohm coils, another of ten 10-ohms coils and a third of ten 100-ohms coils. Sometimes a fourth dial of ten 1000-ohms coils is also provided. At the ends of each of the three arms P, Q and R of the bridge are provided binding screws for connecting the unknown resistance, the battery and the galvanometer as shown. The apparatus is also provided with two spring keys, one included permanently in the battery circuit and another in the galvanometer circuit. Sometimes a special double key is provided by means of which

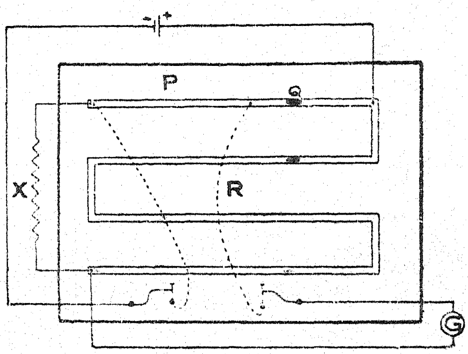


Fig 26·16

when the lever is depressed the battery circuit is first closed and on a further movement of the lever the galvanometer circuit also.

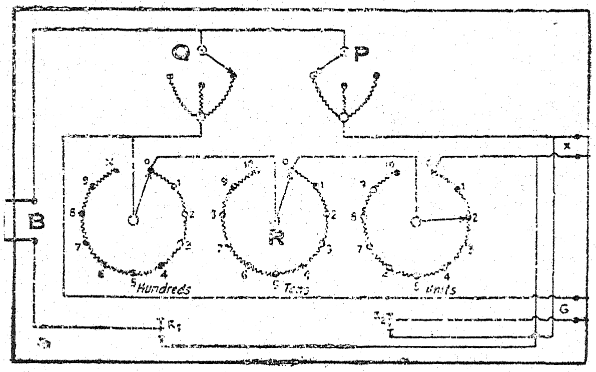


Fig. 26·17

To determine the value of an unknown resistance, the connections are made as shown in the fig. 26·15. The galvanometer which should be highly sensitive is heavily shunted and the ratio arms are adjusted to 10 ohms each. The rheostat arm is set to 1 ohm. The battery key is pressed and then the galvanometer key and the direction of deflection in the galvanometer noted. Next the rheostat arm is set to 10000 ohms and again the direction of deflection noted. If this direction of deflection is *opposite* to the previous one, the unknown resistance X has some value between 1 and 10000 ohms. If, however, the two deflections are in the same direction, there is either some faulty contact or wrong connection, or the value of X is either less than 1 ohm or greater than 10000 ohms. When the unknown resistance does not lie between 1 and 10000 ohms, P must be made smaller than Q if $X < 1$ ohm and greater than Q if $X > 10000$ ohms.

When X lies between 1 and 10000 ohms both P and Q are kept equal to 10 ohms and by varying the resistance in the rheostat arm *systematically*, two resistances in the rheostat arm differing by 1 ohm, say n and $(n+1)$ ohms, with which the deflections in the galvanometer are in *opposite* directions are found out by trial. Now since Q=P, the unknown resistance X must lie between u and $(n+1)$ ohms.

Next to increase accuracy, Q is made equal to 100 ohms keeping P=10 ohms and the value of galvanometer shunt resistance is increased. Since Q=10P, the resistance R of the rheostat arm now required for a balance will be 10 times the unknown resistance. Hence the balance should now occur between $10\,n$ and $10\,(n+1)$ ohms. Intermediate resistances between these limits are then tried until the limits differ by 1 ohm, *i.e.*, two resistances differing by one ohm, say n_1 and (n_1+1) ohms are found by trial which give deflections in opposite directions. Then the unknown resistance X will

be between $n_1/10$ and $(n_1+1)/10$ ohms. It should be noted that with the ratio Q=10 P, the above two resistances in the rheostat arm can be found only if X is less than $\Sigma/10$, where Σ stands for sum of all the resistances in the rheostat arm.

Now to increase the accuracy still further Q is made equal to 1000 ohms keeping P=10 ohms and the galvanometer shunt is also removed. Since Q=100P, R=100X and hence the balance should now occur between $10n_1$ and $10(n_1+1)$ ohms. Intermediate resistances between these limits are tried and a resistance n_2 ohms in the rheostat arm is found out which produces no deflection in the galvanometer. Then the unknown resistance will be equal to $n_2/100$ ohms. If the rheostat arm cannot be adjusted to give the exact balance point, two resistances differing by one ohm in the rheostat arm are found by trial which give deflections in opposite directions and then the unknown resistance can be determined to the third place of decimals by the *method of proportional parts*. If, for example, 258 ohms give a deflection of 6 divisions in one direction and 259 ohms give a deflection of 8 divisions in opposite direction, a difference of 1 ohm in R causes a change of 14 divisions. Hence a resistance of $(258+6/14)$ ohms will produce a balance and therefore the value of the unknown resistance is 2·584 ohms. But there is hardly any justification in going to third place of decimal for such a high degree of accuracy cannot be claimed from a post office box. It should be noted that with Q=100 P, the balance can be obtained only if X is less than $\Sigma/100$, where Σ stands for the sum of all resistances in the rheostat arm

When the unknown resistance lies between 0·1 and 1 ohm, ratios Q=100 ohms, P=10 ohms and Q=1000 ohms, P=10 ohms should be tried while if X lies between 0·1 and 0·01, balance point can be obtained with Q=1000 ohms only.

When the unknown resistance is greater than Σ but less than $10\,\Sigma$, ratios Q=10, P=100 and Q=10, P=1000 can be used and the value of X determined to the nearest 10 ohms. If the unknown resistance is greater than $10\,\Sigma$ but less than $100\,\Sigma$, its value to the nearest 100 ohms can be determined by taking Q=10 and P=1000 ohms.

When $X > 100\,\Sigma$ or $< 0{\cdot}01$ ohms, its value cannot be determined with the post office box and hence it is not suitable for the measurement of very low or high resistances.

Sometimes it is found that when a change from a lower to a higher ratio is made, the resistance required in the rheostat arm for a balance does not lie between the limits expected from the results of the observations with the lower ratio. This may be due to contact resistance or to the inaccuracy in the value of some of the coils. In plug type of post office box contact resistances can be sufficiently reduced by tightening the plugs in the sockets but they cannot be avoided entirely. In the case of inconsistency of limits, those obtained with *higher ratio* should be accepted.

26·15. Kelvin's Method of determining Galvanometer Resistance. The galvanometer, the resistance G of which is to be determined, is connected as an unknown resistance in one of the four arms of the Wheatstone bridge as shown in fig. 26·17. The cell is connected as usual in the branch AC and a key is placed instead of the galvanometer in the branch BD. Let the current distribution in the bridge be as shown in the figure. Then applying *Kirchhoff's second law* to each of the meshes CBAEC, CDAEC, CBD and BDA in turn, we get

$$(I-i_2)\,P+(I-i_g-i)Q+I\,r=E$$
$$i_g\,G+(i_g+i)\,R+Ir=E$$
$$(I-i_g)\,P+iK-i_gG=0$$
and
$$iK+(i_g+i)\,R-(I-i_g-i)\,Q=0$$

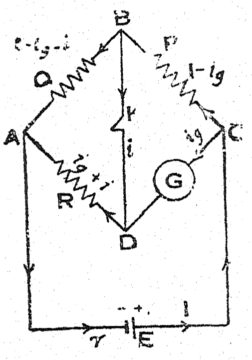


Fig. 26·18

Rearranging we get

$$I\,(P+Q+r)-iQ=E+i_g\,(P+Q) \quad (26{\cdot}18)$$
$$I\,r+iR=E-i_g\,(G+R) \quad (26{\cdot}19)$$
$$I\,P+iK=i_g\,(P+G) \quad (26{\cdot}20)$$
and
$$-IQ+i\,(K+R+Q)=-i_g\,(R+Q) \quad (26{\cdot}21)$$

Now let the current i_g in the galvanometer be *independent* of the resistance K in the branch BD, then it follows from equation (26·18) and (26·19) that I and i are also *constant.* When $K=\infty$, $i=0$, and therefore i is *always* equal to *zero.* Then putting $i=0$ in equations (26·20) and (26·21), we get

$$I\,P=i_g\,(P+G)$$
and
$$I\,Q=i_g\,(R+Q)$$
whence
$$\frac{P}{Q}=\frac{P+G}{R+Q}$$
or
$$\frac{P+G}{P}=\frac{R+Q}{Q}$$
or
$$\frac{G}{P}=\frac{R}{Q}$$

Thus when the values of resistances P, Q and R are so adjusted as to satisfy the above relation, the current in the galvanometer is the *same* whether the key in the branch BD is closed or open.

Experiment 26.2

Object. To determine the resistance of a moving coil (suspended type) galvanometer with *post office box* by *Kelvin's method.*

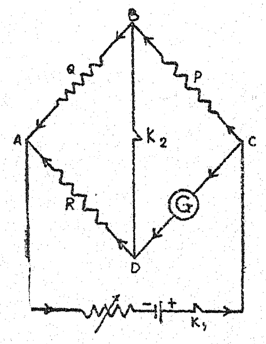
Apparatus. A post office box, Leclanche cell, a moving coil galvanometer whose resistance is to be determined, a variable high

resistance (a resistance box or a U-tube containing water), lamp and scale arrangement and connecting wires.

Theory. Let the galvanometer whose resistance is to be determined be connected in the 'unknown resistance arm' CD of the post office box, *short-circuiting* the *usual* 'galvanometer-arm' BD by a *short* and *thick* piece of copper wire. Then, if the cell be connected in its *usual* arm AC and the resistances of the ratio arms AB and BC and the 'rheostat arm' AD are so adjusted that the deflection of the galvanometer remains the *same* whether the 'usual galvanometer-key' K_2 is pressed or released, the bridge is *balanced* and from §26·15 the resistance of the galvanometer is given by

$$G = \frac{P}{Q} R \qquad \ldots \qquad (26{\cdot}22)$$

Method. Level the base of the galvanometer by means of the levelling screws and release its coil. Throw light on the mirror of the galvanometer and get the spot of reflected light on the scale. Connect the galvanometer in the arm of 'unknown resistance' CD of



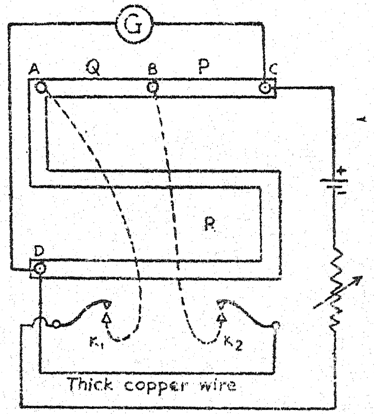


Fig. 26·19

the post office box. Join the *usual* galvanometer terminals B,D *in the post office box* by a short piece of *thick* copper wire and make the rest of the connections as shown in fig. 26·18 inserting *in the cell circuit a variable* high resistance, say a resistance box adjustable up to 10,000 ohms or a U-tube containing water, *in series* with the cell. See that the ends of the connecting wires are *clean* and that all connections have been *firmly* made. If the post office box is of plug type, see also that the plugs and the sockets are clean and the plugs are not loose.

Adjust the ratio arms P and Q to 100 ohms each. Press the battery key. The coil of galvanometer will be deflected and as the galvanometer is very sensitive the spot of light will move *off* the scale. Adjust variable high resistance placed in series with the cell to a high value so as to send only a *small* current through

the galvanometer and hence to produce a small deflection in it and thus bring the spot of light near the middle of the scale in its *deflected state*, *i.e.*, with the battery key K_1 *pressed.*

Adjust the 'rheostat arm' to a few ohms. Press the battery key, and when the deflected spot of light is *stationary* on the scale, press the second key K_2 also. Since the bridge is out of balance, pressing the key K_2 will send a current through the short piece of copper wire connected in the *usual* 'galvanometer arm'. This will alter the galvanometer current and hence will change its deflection. Note down the direction of movement of the *deflected* spot of light.

Next adjust the 'rheostat arm' to a very high resistance, say 10000 ohms and press the battery key. When the deflected spot of light on the scale is stationary, press the key K_2. The deflected spot of light on the scale will move in a direction *opposite* to that obtained in the previous case. Then repeat the operation by alternately adjusting the rheostat arm to a low and high value until two resistances are found, differing by one ohm, with which the directions of change of deflection in the galvanometer when the key K_2 is pressed are *opposite.* This determines the value of galvanometer resistance within one ohm.

If at any stage the arrangement becomes insensitive, *i.e.*, the resistance in the rheostat arm can be varied over a wide range without affecting the spot of light when the key K_2 is pressed, increase the current in the galvanometer by adjusting the value of high resistance placed in series with the cell. This will produce a *larger* deflection in the galvanometer and the arrangement will become more sensitive.

Now keeping $P=100$ ohms, make $Q=1000$. Since $Q=10P$, the resistance in the rheostat arm which will produce balance will now be 10 times that of the galvanometer resistance. Adjust the 'rheostat arm' between 10 times the limits obtained for galvanometer resistances with the *former* ratio and find by trial as above a resistance in the 'rheostat arm' which, with K_1 *pressed*, will produce no shift of the deflected spot of light on the scale when the key K_2 is pressed. The bridge will be *balanced.* Note down the resistance R in the 'rheostat arm' and calculate the galvanometer resistance from equ. (26·22) This gives the galvanometer resistance within one tenth of an ohm. If with $P/Q=100/1000$, the bridge is not exactly balanced, the galvanometer resistance can be computed by the *method of proportional parts* (§ 26·14).

Sources of error and precautions. (1) The base of the galvanometer should be carefully levelled and the coil should be unclamped. This ensures the *free* movement of the coil in the space between the magnet and the soft iron piece.

(2) The post office box should be preferably of *dial pattern* for in it the variable contact resistances are sufficiently reduced. In the case of plug type P. O. box all sockets must be clean and the plugs made tight.

(3) When the bridge is *out of balance*, the degree of alteration in the *steady* deflection in the galvanometer by tapping the *key* K_2 will depend upon the current which flows along the branch BD. Hence to ensure *sensitiveness* of the test of want of balance, the resistance of the branch BD should be made as *small* as possible. B and D should, therefore, be joined by a *short* piece of *thick* copper wire.

(4) To avoid unnecessary heating of the various coils and consequent alteration in their resistances, the battery key K_1 should be pressed only when observations are to be made.

(5) As the galvanometer is generally very sensitive, the deflection in it is very large and hence the spot of reflected light move *off* the scale when the battery key is closed. The deflection in the galvanometer can be decreased by sending in it a *small* current only. A variable high resistance, say a resistance box adjustable up to 10000 ohms should, therefore, be inserted in the cell circuit *in series with the cell* and the value of the resistance adjusted to get the spot of light on the scale in its *deflected* position. When the current through the galvanometer is *very small*, the spot of light can also be brought on the scale by turning back the coil by twisting the suspension strip by rotating the torsion head which carries it, but it is in general inadvisable to interfere with the suspension of the coil.

(6) In order that the bridge may be highly sensitive, the resistances of the four arms should be of the *same* order of magnitude. Hence, if the galvanometer resistance lies between 100 and 200 ohms, each of the ratio arms should be adjusted to 100 ohms.

(7) *The bridge is balanced when there is no change of steady deflection in the galvanometer whether the key K_2 is pressed or released.* Hence the battery key K_1 should be pressed first in order to produce steady deflection in the galvanometer ; and only when the spot of light on the scale is *stationary* with the key K_1 kept pressed that the key K_2 should be pressed to observe any *change* of direction and extent of the steady deflection.

Observations.

S. No.	Ratio arms P ohms	Ratio arms Q ohms	Rheostat arm R ohms	Direction of change of deflection when K_2 is pressed	Galvanometer resistance G
1	100	100	1	Left	Greater than 1 ohm
2	,,	,,	10000	Right	
3	,,	,,	156	Left	Lies between 156 and 157 ohms
4	,,	,,	157	Right	
5	100	1000	1563	Left	$G = 156 \cdot 4$ ohms
6	,,	,,	1564	No change	
7	,,	,,	1565	Right	

Calculations

$$G = \frac{P}{Q} R$$

$$= \frac{100}{1000} \times 1564$$

$$= 156{\cdot}4 \text{ ohms}$$

Result. The galvanometer resistance = ohms

Criticism of the method. When performed carefully, the method gives a fairly satisfactory value of the galvanometer resistance. The main difficulty in the method is that as the galvanometer is usually very sensitive, the steady current in it is large and as a result the spot of reflected light goes off the scale, and if the deflection in the galvanometer is reduced by placing a high resistance in series with the cell, the sensitiveness of the test of proper balance of the bridge decreases and the resistance in the rheostat arm can be varied over a wide range without changing the steady galvanometer deflection when the key K_2 is pressed. The result is also affected by thermo-electric and heating effects in the circuits.

The best method of determining the galvanometer resistance is to clamp its coil and connect it in the unknown resistance arm of the post office box and then to balance the bridge in the usual manner using *another* galvanometer as detector. Kelvin's method is more suitable for moving-magnet mirror galvanometer in which the deflection can be reduced without sacrificing the sensitivity by adjusting the position of the control magnet or if that is not sufficiently strong by an external bar magnet.

Exercise. *To determine the resistance of a weston galvanometer with metre bridge by Kelvin's method.*

Connect the weston galvanometer whose resistance is to be determined in the left gap of the metre bridge by *short* pieces of *thick* copper wire and connect in the right gap a resistance box. Connect a Leclanche cell with a variable high resistance in series with it in the *usual* position of the battery, *i.e.*, between A and C including a plug key in the cell circuit as shown in (fig. 26·19). Join B to the jockey by a *thick* copper wire. Adjust the resistance box in the right gap of the bridge to a suitable value, say 50 ohms. Close the cell circuit and adjust the variable high resistance to produce a convenient deflection in the galvanometer. Press the jockey over the bridge wire; the galvanometer deflection will, in general, change. Shift the jockey and determine by trial the position of the balance point D on the bridge wire such

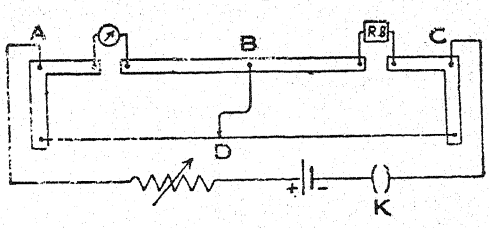


Fig. 26· o

that *when the jockey is pressed at that point there is no change of deflection in the galvanometer.* The balance point should be obtained as near the middle of the wire as possible by adjusting the resistance box in the right gap to a suitable value. Let l be the reading on the scale of the position of the balance point on the bridge wire and let R be the resistance in the right gap of the bridge. Then the resistance G of the galvanometer from § 26·15 is given by

$$G = \frac{l}{100-l} R$$

In this method the galvanometer remains *deflected* the *whole* of the time during the determination of the balance point and often becomes insensitive with the result that the point of contact of the jockey with the bridge wire can be shifted through an appreciable distance without making any noticeable change in the deflection. Consequently the accuracy obtainable by this method in the value of galvanometer resistance is not very great.

26·16. Mance's Method of determining Cell Resistance. The cell, the internal resistance of which is to be determined, forms one of the four arms of the Wheatstone bridge as shown in fig. 26·20. The galvanometer is connected as usual in the branch BD and a key is placed *instead of the cell* in the branch AC. Let the current distribution in the bridge be as shown in the figure. Then applying *Kirchhoff's second law* to the meshes CBD and DAB which do not include K, we have

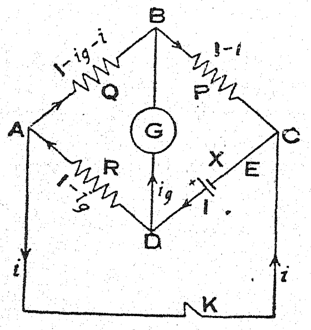


Fig. 26·21

$$IX + i_g G + (I-i) P = E$$

and $(I-i_g) R + (I-i_g-i) Q - i_g G = 0$

Rearranging, we get

$$I(P+X) + i_g G = E + i P \qquad \ldots \qquad (26{\cdot}23)$$

and $$I(Q+R) - i_g (R+Q+G) = iQ \qquad \ldots \qquad (26{\cdot}24)$$

Multiplying equ. (6·) by Q+R and equ. (6·4) (P+X) and then subtracting, we get

$$i_g [G(Q+R) + (R+Q+G)(P+X)] = E(Q+R) + i[P(Q+R) - Q(P+X)]$$

Now let the current i_g in the galvanometer be *independent* of the resistance K in the branch AC. Then the above expression for i_g must be independent of the current, i, for which since *i is not equal to zero,* we must have

$$[P(Q+R) - Q(P+X)] = 0$$

or
$$PR-QX=0$$

whence
$$\frac{X}{P}=\frac{R}{Q}$$

Thus when the values of resistances P, Q and R are so adjusted as to satisfy the above relation, *the current in the galvanometer is the same whether the key in the branch AC is closed or open.* This also follows from the fact that when the above relation is satisfied BD and AC become *conjugate conductors* so that the current in BD must be *independent* of the resistance of the *conjugate* conductor AC.

Experiment 26.3

Object. To determine the internal resistance of a Leclanche cell with *post office box* by *Mance's method.*

Apparatus. A post office box, Leclanche cell, a moving coil (suspended type) galvanometer with lamp and scale arrangement, variable high resistance (a resistance box or a U-tube containing water) a plug key and connecting wires.

Theory. Referring to fig. 26·21, let the cell whose internal resistance is to be determined be connected in the 'unknown resistance arm' CD of the P.O. box, *short-circuiting* the *usual* 'battery arm' AC by a *short* and *thick* piece of copper wire. Then, if the galvanometer be connected in its usual arm BD and the resistances of the ratio arms AB and BC and the 'rheostat arm' AD are so adjusted that the deflection of the galvanometer remains the *same* whether the usual battery key K_1 is pressed or released, the bridge is *balanced* and from § 26·16, the internal resistance of the cell is given by

$$X=\frac{P}{Q}R \qquad (26{\cdot}25)$$

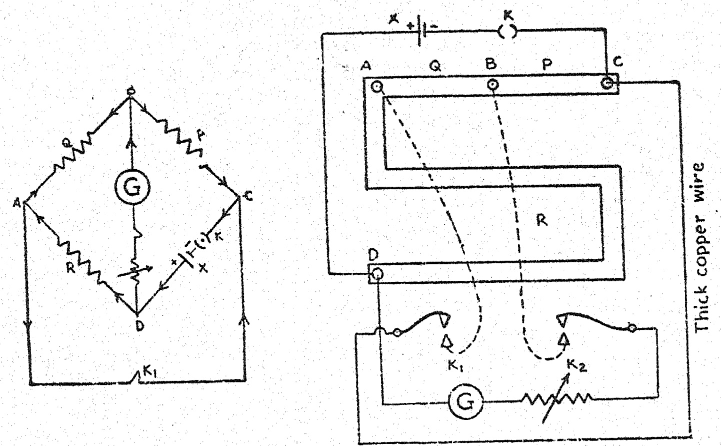


Fig. 26·22

Method. Level the base of the galvanometer by means of the

levelling screws and unclamp its coil. Throw light on the mirror of the galvanometer and get a bright spot of reflected light on the scale. Connect the Leclanche cell whose internal resistance is to be determined in the 'unknown resistance arm' CD of the post office box by *short* pieces of *thick* copper wire including a plug key K in the circuit. Join the usual cell terminals A, C by a *short* piece of *thick* copper wire and make the rest of the connections as shown in fig. 26·21 inserting in series with the *galvanometer*, a variable high resistance, say a resistance box adjustable upto 10,000 ohms or a U-tube containing water. Take care that the ends of the connecting wires are *clean* and that all connections are *tight*. In the case of a plug type post office box see also that the sockets are clean and the plugs are tight.

Adjust the ratio arms P and Q to 1 ohm *each*, close the key K included in the arm CD and the galvanometer key K_2. A current will flow through the galvanometer and its coil will be deflected. As a consequence the spot of reflected light on the scale will move. If the deflection in the galvanometer is *excessive*, the spot of light will move off the scale. In such a case adjust the variable resistance connected in series with the galvanometer to a *high* value so as to send only a *small* current through the galvanometer and hence to produce a small deflection in it and thus bring the spot of light near the middle of the scale in its *deflected* state, *i. e.*, with the keys K and K_2 pressed.

Next adjust the 'rheostat arm' to 1 ohm. Keeping the key K closed, press the galvanometer key K_2 and when the deflected spot of light becomes *stationary* on the scale, press the *usual* 'battery key' K_1 also. If the bridge is out of balance, this will alter the current through the galvanometer and hence will change its deflection. Note down the direction of movement of the *deflected* spot of light. Next adjust the 'rheostat arm' to a *high* value and again press the usual 'battery key' K_1, the keys K and K_2 being already closed. If the *deflected* spot of light now moves in a direction *opposite* to that in the previous case, the cell resistance is greater than 1 ohm. If, however, the deflected spot of light moves in the *same* direction, *the cell resistance is less than 1 ohm* or there is faulty or loose connection somewhere in which case all connections must be made tight and checked up again.

(1) **X>1 ohm.** If the cell resistance is greater than 1 ohm, keep the ratio arm resistances 1 ohm *each* and, after closing the keys K and K_2, find by trial two resistances, *differing by one ohm*, in the 'rheostat arm' with which the *deflected* spot of light on the scale moves in *opposite* directions when the usual 'battery key' K_1 is pressed. This determines the cell resistance within one ohm.

Now keeping P=1 ohm, make Q=10 ohms. Since Q=10 P, the resistance in the 'rheostat arm' which will produce balance will now be 10 times that of the cell resistance. Adjust the 'rheostat arm' between 10 times the limits obtained for cell resistance with the former ratio and find again by trial two resistances in the 'rheostat arm' *differing by one* ohm, which move the *deflected* spot of light in *opposite* directions on the scale when the key K_1 is pressed. This determines the cell resistance within 1/10 of an ohm.

Next keeping P=1 ohm, adjust Q=100 ohms, Since Q=100 P, the resistance in the 'rheostat arm' which will produce balance will now be 100 times that of the cell resistance. Thus find by trial a resistance in the 'rheostat arm', lying between 10 times the limits obtained for cell resistance with the ratio 1 : 10, with which there is *no shift* of the deflected spot of light on the scale when the usual 'battery key' K_1 is pressed. The bridge will then be balanced. Note down this resistance R in the 'rheostat arm' and calculate the cell resistance from equ. (26·25). This gives the cell resistance within 1/100 of an ohm.

(ii) X < 1 ohm. If the cell resistance is less than 1 ohm as is often the case with a *freshly* prepared cell, keep P=1 ohm and adjust Q=10 ohms so that P : Q= 1 : 10. Close the keys K and K_2 and by trying resistances from 1 to 10 ohms *only* in the 'rheostat arm' find two resistances, *differing by one ohm*, which move the deflected spot of light in *opposite* directions on the scale when the *usual* 'battery key' K_1 is pressed. This gives the cell resistance within 1/10 of an ohm.

Now keeping P=1 ohm, make Q=100 ohms so that P : Q= 1 : 100. Then keeping the keys K and K_2 closed, find by trial a resistance lying between 10 and 100 ohms in the 'rheostat arm' which produces *no shift* of the deflected spot of light on the scale when the usual 'battery key' K_1 is pressed. Note down this resistance R in the 'rheostat arm' and calculate the value of cell resistance from equ. (26·25). This gives the cell resistance within 1/100 of an ohm.

Sources of Error and Precautions. (1) The base of the galvanometer should be carefully levelled and the coil should be unclamped. This ensures the *free* movement of the coil in the space between the magnet and the soft iron piece.

(2) The post office box should be preferably of *dial pattern* for in it the variable contact resistances are sufficiently reduced. In the case of plug type P. O. box all sockets must be clean and the plugs made tight.

(3) X represents the resistance of the cell and its connecting wires in the 'unknown resistance arm' CD of the P. O. box. Hence in order that it may be equal to the cell resistance, the cell should be connected by *short* pieces of *thick* copper wire.

(4) A plug key K should be included in the arm CD along with the cell and should be closed only when observations are being made. This would prevent polarisation in the cell as well as unnecessary heating of the various coils and consequent alteration in their resistances.

(5) During the course of the experiment the zinc rod of the Leclanche cell should *never* be taken out or the cell otherwise *disturbed* in any way, for this would alter the internal resistance of the cell. The starting or stopping of current from the cell should be accomplished by the use of the key K only.

(6) When the bridge is *out of balance*, the degree of alteration in the *steady* deflection of the galvanometer by tapping the key K_1 will depend upon the current which flows along the branch AC. Hence to ensure *sensitiveness* of the test of want of balance, the resistance of the branch AC should be made as *small* as possible. A and C should, therefore, be joined by a *short* piece of *thick* copper wire.

(7) As the galvanometer is generally very sensitive, the deflection in it is very large and hence the spot of reflected light moves off the scale when the galvanometer key K_2 is closed. To overcome this difficulty a variable high resistance should be connected *in series with the galvanometer* and its value adjusted to get the spot of light on the scale with the key K_2 kept pressed.

(8) In order that the bridge may be highly sensitive, the resistance of the four arms should be of the *same* order of magnitude. Hence each of the ratio arms should be adjusted to 1 ohm.

(9) *The bridge is balanced when there is no change of steady deflection in the galvanometer whether the usual battery key* K_1 *is pressed or released.* Hence after closing the keys K and K_2, *it is only when the spot of light on the scale becomes stationary* that the key K_1 should be pressed to observe any change of direction and extent of the steady deflection.

(10) While testing the balancing of the bridge by pressing the key K_1, it is often observed in the case of a Leclanche cell that the spot of light first moves with a jerk in one direction and then after a moment begins to move *gradually* in the *opposite* direction on account of polarisation of the cell. In such a case the direction in which the spot of light moves with a jerk *immediately after* the key K_1 is pressed is the correct direction.

Observations.

If X $\gg$ 1 ohm.

S. No.	Ratio arms		Rheostat arm	Direction of *change* of deflection when K_1 is pressed	Cell resistance
	P ohms	Q ohms	R ohms		X
1	1	1	1	Left	Greater than 1 ohm
2	,,	,,	100	Right	
3	,,	,,	2	Left	Lies between 2 and 3 ohms
4	,,	,,	3	Right	
5	1	10	21	Left	Lies between 2·1 and 2·2 ohms
6	,,	,,	22	Right	
7	1	100	215	Left	X=2·16 ohms
8	,,	,,	216	No change	
9	,,	,,	217	Right	

If $X \nless 1$ ohm.

S. No.	Ratio arms		Rheostat arm	Direction of *change* of deflection when K_1 is pressed	Cell resistance
	P ohms	Q ohms	R ohms		X
1	1	1	1	Right	Less than 1 ohm
2	,,	,,	100	Right	
3	1	10	7	Left	Lies between 0·7 and 0·8 ohms
4	,,	,,	8	Right	
5	1	100	76	Left	$X = 0{\cdot}77$ ohms
6	,,	,,	77	No change	
7	,,	,,	78	Right	

Calculations.

$$X = \frac{P}{Q} R$$

=

= ohms

Result. The internal resistance of the given Leclanche cell = ohms.

Criticism of the Method. The method is not very suitable for the determination of the internal resistance of a Leclanche cell owing to polarisation effects and the variation of internal resistance with the current. When the current in the galvanometer is reduced, by adjusting the variable high resistance in series with it, to get the deflected spot of light on the scale, the galvanometer often becomes insensitive. The difficulty can, however, be overcome by the use, of a moving magnet mirror galvanometer or by connecting a condenser in series with the galvanometer as suggested by Lodge.

26·17. Lodge's Modification of Mance's method. Referring to fig. 26·2, in this method a condenser of about 1/2 microfarad capacitance is placed in series with the galvanometer which should preferably have high resistance. The condenser prevents any steady current to flow through the galvanometer. If there is any change of P.D. between B and D due to flow of current in the branch AK_2C when the key K_2 is pressed, the charge on the condenser alters and this produces a momentary kick or throw in the galvanometer. The resistance in the various arms of the bridge are so arranged that there is no kick in the

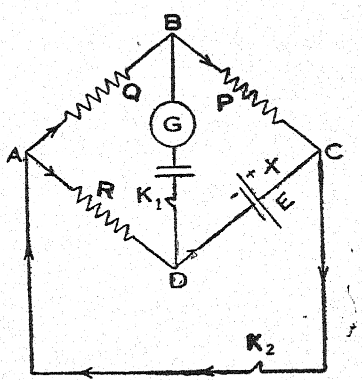


Fig. 26.23

galvanometer when the key K_2 is closed. Then the bridge is balanced and the usual Wheatstone bridge relationship holds. By the use of the condenser the balance point can be accurately determined on account of the now increased sensitivity of the arrangement, and all chances of damaging the galvanometer by the passage of large currents have been avoided.

Exercise. *To determine the internal resistance of a Leclanche cell with metre bridge by Mance's method.*

As shown in fig. 26·23, connect the cell whose internal resistance is to be determined in the *left* gap of the metre bridge by means of *short* pieces of *thick* copper wire, including a plug key K also in the circuit, and in the right gap of the bridge, connect a decimal-ohm box. Connect a weston galvanometer, with a *variable* high resistance in series with it, in the *usual* position of the *cell, i. e.,* between A and C, and join B to the jockey by a *thick* copper wire. See that the end of the connecting wires are *clean* and that all connections have been *firmly* made.

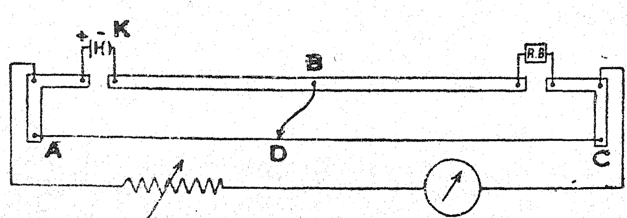


Fig. 26·24

Adjust the decimal-ohm box to a suitable value, say R=2 ohms. Close the cell key K. A current will flow through the galvanometer and hence its needle will be deflected. If the deflection in the galvanometer is *excessive*, reduce it by adjusting the variable high resistance placed in series with the galvanometer. Then press the jockey over the bridge wire. The current in the galvanometer will, in general, be altered and hence the deflection in it will change. Note down whether the deflection increases or decreases. Shift the jockey to another point on the bridge wire and find by trial the position of the balance point D on the bridge wire such that when the jockey is pressed at that point, there is no change of deflection in the galvanometer. Note down the reading l on the scale of the position of the balance point on the bridge wire and calculate the cell resistance X from the formula

$$X = \frac{l}{100 - l}.R$$

Next adjust the decimal ohm box to the above calculated value of the cell resistance. Shift the jockey to near the *middle* of the bridge wire and find as accurately as possible the position of the balance point on the wire. Note down the reading on the scale of the position of *this* balance point on the wire and calculate the value of the cell resistance. Then slightly decrease or increase the

value of the known resistance R and take at least two more sets of observations and calculate the value of cell resistance *separately* for *each* set of observation and take the *mean*.

If with a particular value of known resistance R in the right gap, no balance point is obtained over the entire bridge wire, either there is a faulty contact or wrong connection somewhere in the circuit or the balance point lies on the end strip. In the latter case the known resistance R in the right gap is either very much smaller or greater than the cell resistance.

This method of determining cell resistance is not capable of giving very accurate result, firstly because the internal resistance of the cell varies with the current drawn from it and, secondly, on account of polarisation effects, there is a small change of steady deflection of the galvanometer when the jockey is pressed even if the conjugate relation holds. Another difficulty is that the galvanometer often becomes insensitive. This difficulty can, however, be overcome by using a moving magnet mirror galvanometer in which case the deflection in the galvanometer can be reduced without affecting its sensitivity by adjusting the position of the control magnet. The other sources of error which affect the result are the thermo-electric effects, heating effects, the non-uniformity of the bridge wire, the unknown resistances at the ends of the wire and the non-coincidence of the point of contact of jockey with the pointer on it.

26·18. Carey Foster's Bridge. This is a modification of the metre bridge in which *effective* length of the bridge wire has been considerably increased without actually increasing the normal length of

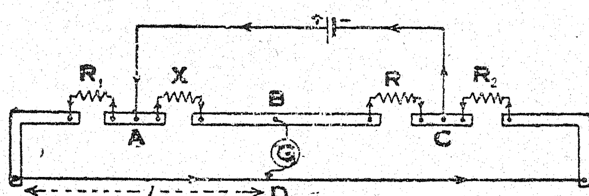


Fig. 26·25

the wire, by connecting two resistances R_1 and R_2 in *series* with the bridge wire, one at *each* end as shown in fig. 26·24. *This increases the accuracy of the result due to any error in reading the position of the balance. Joint on the scale as well as the sensitiveness of the bridge.*

If dX be the small error produced in the value of the unknown resistance X due to a small error dl of reading l, the relative inaccuracy in the result is (prec. 6 expt 26·3) given by

$$\frac{dX}{X}=\frac{L\,dl}{l(L-l)}=\frac{dl}{(1-l/L)l} \quad \ldots \quad (26\cdot26)$$

dX/X is minimum when $l=L/2$. Therefore the *least inaccuracy* in the result will be given by

$$\frac{dX}{X}=\frac{4}{L}\,dl=\frac{2}{5L}$$

since $dl=1$ mm., the *smallest* division on the scale. For *metre bridge* $L=100$ cm., and hence

$$\frac{dX}{X}=\frac{1}{250}$$

For *Carey Foster's bridge* the length is *virtually increased.* Let the resistances in the two outer gaps be equivalent to n times the length of the bridge wire. Then

$$L=100n+100=100\,(n+1)$$

and

$$\frac{dX}{X}=\frac{2}{5L}=\frac{1}{250\,(n+1)}$$

For $n=1$, *i. e.*, when L is virtually increased to *twice* the length the bridge wire

$$\frac{dX}{X}=\frac{1}{500}$$

For $=2$,

$$\frac{dX}{X}=\frac{1}{750}$$

and so on.

Thus *the accuracy in the result increases as* L *increases, being directly proportional to* L.

Now, if dl be the small shift in the position of the balance point on the bridge wire due to a *small* change dR of the *known* resistance R when the bridge is *balanced*, then dl/dR will measure the *sensitiveness* of the bridge. When the bridge is balanced

$$\frac{X}{R}=\frac{l}{L-l}$$

or

$$R=\frac{L-l}{l}\,X=\left(\frac{L}{l}-1\right)X$$

$\therefore$

$$dR=-\frac{L\,dl}{l^2}\,X$$

whence

$$\frac{dl}{dR}=-\frac{l^2}{LX}$$

Let $l=L/n$ where n is any number *greater* than unity. Then

$$\frac{dl}{dR}=-\frac{L}{n^2X}$$

This shows that sensitiveness for the bridge varies directly as the length of the bridge wire, being greater the greater length of the wire. For balance point in the middle of the wire, $n=2$ and then $dl/dR=-L/4X$. The —ve sign

in the above equation shows that when the value of R is increased by dR the value of l will decrease by an amount dl.

26·19. Comparison of two nearly equal resistances with a Carey Foster's Bridge. Carey Foster's bridge is specially suited for the comparison of two *nearly equal* resistances *whose difference is less than the resistance of the bridge wire.* As shown in fig. 26·25, two resistances X and Y to be compared are connected in the *outer* gaps of

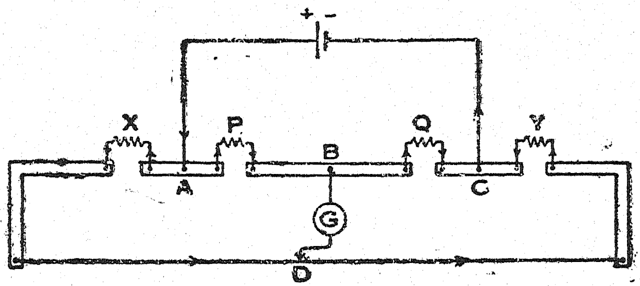


Fig. 26·26

the bridge *in series* with the bridge wire. These two resistances together with the bridge wire form the two arms of the Wheatstone bridge, one composed of X plus a length of the bridge wire up to the balance point and a second composed of Y plus the rest of the bridge wire. The remaining two arms are formed by two *nearly equal* resistances P and Q which are connected in the *inner* gaps of the bridge. If l_1 be the reading on the scale of the position of the null point, we have, from usual Wheatstone bridge formula

$$\frac{P}{Q}=\frac{X+\sigma\,(l_1+\alpha)}{Y+\sigma\,(100-l_1+\beta)}$$

or
$$\frac{P}{Q}+1=\frac{X+Y+\sigma\,(100+\alpha+\beta)}{Y+\sigma\,(100-l_1+\beta)} \quad \ldots \quad (26{\cdot}27)$$

where α and β units of length of the bridge wire are the *end corrections* at the left and right ends of the bridge wire respectively, and σ is the *resistance per unit length* of the bridge wire.

If now X and Y are *interchanged* and l_2 be the reading on the scale of the position of the *new* null point, we have

$$\frac{P}{Q}=\frac{Y+\sigma\,(l_2+\alpha)}{X+\sigma\,(100-l_2+\beta)}$$

$$\frac{P}{Q}+1=\frac{X+Y+\sigma\,(100+\alpha+\beta)}{X+\sigma(100-l_2+\beta)} \quad \ldots \quad (26{\cdot}28)$$

Comparing equations (26·27) and (26·28) we see that the fractions on the right hand side are equal and since their numerators are identical, their denominators must also be equal. Hence equating the denominators of the right hand sides of equations (26·27) and (26·28), we have

$$Y+\rho(100-l_1+\beta)=X+\rho(100-l_2+\beta)$$

whence
$$X-Y=\sigma\ (l_2-l_1) \qquad (26{\cdot}29)$$

Thus *the difference between the resistances X and Y can be obtained by determining the resistance of the bridge wire between the two null points.*

Experiment 26.4

Object. To determine the *resistance per unit length* of a *Carey Foster's bridge* wire and then to compare the resistance of a given one-ohm coil with a *standard* one-ohm resistance.

Apparatus. A Carey Foster's bridge, a Leclanche cell, weston galvanometer, a 1-ohm coil, a decimal-ohm box preferably dial pattern, sliding rheostat of small resistance, a single way plug key, thick copper strips, a shunt wire and connecting wires.

Theory. Let two resistances P and Q of *nearly equal* values be connected in the *inner* gaps of a *Carey Foster's bridge* and let a known resistance R be connected in the *outer* left gap of the bridge. Then, if a *thick* copper strip be connected in the *outer* right gap of the bridge and l_1 and l_2 be the readings on the scale of the positions of the null point on the bridge wire before and after interchanging the known resistance R and the thick copper strip in the outer gaps, we have from equ. (26·29) by putting X=R and Y=0

$$R=\sigma\ (l_2-l_1)$$

$$\therefore \qquad \sigma=\frac{R}{l_2-l_1} \qquad (26{\cdot}30)$$

Now let the coil of *unknown* resistance X be connected in the *outer left* gap and a standard known resistance Y of *nearly the same* value in the *outer right* gap of the bridge. Then, if l_1' and l_2' be the readings on the scale of the positions of the null point before and after interchanging X and Y, we have, from equation (26·29)

$$X-Y=\sigma(l_2'-l_1')$$

whence
$$X=\sigma(l_2'-l_1')+Y \qquad (26{\cdot}31)$$

This equation can be used to calculate X, if σ is determined from equation (26·30).

Method. (*a*) **Determination of σ.** Connect a decimal-ohm

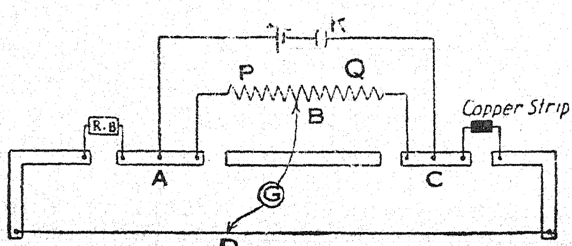


Fig. 26·27

box in the *left* gap of a Carey Foster's bridge and a *thick* copper strip in its outer *right* gap as shown in fig. 26·26. Next connect the two *fixed* terminals at the base of a sliding rheostat MN to A and C and its *sliding contact* B through the weston galvanometer to the jockey. Finally connect the Leclanche cell between A and C including a plug key K in the circuit.

Now adjust the *sliding contact* B of the rheostat MN in the *middle* so that the resistances P and Q introduced with its help in the *inner* gaps of the bridge may be *nearly equal*. Next adjust the decimal ohm box to a value R *slightly* less than the resistance of the bridge wire, say about 2 ohms. Then shunt the galvanometer and close the key K. Shift the jockey towards the left end of the bridge wire and find by trial the null point on the bridge wire, the exact position of the null point being determined with *full* galvanometer sensitivity by removing the shunt. Note down the reading l_1 on the scale of the position of the null point of the bridge wire.

Then interchange the two resistances in the *outer* gaps of the bridge and determine the value of l_2 as for l_1 above. This time the balance point will lie near the right end of the bridge wire. Alter the values of P and Q *slightly* by moving the sliding contact B of the rheostat MN and repeat the observations and determine the mean value of $(l_2—l_1)$. Next alter the value of R and as above, take at least *five* sets of observations for $(l_2—l_1)$ with different values of R, say 1, 1·2, 1·4, 1·6, 1·8 ohms. Calculate the value of σ for *each* set of observations *separately* from equation (26·30) and then find the *mean* value of ρ.

(*b*) **Comparison of resistances.** Next connect in place of the copper strip, the one ohm coil whose resistance X is to be compared with a standard one ohm resistance. Adjust the decimal-ohm box to Y=1 ohm. Shift the jockey to the middle of the bridge wire and find by trial the exact position of the null point l_1' on the bridge wire, the final adjustment being done without any shunt across the galvanometer.

Next interchange the resistances in the *outer* gaps of the bridge and shifting the jockey to the other side of the middle of the bridge wire determine as before the reading l_2' on the scale of the position of the null point on the bridge wire. Then alter the values of P and Q and repeat the observations for l_1' and l_2' at least *five* times. Lastly find the mean value of $(l_1'—l_1')$, and use it to calculate the value of resistance of the given one-ohm coil from equation (26·31).

Sources of error and precautions. (1) The ends of the connecting wires should be *clean* and all connections should be *firmly* made. The decimal-ohm box and the given one-ohm resistance coil should be connected by *thick* copper strips.

(2) A rheostat should be used to introduce the resistances P and Q in the *inner* gaps of the bridge and the *sliding* contact should be adjusted to be approximately in the *middle*. It is not absolutely necessary that P and Q should be exactly equal except for high sensitiveness of the bridge, nor should their values be known. If P=Q, the positions of null point before and after interchanging the resistances in the outer gaps will be at equal distances from the middle point of the bridge wire, provided, of course, the wire is uniform. If P and Q differ very much, it will not be possible to obtain the two positions of the null point on the bridge wire.

The use of rheostat to introduce P and Q in the inner gaps possesses several advantages. Besides being cheap, it is flexible, for it can be used to obtain the null point in any part of the bridge wire and also enables us to take several sets of readings for (l_2-l_1) for the same values of X and Y. With fixed values for P and Q this could not have been possible.

(3) In order that the bridge may have high sensitiveness, the resistances of the four arms should be of the same order.

(4) In order to reduce the inaccuracy in the result due to a small error in reading the position of the null point to minimum, the null points while comparing X and Y should lie as near the middle of the bridge wire as possible.

(5) While determining the value of ρ the value of R should be comparable with the resistance of the bridge wire so that the two positions of the null point before and after interchanging the resistances in the outer gaps lie near the ends of the bridge wire. The value of (l_2-l_1) will then be almost equal to the entire length of the bridge wire and the error in the value of σ due to non-uniformity of the bridge wire will be reduced to minimum.

(6) A plug key should be included in the cell circuit and should be closed when observations are being made.

(7) The galvanometer should be shunted by a low resistance wire to avoid excessive deflection in it when the bridge is out of balance. The exact position of the null point should be determined with full galvanometer sensitivity by removing the shunt wire from it.

(8) The cell circuit should be closed before depressing the jockey over the bridge wire, but when breaking, reverse order should be followed.

(9) The jockey should always be pressed gently and the contact between the jockey and the bridge wire should not be made while the jockey is being moved along.

Observations. (*A*) *Determination of* ρ.

S. No.	R ohms	Position of balance point with copper strip in the Right gap l_1 cm.	Left gap l_2 cm.	(l_2-l_1) cm.	Mean (l_2-l_1) cm.	ohms/cm.
1						
2						
					Mean	

(*B*) *Comparison of given one-ohm coil with standard one ohm*

S. No.	Position of balance point with the given one-ohm coil X in the Left gap l_1' cm.	Right gap l_2' cm.	$(l_2'-l_1')$ cm.	Resistance X of given one-ohm coil ohms
1				
2				
3				
		Mean		

Calculations.

$$\sigma = \frac{R}{l_2 - l_1}$$

$$= \quad \text{ohms/cm.}$$

$$X = \rho\,(l_2' - l_1') + Y, \qquad Y = 1 \text{ ohm}$$

$$= \quad \text{ohms}$$

Result. The resistance of the given one-ohm coil= ohms.

Percentage error =

Criticism of the method. The method is very suitable for the comparison of two *nearly equal* resistances and yields quite accurate results. In the derivation of the expression $X-Y=P\ (l_2-l_1)$ the *end corrections* have cancelled out and hence such unknown factors have no influence on the result. The sensitiveness of the bridge and the accuracy in the result have been increased considerably by virtually increasing the length of the bridge wire.

It is assumed that the bridge wire is uniform in cross-section. This may, however, not be the case in practice. Hence in all exact work the wire should be first calibrated and the value of resistance of the bridge wire between the two positions of the null point before and after interchanging X and Y obtained from the calibration curve. The result is also affected by heating of resistors and the bridge wire by the current and thermo-electric effects.

From the equation (26.31) it is evident that the difference between the two resistances to be compared is equal to the resistance of the bridge wire between the two positions of the null point. From this it follows that the method fails if this difference is greater than the total resistanee of the btidge wire for then the two positions of the null point cannot be obtained. Note that in this method of comparison of resistances, the two resistances are never com ared directly.

26. 0. **Calibration of Carey Foster's bridge wire.** A length of manganin or constantan wire is taken and its resistance measured with a Carey Fosters' bridge. Then a length of it 2 cms. in excess of bridge wire is cut off. If a wire of the same material and cross-section as the bridge wire is available, a piece of it 7 cms. in length can be cut off without measuring its rsistance. This is then soldered to two stout copper connecting strips as shown in fig. 26.27, the two cm. excess of the wire being soldered to the strips. Then the resistance X of the wire between the strips will be nearly equal to that of the bridge wire. This reistance X is connected in the outer left gap of the Carey Foster's bridge (Fig. 26.29) and a *thick* copper strip Y of practically zero resistance is connected in the outer right gap of the bridge. Theen the two fixed terminals at the base of a suitable rheostat are connected to the points A and C of the bridge, its *sliding* contact B being connected through a galvanometer to the jockey. The cell is connected in its usual place between A and C.

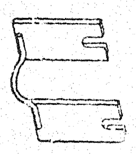


Fig. 26·28

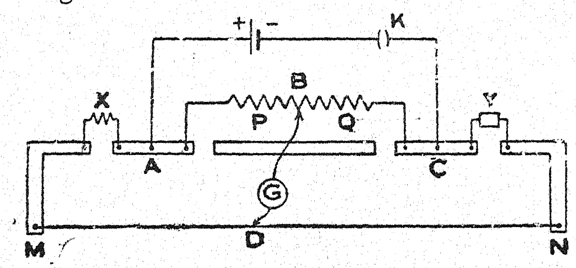


Fig. 26·29

The cell circuit is closed and the rheostat adjusted so that the null point lies as close to the left end M of the bridge wire as possible and the reading l_1 on the scale of its position on the bridge wire noted. X and Y are then interchanged and the reading l_2 on the scale of the position of the new null point determined. Then (l_2-l_1) is the first section of the wire whose resistance is equal to X. Next keeping the jockey *undisturbed* X and Y are placed in their *first* position and the rheostat adjusted again until the null point lies at the point of contact of jockey on the bridge wire, *i.e.*, at l_2. Then X and Y are again interchanged and the process repeated until the vight end N of the bridge wire is reached, noting down the readings on the scale of the positions of the successive null points on the bridge wire.

Then successive differences $x_1, x_2, x_3, \ldots x_{20}$ between the reading l_1, l_2, l_3, etc., on the scale of the position of the successive null points are found out. Let their *mean* value be x. Each successive difference x_1, x_2, etc., is then subtracted from this mean value x which gives the error over each section. To find out the correction which is to be applied at a point, say P distant l_1 from the left end of the bridge wire, these errors are added up *algebraically* from the left end of the bridge wire to the point P. Next a graph is plotted taking the lengths of the wire as abscissae and the corrections at the 21 points including the two ends at which the corrections are supposed to be zero, as ordinates. The graph will be the *calibration curve* of the bridge wire and from it the correction at any point on the bridge wire may be found out. Now, if the *end corrections* α and β with their sign *reversed* are also plotted on the same graph and the points corresponding to them joined by a straight line, the total correction at any point of the bridge wire can then be read off on the curve with reference to this line.

26·21. Effect of temperature and other factors on resistance. The resistance of a conductor also depends upon the purity and density of its material, drawing and annealing of the material and its temperature. The presence of even a slight trace of another metal has an enormous effect on the resistance, and hence copper used for electrical purposes must be exceptionally pure. The resistivity of an alloy is much greater than that of any of its constituent metals. This is a characteristic property of alloys and is used in the preparation of wires of high resistivity. Annealing diminishes the resistance of metals but gives it permanence, and drawing increases the resistance.

The. resistance of metals is also affected by magnetic fields but the effect in most cases is very small. In the case of bismuth the resistance can be doubled if a strong magnetic field is applied perpendicular to the direction of the current. This remarkable property of bismuth can be used to measure strengths of magnetic fields. The resistance of solid crystals, *e. g.*, of selenium, tellurium and carbon are affected by light or X-rays. On account of this property of selenium, its cells are used in talking films and telephony by means of light beam. In the case of the single cystals not belonging to the regular system, *e.g.*, tin (tetragonal) the resistivity changes with direction also.

The resistivity and hence the resistance varies with temperature. For pure metals the resistance increases with rise of temperature. If R be the resistance of a metallic conductor at t°C and R_0 that at 0°C, then $R=R_0\ (1+\alpha t)$, where α is called the *temperature coefficient of resistance* for the material of the conductor and is numerically equal to the increase in unit resistance at 0°C for 1°C rise in temperature. Here the original resistance has been taken at 0°C, but in the general case it can be at any temperature and then *the temperature coefficient is numerically equal to the fractional increase in resistance per degree C rise in temperature.* The value of α for common metals (except mercury) is about 1/273 or $3{\cdot}76 \times 10^{-3}$ per °C, *i.e.*, the same as the volume or pressure co-efficient for gases, and hence *the resistivity for pure metals is almost directly proportional to their absolute temperature.* The value of α for iron is very high and increases suddenly at red heat to an abnormally high value. On account of this property of iron, it is used for automatic regulation of current in Nernst lamps. The presence of an impurity decreases the value of α but the product of temperature coefficient and the specific resistance at any temperature is the same for the pure and the impure metal.

The simple expression $R=R_0\ (1+\alpha t)$ for variation of resistance with temperature in the case of pure metals is true for *small* ranges of tempereture only, for the graph between R and t is not really a straight line but a *parabolic* curve. For large ranges of temperature or for accurate work, the variation of R with t should, therefore, be represented by more exact expression $R=R_0\ (1+\alpha\ t+\beta t^2)$, where β is a very small additional constant depending upon the material of the conductor. The variation of resistance with temperature is used to measure temperature. The metal usually employed is platinum, *e.g.*, in platinum resistance thermometer and bolometer. For lower temperature lead and gold are also employed.

As the resistivity of pure metals is almost proportional to their absolute temperature, it was surmised that the resistance of pure metals would vanish at absolute zero (—273°C), but the recent researches of Kamerlingh Onnes, Dewar and others have shown that the above rule changes rapidly near absolute zero for the resistance of many pure metals drops rather suddenly near absolute zero to a value too small to be measurable. *The state at which the metal offers no measurable resistance to the passage of current is called* **superconducting state.**

The resistance of alloys also increases with temperature but the temperature coefficient is much smaller in alloys than in pure metals, especially in alloys containing nickel and manganese in certain proportion, *e. g.*, constant, manganin, etc. The temperature coefficient of manganin becomes *negative* temperature increases, the point at which the change occurs depending upon the iron content. On account of *negligible temperature coefficient* and *high resistivity*, manganin and constantan are chiefly used in the construction of standard resistances.

The resistance of non-metallic conductors, *e. g.*, carbon, silicon, tellurium, etc., electrolytes, and insulators like glass, marble, slate, etc.

which become partial conductors at high temperatures, *decreases* with rise of temperature, *i.e.*, they have a *negative* temperature coefficient. The resistance of a carbon filament lamp at room temperature is about 1·6 to 2·4 times its resistance when heated under full voltage. The resistance of electrolytes decreases by about 2·5 per cent per degree near room temperature. The non-metallic element boron has an abnormally high negative temperature coefficient. Very pure graphite, however, has a *positive* temperature coefficient.

26·23. Standard Resistances, Resistance Boxes and Rheostats. For many practical purposes, *e.g.*, measurement or comparison of resistances, standard resistances are required. The mercury standard described in § 26·2 is not convenient for general use in the laboratory and hence standard resistances consisting of wires of suitable materials are constructed. The most important properties of such standard resistances should be—

(*a*) permanence, *i. e.*, no variation of the resistance value of the finished standard with time,

(*b*) robust construction and convenient size,

(*c*) negligible variation with temperature,

(*d*) very small thermo-electric effect,

(*e*) very low inductance and self-capacitance, and

(*f*) capacity for carrying appreciable current without over-heating.

A consideration of these requisites of standard resistances shows that the material to be used in their construction should be of *high resistivity* so that a reasonably compact standard of any given value with a convenient size can be made up without using either an abnormally great length or dangerously thin section. The *temperature coefficient of resistance* for the material should be *practically nil* and it should have a very small thermo-electric effect with copper. In addition it should not easily oxidise, should be unaffected by moisture, acids, etc. and should be easily worked and jointed.

These requirements are best satisfied by alloys manganin, therlo constantan, eureka, etc. The alloy manganin of the composition 84% Cu, 12% Mn, 3·5% Ni and 0·5% Fe has an extremely low temperature coefficient 0·000004 per °C at 20 °C, a very small thermo-electric E. M. F. against copper 3 to 8 micro-volts per °C, and a high resistivity 50 micro-ohms × cm. at 20 °C and is, therefore, used in high-grade standard resistances.

Properties of Resistance Materials

Material	Resistivity ρ micro-ohms × cm.	Temp. coefficient α per °C	Thermo-electric E.M.F. against copper micro-volts	Remarks
METALS				
Copper ...	1·7	0·0040		
Iron ...	9—15	0·0045 to 0·006		
Mercury ...	95	0·00092		
Platinum (pure) ...	10·8	0·0039		
Steel ...	15—50	0·0052 to 0·006		
ALLOYS				
Brass ...	7—9	0·0015		
German Silver (60 % Cu, 21% Ni, 19% Zn) ...	15—40	0·00022 to 0·0007	35	Zn produces unstable properties.
Nickelin (58% Cu, 41% Ni, 1% Mn) ...	42	0·00023		
Constantan (35—55 Ni, 65—45 % Cu, 0—20% Zn)	49—52	0·00001	40	Cheap, high thermo-electric E. M.F.
Eureka (60%, Cu 40%) ...	50	,,	40	
Platinum silver (1 part platinum, 2 parts silver) ...	31·6	0·0003	Small	High temp. coefficient.
Platinoid (German silver with addition of about 1% tungsten) ...	34—40	0·0003	20	High temp. coefficient. Tungsten improves permanence.
Manganin (70% Cu, 30 % Mn)	42	0 to 0·00003	Very low	High resistivity and very low temp. coefficient.
Therlo (71%Cu, 16·5% Al, 10·5 Mn, 2% Fe) ...	47	0·00005	Very low	Properties similar to manganin.
Nichrome ...	95	0·0004		Very high resistivity

Various forms of standard resistances have been designed. In one form a coil of manganin wire is wound *non-inductively* on a metal bobbin. In non-inductive winding the wire is first doubled on itself and then wound (Fig. 26·30) which gives the effect of two coils, side by side, carrying currents in *opposite* directions so that the magnetic flux in one coil neutralises the magnetic flux through the other. The *residual* flux is, therefore, *zero* or so small that the self inductance may be neglected. In this type of winding the *self-capacitance* is also *small*. This wire is always silk covered in order to save space and is insulated from the metal bobbin by a double layer of shellaced silk which is baked before the wire is wound on to remove moisture. The wire is laid in a single layer so that the heat produced by the current in it may be removed as efficiently as possible. After winding the coil is shellaced and baked at 140°C. This dries the coil as well as anneals the wire. Annealing of the material is also done during its manufacture. Annealing removes bending strains in the wire and hence ensures greater permanence of the resistance of the coil. The bobbin and the coil are enclosed in a cylindrical case with an ebonite cover, the space between the coil and the wall of the cylinder being filled with paraffin wax to prevent the absorption of moisture. The leads from the coil are two copper rods which are silver-soldered to the coil and serve as the terminals. Their ends are amalgamated, and when electric connections are made, they are dipped in mercury cups to establish good contact.

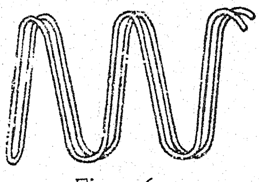


Fig. 26·30

In the construction of low resistances (Fig. 26·31) *strips* of suitable alloys are used instead of coils of wires, and since they are often required to carry large currents as is the case with those used for potentiometer work, adequate cooling must be provided which is done by immersing the resistance in first grade paraffin oil. This oil should be free from acid and water in order to avoid corrosion of the resistance alloy. The oil should be stirred by a motor-driven stirrer and cooled by some water-cooling arrangement. These low resistance standards used with large currents are provided with potential terminals as well as current terminals. The nominal value of the standard is measured between the potential terminals and the standard is connected to a circuit by the current terminals.

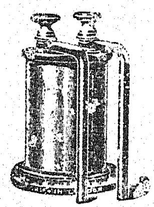


Fig. 26·31

For varying the resistance of a circuit in *steps* of known amount, **resistance boxes** are used. In the plug type arrangement resistance coils of various values are arranged in series like the weights in a weight-box by means of which any multiple of the unit of the box up to the sum of all the coils can be obtained by removing the appropriate plugs

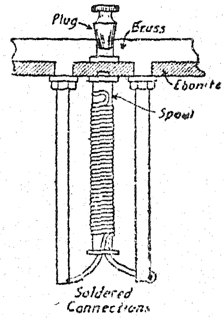


Fig. 26·32

whereby the corresponding coils will be put into the circuit. The values of coils are such as to give a particular resistance with the smallest number of coils. The series commonly adopted is 1, 2, 2, 5, 10, 20, 20, 50, up to 500 or 5,000. There is no coil between blocks separated by the gap marked 'infinity' so that by withdrawing the 'infinity plug' the circuit which includes the box is broken. Fig. 26·32 shows the method of mounting coils in such boxes. As the spacs available for each coil is the same, finer wires are used for large resistances. For accurate work it is necessary that the *contact resistances* in these boxes should be *small* and constant in value. This is achieved by making the plugs *tapering, i.e.*, conical in shape, and pressing them firmly in the sockets before any measurements are made. Further they should always be inserted (or withdrawn) with a screwing motion which ensures good contact and prevents the plug from fastening tight in the socket.

In the dial type arrangement there is a group of ten 1-ohm coils in series, a group of ten 10-ohm coils in series and so on. The coils in a group are arranged in a circle (Fig. 26·33) and the junctions between the *consecutive* coils are connected to brass studs fixed in circle over the ebonite cover of the box. By rotating a laminated copper brush or arm round each dial and making contact with the appropriate stud, any required number of units, tens and hundreds can be obtained. This pattern of resistance box has almost *negligible contact resistances* and hence is preferable to the plug type arrangement in which the contact resistances are appreciable and depend upon the pressure with which the plugs are inserted in the sockets.

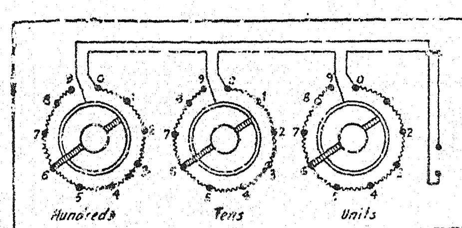


Fig. 26·33

When the resistance of a circuit is varied *continuously* instead of the steps, a **rheostat** is used. These rheostats are of various types, *e. g.*, sliding rheostat, rotatory rheostat, etc. In one form of sliding rheostat (Fig. 26·34) a wire of the suitable material such as constantan, manganin, etc., is wound on a cylinder of slate or enamelled

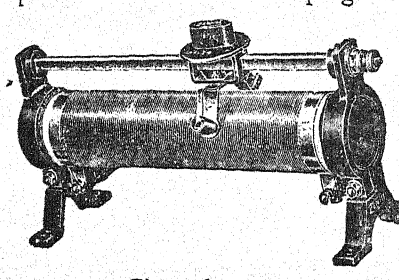


Fig. 26·34

iron so that its turns are close together but *not touching* each other. The two ends of the wire are connected to two binding screws at the two ends of the cylinder. A third binding screw is fixed to metal rod along which slides the metal slider which makes contact with the turns of wire. By connecting the binding screw fixed to the metal rod and one of the screws at the base and then sliding the metal slider, any desired part of the total resistance of the rheostat can be introduced in the circuit. For introducing the *whole* of the resistance, the two binding screws *at the base* may be used. The rheostat can also be used as a *potential divider*. For this the current is passed through the whole rheostat by using the screws at the base when there is a fall of potential from one end of the rheostat to the other, from which a portion may be tapped off by using one of the terminals at the base and the screw attached to the rod along which the metal slider moves.

In using the resistance boxes and the rheostats, care should be taken never to pass currents of values greater than the maximum which the coils in the boxes or rheostats can safely carry otherwise the coils or the wire in the rheostat will be over-heated and burnt.

Bulb or Lamp Resistances. When the current is taken directly from electric mains, the strength of the current in the circuit is regulated by lamp resistances. These lamp resistances consist of several lamps connected *in parallel* (Fig. 26·35) the combination being inserted in series with the circuit.

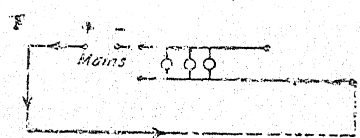


Fig. 26.35

The lamp resistances are very cheap and convenient.

26·24. Potentiometer. A potentiometer is a device for comparison or measurement of E. M. F. or P. D. by balancing the unknown against a variable potential difference the value of which may be known in terms of a standard of E. M. F. In its simplest form it consists of a long piece of *uniform wire* of fairly *high resistance* stretched over a scale of equal divisions. The ends of the wire are connected to a secondary cell whose function is to maintain a *perfectly steady* P. D. between the ends of the wire, which must always be greater than the E. M. F. or the P. D. to be measured.

To understand the principle of working of a potentiometer let us first consider the flow of water through a pipe AB in consequence of a difference of level or *hydrostatic pressure* between A and B, the higher pressure being at A. If the pipe AB be tapped at any two points between A and B, say at C and D (Fig. 26·36) and a *branch pipe* CPD attached, the current of water flowing through the pipe AC will now divide at C into two parts both flowing towards

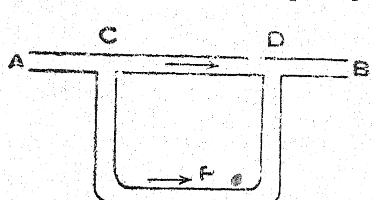


Fig. 26·36

D, one through the *main pipe* CD and the other through the *branch pipe* CPD, and *the greater the distance between C and D greater will be the pressure difference tending to urge the water through the branch pipe.*

Now let us consider the flow of a current of electricity along a conductor AB in consequence of a P. D. between A and B, the potential at A being *higher*. Then, if at any two points C and D between the ends of the conductor, a branch conductor CPD is connected to the conductor AB (Fig. 26·37), the current flowing along AC will now divide at C into two portions both flowing towards D, one along CD and the other along the new path CPD, and *the greater the distance between C and D greater will be the P. D. tending to urge the current along the branch conductor.*

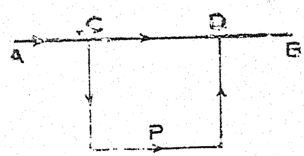


Fig. 26·37

Next referring again to fig. 26·36, let us suppose that something of the nature of a rotatory or force pump be inserted in the branch CPD (Fig. 26·38) and let it be driven at constant speed by some power and thus maintain a *steady* pressure difference between N and P. The pump may evidently be driven in either direction and will assist or oppose the flow of water through the branch pipe in the direction CPD according as the pressure at N is maintained lower or higher than that at P. Now let the pressure at N be higher than that at P. Then as the pump now opposes the flow of water through the branch pipe in the direction CPD, the current of water in the branch pipe will flow in the direction CPD or in the opposite direction according as the pressure difference between N and P is smaller or greater than that between C and D. If the current of water through the pipe A B be of such strength that the pressure difference between C and D is exactly balanced by the pressure difference between N and P maintained by the pump, then *the tendency for the current of water to flow in one direction is neutralised by the tendency to flow in the opposite direction and hence there will then be no flow of water through the branch pipe.* This absence of current of water in the branch pipe can be shown by some sort of flow indicating instrument inserted in the branch CPD.

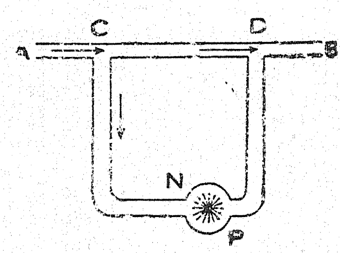


Fig. 26·38

Now referring to fig. 26·37, let us suppose that a cell of constant E. M. F. be inserted in the branch CPD (Fig. 26·39) and let N be the *positive* pole of the cell. Then, since the potential at N is higher

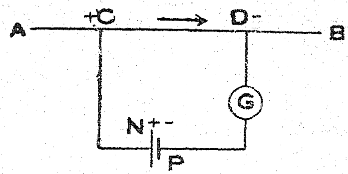


Fig. 26·39

than that at P, the current flowing through the branch conductor in consequence of the P.D. between C and D will be opposed by the P.D. between N and P, and evidently the current in the branch conductor will flow in the direction CPD or in the opposite direction according as the P. D. between N and P is smaller or greater than that between C and D. If there be no flow of current through the branch conductor CPD, *the tendency for the current to flow in the direction CPD is neutralised by the tendency to flow in the opposite direction,* and hence in that case, from the analogy of flow of water through a pipe as given above, we conclude that *the potential difference between C and D tending to urge the current in direction CPD is exactly balanced by the potential difference between N and P tending to urge the current in the opposite direction.* The absence of current in the branch conductor can be shown by a galvanometer inserted in the branch CPD.

Now let r be the resistance of the portion of the potentiometer wire between C and D and let i be the *steady* current flowing through it. Then the potential difference between C and D is equal to ir. When there is no deflection in the galvanometer, *the potential difference between C and D is exactly balanced by the potential difference between N and P.* Hence P. D. between N and P is given by

$$V=ir$$

But

$$r=\rho\frac{l}{S}$$

where l is the length of the portion of the wire between C and D, S its area of cross-section and ρ the *resistivity* of the material of the potentiometer wire.

Thus

$$V=\frac{i\rho l}{S}$$

If the wire be of *uniform* cross-section throughout and the current in the wire is of constant strength, $i\rho/S$ will be constant. Since ρ/S is the resistance of unit length of the wire, $i\rho/S$ will be the fall of potential per unit length of the wire, *i.e.*, the *potential gradient* of the potentiometer wire. Hence, if K be the *potential gradient* of the wire

$$V=i\rho l/S=Kl$$

When there is no current in the branch CPD, the cell is on *open circuit* so that the potential difference between N and P equals the E. M. F. of the cell. Thus when there is no deflection in the galvanometer

E. M. F. of the cell NP$=Kl=$potential gradient $\times$ length CD.

The potentiometer is the *most accurate* device for the comparison or measurement of E. M. F. or P. D. The accuracy of the instrument depends upon, (*i*) the *constancy* of the difference of potentials between its ends and (*ii*) the *uniformity* of cross-section of the wire. The accuracy in obtaining balance evidently depends upon the *sensitiveness* of the galvanometer inserted in the branch CPD. For a given potential difference between the ends of the potentiometer wire, *the sensitiveness of the potentiometer increases with the length of its wire*, for *the greater the length of the wire smaller will be the potential gradient* and hence greater will be the sensitiveness.

But it is very inconvenient to use a very long wire. Hence instead of one wire several parallel wires, connected in series are often used, the first and second wires being connected by copper strips fixed at one end of the board and the second and third similarly connected at the other end, and so on. But the use of many wires involves two difficulties, (*a*) the apparatus becomes cumbersome and (*b*) it is difficult to get a very long wire of absolutely *uniform* cross-section throughout. These difficulties are overcome by using a short wire connected in series with a number of resistance coils, the resistance of *each* coil being *equal* to that of the wire. A potentiometer of this type is shown in fig. 26·40.

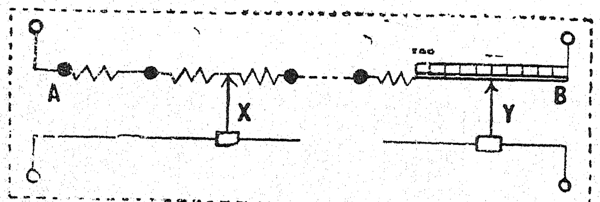


Fig. 26·40

It is very important that there shall be no appreciable thermo-electric E. M. F. within the potentiometer itself. Accordingly *manganin which has a very low thermo-electric E. M. F. with copper* is usually selected as the material for the construction of potentiometer coils and slide wire. *Manganin has also negligible temperature coefficient of resistance and high resistivity.* The contacts and joints in the potentiometer are often made of a special gold-silver alloy which is not affected by acids. To protect the contacts from atmosphere they are in some cases included within the case of the instrument. This also ensures a uniform temperature of all parts.

26·25. Crompton Potentiometer. It is a *compact* and *precision* type of potentiometer in which *the sensitivity of the instrument is considerably increased without sacrificing its accuracy.* As illustrated in fig. 26·41, it consists of a *uniform* manganin or constantan wire AB, of 50 cm. in length, connected in series with fourteen coils each of which has a resistance *equal* to that of the wire AB. By means of a rotating switch S any number of coils can be put in series with the wire AB. The main circuit battery is connected at M. In order to alter the sensitivity of the instrument, the main circuit is provided with two adjustable rheostats R_1 and R_2 for rough and fine adjustments.

The sources whose P. D. or E. M. Fs. are to be measured or compared are connected at E_1, E_2, E_3, etc., and can be successively

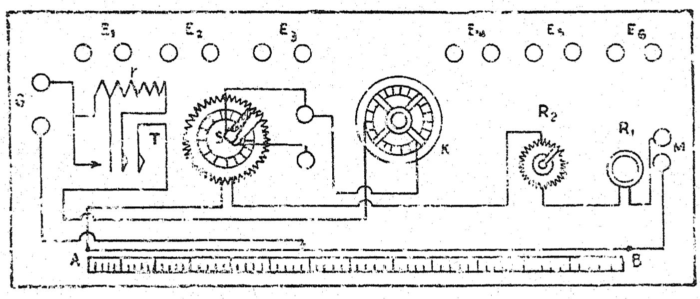


Fig. 26·41

brought into the circuit by rotating the switch K. The tapping key T brings the galvanometer G into the circuit through a resistance r which is reduced to zero while determining the exact balance point.

To calibrate the potentiometer a standard weston cadmium cell (E. M. F.$=1{\cdot}0183$ volts) is connected at E_1 and, brought into the circuit by rotating the switch K. By means of S, ten coils are switched on and the jockey is set at 18·3 divisions from the end A. Then the current in the main circuit is varied by means of the rheostats R_1 and R_2 until there is no deflection in the galvanometer *i.e.*, the exact balance point is obtained. This calibrates the potentiometer, each smaller division of the scale corresponding to one millivolt. Any P. D. or E. M. F. can now be measured by connecting it say, at E_2 and then obtaining the balance point.

26·26. Comparison of E. M. Fs. of two cells. As shown in fig. 26·42, the potentiometer wire is connected to a battery of constant E. M. F. in series with a suitable rheostat so that a *steady potential difference* is maintained between the ends of the wire. The +ve poles of the two cells whose E. M. Fs. E_1 and E_2 are to be compared are joined to the end A of the potentiometer wire which is at *higher* potential while their —ve poles are connected, through a two-way key, to a galvanometer, the latter being connected to a jockey which can be slid over the potentiometer wire. The two cells are connected in turn to the galvanometer by means of the two-way key and the point of contact of jockey with the potentiometer wire altered until the galvanometer shows no deflection. Let the points of contact be C_1 and C_2 for cells of E. M. Fs. E_1 and E_2 respectively when there is no deflection in the galvanometer. Then

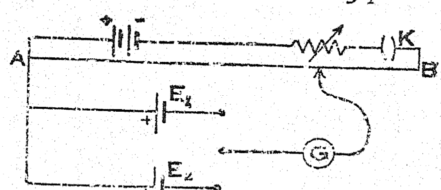


Fig. 26·42

$$E_1 = \text{P. D. between A and } C_1$$
$$= \text{Potential gradient} \times \text{length of } AC_1$$

Similarly $E_2 = \text{Potential gradient} \times \text{length of } AC_2$

$$\therefore \quad \frac{E_1}{E_2} = \frac{\text{length of } AC_1}{\text{length of } AC_2} = \frac{l_1}{l_2}$$

It is evident that *the P. D. between the ends of the potentiometer wire must be greater than the E. M. F. of each of the two cells under examination.* The advantages of this method are:

(*a*) When there is no deflection in the galvanometer, the cells are on *open circuit* and hence we compare the true values for their E. M. Fs.,

(*b*) Since the cells are not giving out current during the measurements except *very small* currents during adjustments, we get accurate results even with those cells which polarise readily, and

(*c*) It is a *null method, i.e.*, it depends upon reducing the deflection in the galvanometer to zero, a process which can be carried out with much greater exactness than the reading of a deflection.

26·27. Internal Resistance of a cell. When a resistance is connected across a cell, the current which results circulates right round the circuit from the positive plate to the negative plate in the *external* circuit and back through the liquid inside the cell from the negative plate to the positive plate. During the passage of the current *inside* the cell the liquid conductor offers some resistance to the flow of the current. This resistance is called the *internal resistance* of a cell. The internal resistance r of a cell is made up of two parts, one r_1 being dependent upon (*i*) the area of the plates *immersed* in the electrolyte, (*ii*) the distance between them and (*iii*) the nature of the electrolyte, and the other r_2 being dependent upon the strength of the current only. If S be the area of the plates immersed and l the distance between them, then $r_1 = l/kS$ where k is a constant which depends only on the nature of the electrolyte and is known as its *specific conductivity*; r_2 is associated with the passage of current out of the plates into the electrolyte and *varies directly as the strength of the current.* For *very small* current r_2 may be assumed to be *negligible.*

Thus, if R be resistance of the external circuit, the total resistance of the circuit is $(r+R)$ and hence the strength of the current in the circuit is given by

$$i = \frac{E}{r+R}$$

when E is the E. M. F. of the cell.

Now, if V be the *terminal* P. D., *i.e.*, P.D. across the external resistance R, we have from *Ohm's law.*

$$V = iR$$

or
$$i = V/R$$

Equating the above two expressions for i, we have

$$\frac{V}{R}=\frac{E}{r+R}$$

or
$$\frac{r+R}{R}=\frac{E}{V} \quad \ldots \quad \ldots \quad (26{\cdot}36)$$

whence
$$r=\frac{E-V}{V}R \quad \ldots \quad \ldots \quad (26{\cdot}37)$$

This equation enables us to calculate *internal resistance* of the cell, if E and V are determined.

Experiment 26.6

Object. To determine the *internal resistance* of a *Leclanche cell* by means of a potentiometer.

Apparatus. A potentiometer, a storage battery, a rheostat, a resistance box, a high resistance, a weston galvanometer, a plug key, a tapping key and connecting wires.

Theory. Let a *constant* P. D. be maintained between the two ends A and B of a potentiometer wire, the potential of A being *higher* than that of B. Let the +ve terminal of the cell whose internal resistance is to be determined be connected to A and the —ve terminal through a galvanometer to a jockey which slides over the potentiometer wire. Further, let l_1 be the length of the potentiometer wire for no deflection in the galvanometer. Then, since the cell is on *open circuit*, we have

$$E=Kl_1 \quad \ldots \quad \ldots \quad (26{\cdot}38)$$

where E is the E. M. F. of the cell and K the *potential gradient* of the potentiometer wire.

Now let the cell be *short-circuited* by a resistance R. Since the cell is now supplying current, the potential difference V between its terminals is less than the E. M. F. of the cell and hence the length of the potentiometer wire for balance will be *less* than l_1. Let l_2 be the length of the potentiometer wire in this case when the galvanometer shows no deflection. Then

$$V=Kl_2 \quad \ldots \quad \ldots \quad (26{\cdot}39)$$

Now, if r be the internal resistance of the cell, we have, from equation (26·37)

$$r=\left(\frac{E}{V}-1\right)R$$

Substituting the values of E and V from equations (26·38) and (26·39) in the above equation, we get

$$r=\left(\frac{l_1}{l_2}-1\right)R \quad \ldots \quad (26{\cdot}40)$$

from which the internal resistance of the cell can be calculated if l_1 and l_2 are determined.

Method. Referring to fig. 26·43, connect the two ends A and B of the potentiometer wire to a storage battery S in series with a rheostat R_1 including a plug key K_1 in the circuit. Connect the +ve pole of the Leclanche cell E whose internal resistance is to be determined to the *higher* potential end A of the potentiometer wire and connect its—ve pole to the jockey of the potentiometer through a weston galvanometer and a high resistance R_2 all in series. Finally connect a resistance box through a tapping key across the poles of the Leclanche cell E.

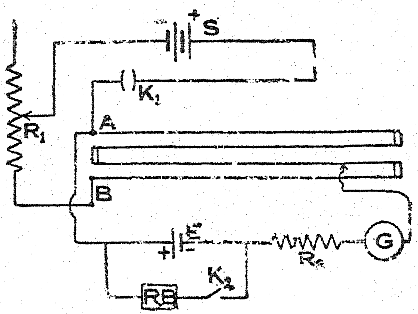


Fig. 26·43

Close the key K_1. Press the jockey over the first wire near the end A and then on the last wire near the end B and see that the deflections of the galvanometer are in *opposite* directions. If it is not so, then, if the connections are correct, the potential difference between the ends of the potentiometer wire is less than the E. M. F. of the cell E. In that case reduce the resistance in the battery circuit by means of the rheostat R_1 and adjust its value until the null point is obtained roughly on the last wire. Now remove the high resistance R_2 connected in series with the galvanometer and determine the *exact* position of the null point. Open K_1 and note down the length of the wire from the end A up to the position of the null point on the wire which gives the value of l_1.

Next adjust the resistance box connected across the Leclanche cell E to a suitable resistance R, say, 1, 2, 5, 10,20, 30, 50 or 100 ohms. Close K_1. First press K_2 and then the jockey over the potentiometer wire and determine as before the exact position of the new null point keeping the current in the battery circuit the same. Open K_1 and note down the length of the wire from the end A to the position of the new null point on the wire which gives the value of l_2.

Repeat the experiment with different values of R *taking observations for open and closed circuits of the Leclanche cell alternately.* Then slightly increase the resistance in the battery circuit and again repeat the experiment with the same values of R. Finally calculate the value of internal resistance r of the Leclanche cell from equation (26·40) for each set of observations separately.

Sources of error and precautions. (1) The battery in the main circuit must be of practically *constant* E. M. F. and preferably

of large capacity so that *the current in the potentiometer wire may remain constant throughout the test.*

(2) The magnitude of the P. D. between the ends of the potentiometer wire must be *greater* than the E. M. F. of the cell whose internal resistance is to be determined.

(3) A suitable rheostat should be connected in series with the battery in the main circuit *in order to increase the sensitiveness of the potentiometer* by decreasing the total fall of potential between the ends of the potentiometer wire, and should be adjusted so that the null point lies roughly on the last wire.

(4) A key should be included in the main circuit *which should the closed only when observations are to be made.*

(5) A high resistance, say of 10000 ohms should be connected in series with the galvanometer and should be kept until the approximate position of the null point has been found. This will reduce the current in the galvanometer thus preventing it from excessive deflections as well as *minimise polarisation in the cell* while the approximate position of the null point is being determined. The exact position of the null point should be determined by finally removing this resistance which increases the sensitiveness of the test of null point by increasing the current in the galvanometer. Note that the presence of high resistance in the galvanometer circuit does not in any way alter the position of the null point.

(6) A tapping key should be connected in series with the resistance box connected across the cell E and should be pressed only when observations are to be made. This will avoid continuous flow of current from the cell into the box thus minimising polarisation effects in the cell.

(7) The cell whose internal resistance is to be determined *should not be disturbed during the course of the experiment* as it may alter its internal resistance.

(8) The contact between the jockey and the potentiometer wire should be momentary and should not be made while the jockey is being moved along, otherwise the wire will be worn unevenly in various parts and no longer remain uniform in cross-section.

(9) If the balance point is obtained on the connecting strip between the wires, it should be shifted on to any of the two wires connected to it by slightly changing the resistance in the battery circuit. But note that the total resistance of the battery circuit for *same* set of observations of the P. D. between the terminals of the cell on open or closed circuit must remain the same.

Observations

S. No.	Length* of the potentiometer wire with the cell on: Open circuit — No. of complete wires	Open circuit — Position of null pt. cm.	Open circuit — Total length of wire l_1 cm.	Closed circuit — No. of complete wires	Closed circuit — Position of null point cm.	Closed circuit — Total length of wire l_2 cm.	R ohm	Internal resistance r ohm
1.							1	
2.							2	
3.							5	
4.							10	
5.							20	
6.							30	
7.							50	
8.							100	

Calculations. Set I.

$$r=\left(\frac{l_1}{l_2}-1\right)$$
$$= \quad \text{ohms.}$$

[Similarly calculate r from other sets of observations]

Result. The resistance of the Leclanche cell *varies with the current drawn* from it and lies *between*...and...ohms.

Criticism of the method. When the Leclanche cell is short-circuited, it discharges at a considerable rate and rapidly becomes polarised. The result is that the internal resistance of the cell increases. The greater the current drawn from the cell, the greater is the polarisation in it and hence greater is the increase in its internal resistance. The value of internal resistance of the cell will, therefore, be different for different values of resistance short-circuiting the cell. If the time taken to determine the null point when the cell is short-circuited is large, *the null point will continuously shift on the slide wire due to its internal resistance changing on account of polarisation.*

The value of E. M. F. of the cell can, however, be determined quite accurately as the cell is then on open circuit and in this respect this method is superior to voltmeter method of measuring the E. M. F. of the cell. It is a null method and is, therefore, comparatively more accurate than the voltmeter method. But since the method possesses all the defects due to polarisation, it does not yield satisfactory results.

Experiment 26·7

Object. To calibrate a given *voltmeter* of 1-volt range by means of a potentiometer.

Apparatus. A potentiometer, two storage batteries, two rheostats, a standard cadmium cell, a weston galvanometer, a

*In case of a coil potentiometer, no. of coils and length of wire should be noted.

voltmeter to be calibrated, a two-way key, two single-way keys connecting wires.

Theory. Referring to fig. 26·43, let a *constant* P. D. be maintained between the two ends A and B of potentiometer wire, the potential of A being *higher* than that of B. Let the +*ve* pole of a cadmium cell D be connected to the end A and the —*ve* through a galvanometer to the jockey of the potentiometer. Then, if l_1 is the length of the potentiometer wire for no deflection in the galvanometer, we have

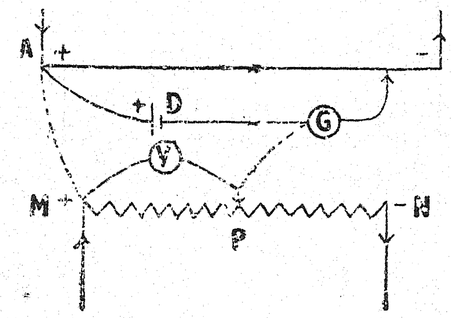


Fig. 26·44

$$E=Kl_1$$

where E is the E. M. F. of the cadmium cell and K is the *potential gradient* of the potentiometer. This gives the value of K.

Now let the cadmium ce be replaced by all portion MP of a rheostat MN through whole of which a *steady* current is maintained, the *higher* potential point M of the rheostat being connected to the end A of the potentiometer. Then, if l_2 is the length of the potentiometer wire at the null point, the P. D. across the portion MP of the rheostat is given by

$$V=Kl_2 \quad \dots \quad (26\cdot41)$$

If the P. D. across the *same* portion MP of the rheostat as measured by a voltmeter is V′, the error in the reading V′ of the voltmeter is (V′—V).

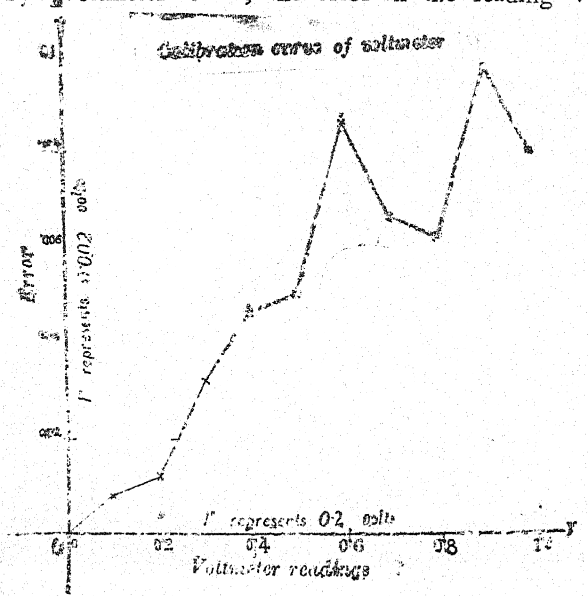


Fig. 26·45

Now if the P. D. between the points M and P of the rheostat is made *variable* by making one of the points, say P variable, and the errors corresponding to the various readings of the voltmeter *thus* obtained be determined by the potentiometer, a *calibration curve* of the voltmeter may be plotted by taking V′ as abscissae and (V′—V) as ordinates.

Method. Referring to fig. 26·46, connect the two ends A and B of the potentiometer to a storage battery S_1 in series with a rheostat R_1, including a plug key K_1 in the circuit. Connect the two *fixed* terminals M and N of a rheostat R_2 to a battery S_2, including a plug key K_2 in the circuit. Connect one terminal of the voltmeter V to be calibrated through a key K_3 to the *higher* potential *fixed* terminal M of the rheostat R_2 and the second terminal of the voltmeter to the *sliding* terminal P of the rheostat.

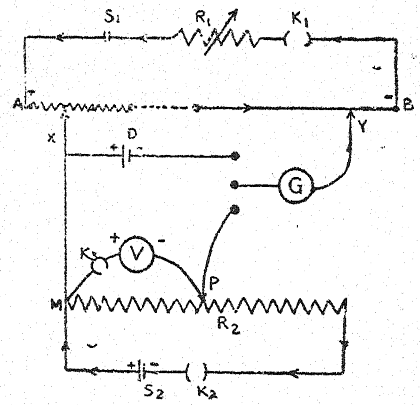


Fig. 26·46

Then connect the +*ve* pole of a cadmium cell and the higher potential fixed terminal M of the rheostat R_2 to the sliding contact maker X of the potentiometer which is nearer to the higher potential end A of the potentiometer. Finally connect the —*ve* pole of the cadmium cell and the *sliding* terminal P of the rheostat R_2 through a two-way key and a galvanometer to the jockey Y which slides over the potentiometer wire.

Close the key K_1 and connect the —*ve* pole of the cadmium cell by means of the two-way key to the galvanometer. Place the slider X at the end of the 10th coil and the jockey at 18·3 div. on the slide wire. Adjust the rheostat R_1 in the *main circuit* till there is no deflection in the galvanometer. *This gives a potential gradient of 1 millivolt/div. along the potentiometer slide wire.*

Next connect the sliding terminal P of the rheostat R_2 by means of the two-way key to the galvanometer and determine the *equivalent* length l_2 of the potentiometer wire corresponding to the P. D. across the portion M. P. of the rheostat R_2.

Note down the reading V′ of the voltmeter, open the keys K_1 and K_2 and calculate the *true* value V of P. D. corresponding to the reading V′ of the voltmeter and then the error (V′—V) in the voltmeter reading.

Next alter the P. D. between M and P by moving the *sliding* terminal P of the rheostat R_2 and determine as before, the errors (V′—V) corresponding to the various readings V′ of the voltmeter thus obtained covering the *entire range* of the instrument. Finally, taking voltmeter readings as abscissae and corresponding errors as ordinates, plot the calibration curve of the voltmeter as depicted in fig. 26·45.

Sources of error and precautions. (1) The main circuit battery S_1 must be of practically *constant* E. M. F. and preferably of large capacity so that *the current through the potentiometer coils and wire may remain constant throughout the expt.*

(2) The magnitude of the P. D. between A and B must be *greater* than the *maximum* P. D. to be measured by means of the potentiometer.

(3) The E. M. F. of the battery S_2 in the auxiliary circuit should be greater than the range of the voltmeter to be calibrated.

(4) A plug key should be included in both the main and auxiliary circuits and should be closed only when observations are to be made.

(5) The resistance R_1 included in the main circuit should be chosen carefully.

(6) The galvanometer should be shunted during the initial stages of determination of null point, the exact null point being determined with full galvanometer sensitivity by removing the shunt.

(7) Change-over from cadmium cell to the portion MP of the rheostat R_2 should be effected by the use of a two-way key.

(8) The contact between the jockey and the potentiometer wire should be momentary and should not be made while the jockey is being moved along.

(9) A key should be connected in series with the voltmeter and should be kept open when P. D. across the portion MP of the rheostat is measured by the potentiometer. In such a case no current will be drawn by the voltmeter and the potentiometer will measure *accurately* the P. D. across MP.

(10) The voltmeter should be calibrated over its *entire* range.

Observations. [*A*] *Standardisation of potentiometer wire.*

E. M. F. of standard cadmium cell=1·0183 volts.

Length of potentiometer wire corresponding to E. M. F. of cadmium cell

	No. of coils	Length of slide wire div.	*Equivalent* length l_1 div.
At start of expt.	10	18·3*	1018·3†
At end of expt.	10	18·3	1018·3†

* Length of slide wire=100 divisions.

† These must be *equal.*

[*B*] *Calibration of voltmeter.*

Least count of voltmeter= volts.

S. No.	Length of potentiometer wire corresponding to P. D. across the portion M P of the rheostat R_2			*Accurate* value of P.D. V volts	Reading of voltmeter V′ volts	*Error* in the reading of voltmeter (V′—V) volts
	No. of coils	length of slide wire div.	*Equivalent* length l_2 div.			
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						

Calculations. $K=\frac{1\cdot0183}{1018.3}=10^{-3}$ volts per div.

Reading No. 1 $V=Kl_2$ = volts

(N. B. Make similar calculations for other readings).

Result. The graph so obtained by plotting the *errors* against the *voltmeter readings* is the *calibration curve* of the given voltmeter.

Criticism of the method. The method gives fairly accurate results. The accuracy in the calibration of voltmeter depends upon the constancy of *potential gradient* of the potentiometer, accurate knowledge of E. M. F. of the cadmium cell and *uniformity* of the slide wire. With the use of a rheostat R_2 as described above, the current in the *auxiliary circuit* can be kept *very low* and still the *entire* range of the voltmeter can be calibrated. This avoids unnecessary heating of conductors in the auxiliary circuit as well as prevents waste of energy of the battery. By standardising the potentiometer to give a potential gradient of 1 milli-lt/div., the calculations are very much simplified.

26·28. Measurement of current by means of a potentiometer. The potentiometer can be used to measure currents. In fact ammeters are accurately calibrated by means of a potentiometer. The current I to be measured passes through a standard resistance R usually of one ohm *already* included in the circuit in which the current I flows and the potential difference V across it is determined by means of a potentiometer accurately standardised in terms of the E. M. F. of a standard cell. Then the current through the standard resistance is given by I=V/R.

Experiment 26.8

Object. To calibrate a given *ammeter* of 1 ampere range by means of a potentiometer.

Apparatus. A potentiometer, two storage batteries, two rheostats, a standard cadmium cell, a weston galvanometer, standard one-ohm resistance, ammeter which is to be calibrated, a two-way key, two single-way keys, shunt wire and connecting wires.

Theory. Let a steady P. D. be maintained between the two ends A and B of a potentiometer wire, the potential of A being *higher* than that of B. Let +ve pole of cadmium cell be connected to the end A and the —ve through a galvanometer to the jockey of the potentiometer. Then, if l_1 be the length of the potentiometer wire for no deflection in the galvanometer, we have

$$E=Kl_1$$

where E is the E. M. F. of cadmium cell supposed to be known and K is the *potential gradient* of the potentiometer wire. Thus the value of K is known.

Now let the cadmium cell be replaced by a *one-ohm standard resistance* through which a *steady* current I is flowing, its higher potential terminal being connected to the end A of the potentiometer wire, and let the current I *also* flow through the ammeter to be calibrated connected *in series* with the standard resistance. Then, if l_2 be the length of the potentiometer wire for no deflection in the galvanometer, the P. D. between the terminals of the standard one-ohm resistance is given by

$$V=Kl_2$$

and the current through it by

$$I=V/R=Kl_2 \qquad \ldots \qquad (26{\cdot}42)$$

Let the reading of the ammeter be I′. Then the *error* in it is (I′—I). If now the current through the standard resistance be *varied* and the values of (I′—I) corresponding to different readings I″ of the ammeter be determined, *calibration curve* of the ammeter may be darwn.

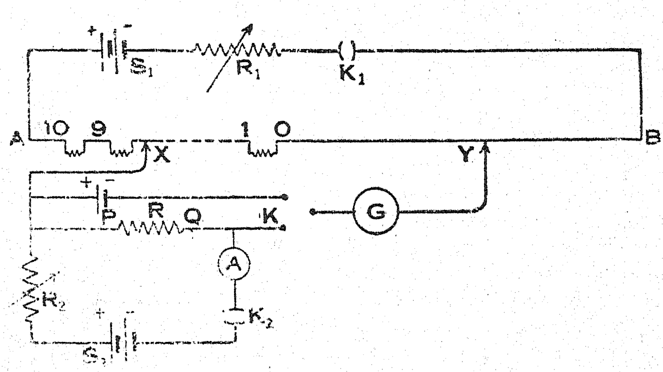


Fig. 26·47

Method. Referring to fig. 26·49, connect the two ends A and B of the potentiometer to one of the storage batteries S_1 in series with a rheostat R_1, including a plug key K_1 in the circuit. Connect the *standard one-ohm resistance* R to the other storage battery S_2 in series with a rheostat R_2 and the ammeter which is to be calibrated, including a plug key K_2 in the circuit. Finally connect the +ve pole of the cadmium cell and the *higher* potential terminal P of the standard one-ohm resistance together to the *sliding* contact maker X of the potentiometer which is nearer to the *higher* potential end A of the potentiometer and, the —ve pole of cadmium cell and the lower potential terminal Q of the standard one-ohm through a two-way key K to one terminal of the galvanometer the second terminal of which is connected to the jockey Y which slides over the potentiometer wire.

First *standardise* the potentiometer wire with the cadmium cell. For this connect —ve pole of cadmium cell by means of the two-way key K to the galvanometer, put the slider X at the and oı the 10th coil and the jockey at 18·3 div. on the slide wire and adjust the rheostat R_1 in the *main circuit* till there is no deflection in the galvanometer. Now the *potential gradient* K of the poten iometer equals 10^{-8} volts/div.

Next lose K_2 also. Connect the lower potential terminal Q of the standard one-ohm resistance by means of the two-way key K to the galvanometer and determine the equivalent length l_2 of the potentiometer wire corresponding to the potential difference V between the terminals of the standard resistance. Note down the reading of the ammeter. Open K_2 and K_1, and calculate the *true* value of current corresponding to reading I′ of ammeter from equation (26·42). Then calculate the *error* (I′—I) in the ammeter reading.

Next alter the current through the standard resistance by means of the rheostat R_2 and take several sets of observations for different values of P. D. across the standard resistance at the same time noting the readings of the ammeter and thus determine the error (I′—I) corresponding to various readings of the ammeter covering its *entire range*. Finally plot *calibration curve* of the ammeter taking the readings of the instrument as abscissae and the corresponding errors (I′—I) as ordinates.

Sources of error and precautions. (1) The main circuit battery S_1 must be of practically *constant* E. M. F. and preferably of large capacity so that *the current in the potentiometer coils and wire may remain constant throughout the test.*

(2) The magnitude of the P. D. between A and B must be *greater* than the *maximum* P. D. to be measured by means of the potentiometer.

(3) The battery S_2 in the auxiliary circuit should be of constant E. M. F. and ample capacity. Its E. M. F. should be slightly *higher* than that necessary to produce *full-scale* deflection in the ammeter.

(4) The rheostat R_1 included in the main circuit should be carefully chosen.

(5) A key should be included in both the main and the auxiliary circuits and should be closed only when observations are to be made.

(6) The galvanometer should be shunted. The shunt should be removed when the exact position of the null point is to be determined in order to increase the sensitiveness of the test of null point by increasing the current in the galvanometer.

(7) The calibration of the potentiometer wire with the cadmium cell and determination of different values of P. D. across the standard one-ohm resistance corresponding to various readings of the voltmeter should be made *in as short a time as possible* in order to minimise the chance of error due to variation of steady current in the potentiometer wire. Accordingly it is convenient to use a two-way key so that the change from the cadmium cell to the standard resistance or vice versa may be affected quickly.

(8) The contact between the jockey Y and the potentiometer wire should be momentary and should not be made while the jockey is being moved along.

(9) The ammeter should be calibrated over its *entire* range.

Observations. [*A*] *Standardisation of potentiometer wire.*

E. M. F. of standard cadmium cell=1·0183 volts

	Length of potentiometer wire corresponding to E. M. F. of the cadmium cell		
	No. of coils	Length of slide wire div.	*Equivalent* length l_1 div.
At start of expt.	10	18·3*	1018·3†
At end of expt.	10	18·3	1018·3†

[*B*] *Calibration of ammeter*

Least count of ammeter = amp.

S. No.	Length of Potentiometer wire corresponding to P. D. (V) across the standard one-ohm resistance			*Accurate* value of		Reading of ammeter	*Error in* the reading of ammeter
	No. of coils	Length of slide wire div.	*Equivalent* length l_2 div.	P. D. V volt	Current I amp.	I' amp.	$(I'$ amp.
1							
2							
3							
4							
9							
10							

* Length of slide wire=100 divisions.
† These must be equal.

Calculations. $K = \frac{1{\cdot}0183}{1018{\cdot}3} = 10^{-3}$ volts per div.

Reading No. 1. $V = Kl_2 =$ $=$ volts

and $I = V =$ $=$ amp.

(*N. B.* Make similar calculations for other readings.)

Result. The graph so obtained by plotting the *errors* in the *readings of ammeter* against the corresponding *ammeter readings* is the *calibration curve* of the given ammeter.

Criticism of the method. The accuracy in the calibration of ammeter depends upon the constancy of the *current* through the standard one-ohm resistance during the measurements, the constancy of the *potential gradient* of the potentiometer, the *uniformity* of the slide wire and the accurate knowledge of the value of standard resistance and the E. M. F. of the cadmium cell. The method yields sufficiently accurate results. For greater accuracy, a more sensitive potentiometer, *e.g.*, *Crompton potentiometer* should be used and the ordinary one-ohm resistance should be replaced by a standard resistance provided with separate current and potential terminals.

The method can also be used to calibrate a *voltmeter*, if the P. D. across the one-ohm standard resistance is measured by both the voltmeter and the potentiometer.

26.29. Measurement of resistance by a potentiometer. The potentiometer is also used to determine the value of an unknown resistance. A *steady* current of *known* strength I is allowed to pass through the resistance R to be measured and the potential difference V across it is determined by means of an accurately calibrated potentiometer. Then obviously $R = V/I$.

Experiment 26·9

Object. To compare two resistances by means of a potentiometer.

Apparatus. A potentiometer, two resistances to be compared, a *special key* K as shown in fig. 26·48, a weston galvanometer, a shunt wire, two storage batteries, two rheostats, two single-way plug keys and connecting wires.

Theory. Let a *steady* potential difference be maintained between the two ends A and B of a potentiometer wire, the potential of A being *higher* than that of B. Let a *steady* current of strength I be allowed to pass through the two resistances R_1 and R_2 to be compared connected *in series*. Further, let the higher potential terminal of one of the resistances, say R_1, be connected to the end A of the potentiometer wire and the lower potential terminal through a galvanometer to the jockey of the potentiometer. Then, if l_1 be the length of the potentiometer wire for no deflection in

the galvanometer, the potential difference between the terminals of R_1 is given by

$$V_1 = IR_1 = Kl_1 \qquad \ldots \qquad (26{\cdot}43)$$

where K is the *potential gradient* of the potentiometer.

Now let the experiment be repeated with the other resistance R_2, and if l_2 be the length of the potentiometer wire corresponding to the P. D. V_2 across its terminals when there is no deflection in the galvanometer. Then

$$V_2 = IR_2 = Kl_2 \qquad \ldots \qquad (26{\cdot}44)$$

Dividing equ. (26·43) by equ. (26·44), we get

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \qquad \ldots \qquad (26{\cdot}45)$$

from which the ratio of two resistances may be calculated. If a graph is plotted between l_1 and l_2, the *slope* of the straight line thus obtained will also give the ratio of the two resistances.

Method. Referring to fig. 26·48, connect the two ends A and B of the potentiometer to the storage battery B_1 in series with a rheostat R, including a key K_1 in the circuit. Then connect the two resistances R_1 and R_2 to be compared *in series* and send through them a steady current from the storage battery B_2 including a rheostat R′ and a key K_2 in the circuit. Next connect the terminal *e* of the key K to *sliding* contact maker X which is nearer to the

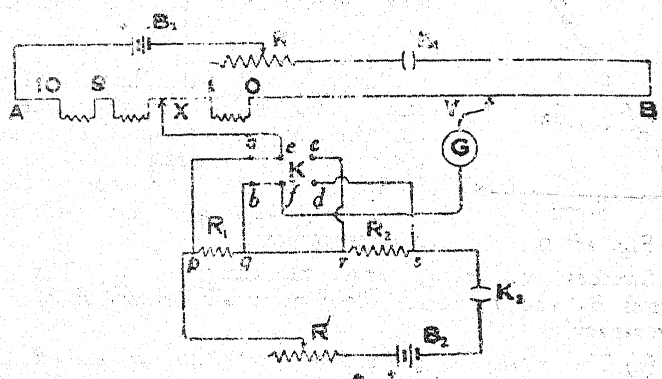


Fig. 26·48

higher potential end A of the potentiometer and the terminal *f* through the galvanometer to the jockey Y of the potentiometer. Finally connect the *higher* terminals *p* and *r* of the two resistances R_1 and R_2 to the terminals *a* and *c* of the key K respectively and the lower potential terminals *q* and *s* to the terminals *b* and *d* respectively.

Next close K_2 and adjust the rheostat R' to produce small potential differences across the resistances R_1 and R_2. Now shunt the galvanometer, close K_1 and connect *e* to *a* and *f* to *b*. Determind the approximate equivalent length of the potentiometer wire corresponding to the P. D. across R_1. Then connect *e* to *c* and *f* to *d* and determine the approximate equivalent length of the potentiometer wire corresponding to the P. D. across R_2 and thus determing which of the resistances R_1 and R_2 has the *greater* value.

Next connect the terminals *e* and *f* to the *larger* resistance and adjust the rheostat R in the *main circuit* so that when the contact maker X lies near the end A of the *last* coil of the potentiometer, an approximate balance point is obtained on the slide wire. *This adjusts the P. D. between A and B to be greater than the potential differences V_1 and V_2 across the resistances R_1 and R_2, at the same time making the potentiometer to have maximum sensitiveness for the given value of V_1 and V_2.*

Now remove the shunt from the galvanometer and determine by connecting R_1 and R_2 *in turn* to *e* and *f*, the *exact* positions of the null point corresponding to the potential differences V_1 and V_2 and after noting down the number of coils and the length of the slide wire between X and Y, calculate the *equivalent* lengths l_1 and l_2 of the potentiometer wire corresponding to the potential differences V_1 and V_2 across the resistances R_1 and R_2 respectively. Next increase R' or decrease R and take at least six more sets of observations for l_1 and l_2. Then calculate the ratio of the resistances R_1 and R_2 from equation (26·45) and find the *mean* value of R_1/R_2.

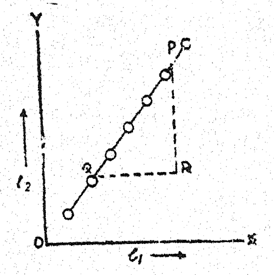


Fig. 26·49

Finally plot a graph between the different values of l_1 and the values of l_2 corresponding to the same set. This will come out to be a straight line as shown in fig. 26·49. Find the *slope* of this line which will also give the ratio of the two resistances.

Sources of error and precautions. (1) The supply batteries B_1 and B_2 should be of practically *constant* E. M. Fs. and ample capacities.

(2) The steady P. D. between the ends A and B of the peteniometer must always be *greater* than the potential differences V_1 and V_2 across the resistances R_1 and R_2.

(3) A suituld rheostat R should be included in the main circuit and should be adjusted to have the potentiometer *maximum sensitiveness* for given values of V_1 and V_2 ensuring at the same time that the P. D. between A and B is *greater* than V_1 and V_2.

(4) A key should be de. included in *each* of the main and auxiliary circuit and should babe closed only when observations are to be made.

(5) The two resistances R_1 and R_2 to be compared should not be handled or disturbed during the experiment, and while determining the potential differences V_1 and V_2 across them, change over from one resistance to the other should be effected *in a very short time* with the help of a special key as described above, in order to diminish the effects of heating and any variations in the batteries.

(6) The *higher* potential terminals p and r of the resistances R_1 and R_2 should be connected to the terminals a and c respectively of the key K which are *collinear* with the terminal e connected to the sliding contact maker X near to the *higher* potential end A the potentiometer ; and the lower potential terminals q and s of R_1 and R_2 should be connected to the terminals b and d respectively which are *collinear* with the terminal f which has been connected to the galvanometer.

(7) The galvanometer should be shunted with a low resistance wire. The exact position of the null point should be determined with the full galvanometer sensitivity without any shunt across it.

(8) While determining the values of l_1 and l_2 for a *particular* set of observations, both R and R′ should be kept *constant*.

(9) The contact between the jockey Y and the slide wire should be momentary and should not be made while the jockey is being moved along.

(10) The graph between l_1 and l_2 will come out to be a straight line and should be smoothly drawn.

Observations.

S. No.	Length of potentiometer wire corresponding to the P. D. across the resistance						$\frac{l_1}{l_2}$
	R_1			R			
	No. of coils	Length of slide wire div.	*Equivalent* length l_1 div.	No. of coils	Length of slide wire div.	*Equivalent* length l_2 div.	
1							
2							
...							
5							
8							
						M	

Calculations. I set $\frac{R_1}{R_2}=\frac{l_1}{l_2}$

(*N. B.* Make similar calculations for other sets).

$\therefore$ *Mean* value of $\frac{R_1}{R_2}=$

From the graph (26·49)

$$PR= \qquad QR=$$

$$\therefore \qquad \frac{R_1}{R_2}=\frac{QR}{PR}=$$

Result. The ratio of the values of the given resistances R_1 and R_2 as obtained by calculations=

and as obtained from graph =

Criticism of the method. This method of comparing resistances gives a very accurate result. The slight inaccuracy in the result may be due to the inconsistency of the E. M. Fs. of the batteries, the non-uniformity of the slide wire and the heating of the resistances. The method is especially suited to the comparison of two low resistances for which the Wheatstone bridge method as ordinarily employed is not suitable. But the two low resistances to be compared should be of the *same order* of magnitude. To compare two resistances of different orders, say one of the order of 1/100 ohm and the other of the order of 1 ohm, the method has to be slightly modified as described below. For very accurate results, the comparison should be made by means of *Crompton's potentiometer.*

Exercise. *To determine a low resistance by means of a potentiometer.*

Connect the two ends A and B of the potentiometer to a storage battery S_1 in series with a rheostat R_1, including a key K_1 in the circuit. Next connect the low resistance *r* to be measured *in series* with a known resistance R of about 1 to 5 ohms, and then insert the *combination in series* with a circuit containing a battery S_2, rheostat R_2 and a key K_2. Connect the *higher* potential terminal M of the resistance R to the *higher* potential end A of the potentiometer. Finally connect the

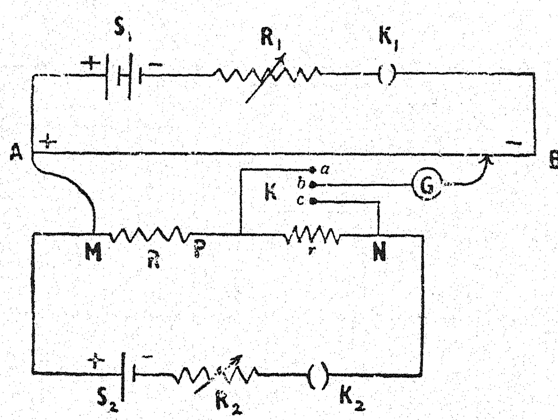


Fig. 26·50

lower potential terminals P and N of the resistances R and r respectively through a two-way key and a galvanometer, to the jockey of the potentiometer, as shown in fig. 26·50.

Next connect a to b and determine the length l_1 of the potentiometer wire at the null point corresponding to the P. D., V_1 across the resistance R. Then connecting c to b, determine the length l_2 of the potentiometer wire corresponding to the P. D., V_2 across the *series combination* of the two resistances R and r. If I is the *steady* current flowing through the resistances R and r, we have

$$\frac{V_1}{V_2}=\frac{I\,R}{I(R+r)}=\frac{l_1}{l_2}$$

whence $$r=\left(\frac{l_2}{l_1}-1\right)R$$

from which r may be calculated.

Repeat the experiment with at least three values of R taking at least three sets of observations with each value of R by varying the P. D. across the resistances with the help of the rheostat R_2. The *mean* value of r thus obtained will give the low resistance.

Alternatively. A graph may be plotted between l_2/l_1 and $1/R$. This will come out to be a straight line and its *negative* intercept on the X-axis will give $1/r$. This is illustrated in fig. 26·51.

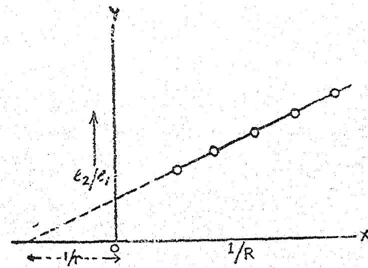


Fig. 26.51

26·30. Leeds and Northrup student's potentiometer. This is a compact potentiometer of *high precision* and *sensitivity* and works on the same principle as that of a Crompton potentiometer. As illustrated in fig. 26·52, the potentiometer consists of 15 manganin coils C, *each* of 10 ohms resistance, connected *in series* with a manganin slide wire W of 10 ohms resistance. The *positive* pole of the main-circuit battery is connected directly to the terminal marked +B and its negative pole to the decade resistance box R. The other terminal of the resistance box R is joined to the double-pole double-throw switch K. The *positive* pole of the standard cell and that of the source of unknown P. D. are connected through a sensitive galvanometer to the +E marked terminal of the instrument which is joined to the slide wire W. The —E marked terminal of the instrument which is joined to the coils C is connected to the switch K. The D. P. D. T. switch K, when in one position as shown, connects the resistance box R to the terminal marked S and the negative pole of the standard cell to the terminal marked — E of the instrument. When

in the other position, the switch K connects the resistance box R to the terminal marked o.t or o. ·1 and the negative pole of the source of unknown P. D. to the terminal marked—E of the instrument. The dial C is graduated in steps of 0.1 volt upto 1.5 volts : and the

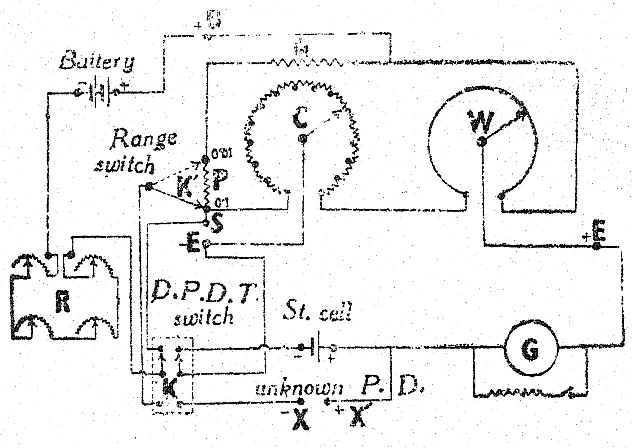


Fig. 26.52

dial W of the slide wire has a continuous scale, the maximum reading being 0.1 volt. The instrument, when standardised, reads P. D. *directly* in volts.

For standardisation of the instrument, the electric connections are made as shown in fig. 26.52. The range switch K' is adjusted to point towards the terminal marked c.· and the D. P. D. T. switch K is thrown to the standard cell side as shown. The dials C and W of the coils and the slide wire are adjusted to read the E. M. F. of the standard cell. For instance, if there are 100 divisions on the slide wire scale, then for a standard Cadmium cell of E. M. F. 1.0183 volts, the dial C is set to include 10 coils and the dial W to 18.3 div. Next the decade resistance box R is adjusted such that there is no deflection in the galvanometer. This standardises the potentiometer giving a P. D. of 0.1 volt across each coil or the slide wire. Since the resistance of each coil or the slide wire is 10 ohms, it is evident that after standardisation the current passing through the coils and the wire is 10 milliamp.

Having standardised the potentiometer, if the range switch K' is adjusted to point towards the terminal marked 0.01, the *potential gradient* of the potentiometer is reduced to 1/10 of its previous value, *i.e.*, it now gives a voltage of 0·01 volt across each coil or the slide wire so that the *maximum* reading of the instrument is 0·16 volts or 160 millivolts. When the switch K' points towards the terminal marked 0·1, the series combination of the resistances P and Q is in parallel with the series combination of the coils and

the slide wire When the switch K' is adjusted to point towards the terminal marked 0·01, the resistance P becomes in series with the coils and the slide wire and the resistance Q shunts the series combination of the resistance P, the coil and the slide wire. This shifting of the resistance results in a reduction of the current through the potentiometer coils and the slide wire to 1/10 of its previous value, *i. e.*, to the value 1 milliamp. and hence the potential gradient to 1/10 of its former value. A little calculation will show that the resistance P=1440 ohms and the resistance Q=160 ohms.

After standardisation, the student's potentiometer can be used for the following purposes.

(i) **Measurement of E. M. F. of a cell.** The cell under test is connected between the terminals XX' meant for source of unknown P. D. and the D. P. D. T. switch K is thrown on the unknown P. D. side. Next the dials C and W of the coils and the slide wire are adjusted so that there is no deflection in the galvanometer. The sum of the readings on the two dials C and W gives the E. M. F. of the cell under test. In case the unknown E. M. F. is less than 160 millivolts, the range switch K' should be adjusted to point towards the terminal marked 0·01.

(ii) **Calibration of ammeter and voltmeter.** An auxiliary circuit (Fig. 26.53) consisting of a storage battery B, a rheostat R, the given ammeter A and a standard one-ohm resistance *r*, *all in series* is made, including a plug key K in the circuit. The given voltmeter P is placed *in parallel* with the one-ohm resistance. Next the terminals of the one-ohm resistance are connected to the potentiometer terminals XX' meant for source of unknown P. D. and the P. D. across the one-ohm resistance is measured by the potentiometer in the usual way. The difference between the reading V' of the voltmeter and P. D. (V) across the one-ohm resistance as measured by the potentiometer gives the error in the voltmeter reading V'. Since the resistance of the standard resistance *r* is one ohm, the current through it equals the P. D. across it. Hence (I'—V) gives the error in the reading I' of the ammeter. Next the current in the *auxiliary circuit* (Fig. 26.53) is altered by means the rheostat R and the errors corresponding to the various readings of the ammeter and the voltmeter covering the entire range are determined as above by means of the potentiometer. A *calibration curve* of the voltmeter is plotted taking the readings of the voltmeter as abscissae and the corresponding *errors* as ordinates. Similarly a calibration curve of the ammeter is drawn.

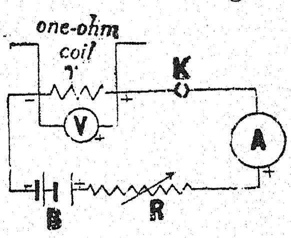


Fig. 26.53

(iii) **Measurement of P.D. higher than the range of the poten-**

tiometer. This can be done with the help of a *volt-box*. The fig. 26·54 illustrates the principle of a volt-box which consists of a high resistance MN which can be tapped in desired steps at a number of points. As an illustration, let the resistance between the terminals PP′ of the volt-box be 1/10 of the resistance between the terminal MA, 1/100 of the resistance between the terminals MB, 1/200 of the resistance between the terminals MN. Then to such a volt-box if we apply a voltage less than 16 volts between the terminals MA, the P. D. across the volt-box terminals PP′ will be less than 1·6 volts, which being within the range of the instrument can be measured by the student's potentiometer. Similarly, if a high voltage is applied between the terminals MB, the P. D. available between the terminals PP′ will be 1/100 of this high voltage.

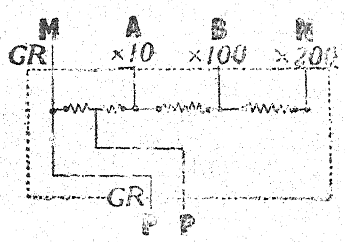


Fig. 26·54

To determine a high voltage, the terminal PP′ of the volt-box are connected to the terminals XX′ of the potentiometer meant for source of unknown P. D. The high voltage to be measured is applied between the terminals MA of the volt-box if it is less than 16 volts, or between the terminals MB if it lies between 16 volts and 160 volts, or between the terminals MN if it is greater than 160 volts but less than 320 volts. The potentiometer is standardised as usual and the D. P. D. T. switch K is thrown on the unknown P. D. side. The dials C and W of the coils and the slide wire are adjusted to give no deflection in the galvanometer. Then the sum of the readings of the two dials C and W will give the P. D. between the terminals PP′ of the volt-box. To get the value of the unknown high voltage, this P. D. between the volt-box terminals PP′ has to be multiplied by 10, 100 or 200 according as the high voltage was applied between the terminals MA, MB or MN.

Oral questions

RESISTANCE

How does resistance vary with (*a*) length, (*b*) the cross-section, (*c*) the material and (*d*) temperature of the conductor ? Do you know any case in which resistance decreases with temperature ? What are the laws of resistances in series and in parallel ? How is the resistance in the box made up ? Why is the wire first doubled and then wound ? What is the material of which the resistance coils are usually made ? What is the composition of manganin and constantan ? Why do you select this particular material for the coil ? Of what material are the coils of galvanometer made ? Why do you not use manganin or constantan there ? What is meant by S. W. G ? Of what materials are the plugs of the box made ? Why are the studs put in ebonite board ? Why are the plugs made tapering ? What is the length of the wire connecting the blocks where infinity is written ? What are the uses of slide wire rheostat ? On a rheostat the following specifications are given 1·4 amps. 2·5 ohms ; accuracy 1 in 100 at 16°C : how do you interpret this ? What type of resistance is ceiling fan regulator ? How many methods of measuring resistance do you know ? Define specific resistance and state its units ? What precaution is to be taken in the measurement of length of a wire given for finding out its specific resistance ?

METRE BRIDGE

What is a Wheatstone's bridge ? What is a metre bridge and why is it so called ? Show the four arms in your apparatus. Point out the two points which are of the same potential. What do you understand by conjugate conductors ? What is the advantage in taking observation by interchanging the positions of the unknown resistance and the resistance box ? What advantage will you get by increasing the length of the wire ? What is the material of the bridge wire ? Why is it made of this particular substance ? Is it essential that the wire should be of uniform cross-section ? If it is not, how will you modify your formula to get correct result ? What condition should be satisfied to get a null point ? Why does the deflection change direction in the galvanometer when the jockey passes over the null point ? Why is it desirable to get the null point as near the centre as possible ? What do you understand by end-corrections of a metre bridge ? What are they due to ? Is the value of end-correction always positove ? If not in what case can it be negative ? Can it be ever zero ? If so, when ? Hw can you determine the end corrections of a metre bridge ? Why should theresistance boxes be preferably of the dial pattern ? What is the advantage of using the interchanging commutator ? Explain its working. Why is it connected by thick wires to the metre-bridge ? What should be the ratio of P and Q while determining the end-corrections and why ? Sometimes there is no null point obtained on the whole of the wire; what are the reasons ? How will you readjust your apparatus to get it ? Trace the direction of current (*i*) at the null point (*ii*) when the deflection in the galvanometer is to the right and (*iii*) when it is to the left. Which key do you press first and why ? How is null point affected by introducing a resistance R at null point in the cell circuit or in the galvanometer circuit ? What is the use of the battery key ? How can thermo-electric effects be got rid of in the metre bridge ? Why do you shunt the galvanometer in the beginning and then cut it off when the approximate null point is obtained ? Define sensitivity of a galvanometer ? What kind of galvanometer do you use ? Explain its working. How is it that the needle of a galvanometer comes to a dead stop immediately and does not oscillate ? If your galvanometer is not dead beat how can you make it so ? Can you use a tangent galvanometer here ? Can you replace your galvanometer by a telephone ? If so, is there any advantage in doing so ? Why do you use Leclanche cell here ? What maximum resistance can you measure with a metre-bridge ? Can you find the resistance of a galvanometer or a cell with metre-bridge by using no extra galvanometer or a cell ? If another galvanometer were used what care will you take to avoid damage to the galvanometer ? What do you understand by the sensi tiveness of a Wheatstone's bridge ? On what factors does it depend ? When is it most sensitive? Are the positions of the battery and the galvanometer interchangeable ? When the positions of the battery and the galvanometer are interchanged does the arrangement remain equally sensitive ? If not, which of the two arrangements is more sensitive and why ?

POST OFFICE BOX

Why is the instrument called P. O. box ? Where are the four arms of the bridge ? How are the resistances in P. O. box arranged ? How can you find an unknown resistance with a P. O. box ? What is the minimum and maximum resistance that can be measured with your P. O. box ? How can you measure the resistance of a galvanometer with a P. O. box by Kelvins' method ? Can you get a true null point in this case ? When is the bridge said to be balanced ? Why is there no change of deflection in the galvanometer at the balance point when the key short-circuiting the usual galvanometer arm is pressed ? Why should the galvanometer be properly levelled ? Why does the spot of light shift on changing the resistance in the rheostat arm of the bridge ? Why should a thick copper wire be used to short-circuit the usual galvanometer arm ? Which of the two keys should be pressed first and why ? If we take the ratio 10 : 10 in one case and 100 : 100 in the other, will it make any difference ? If at any stage the arrangement becomes insensitive, how can you make it more sensitive ? If the bridge is not exactly balanced in the ratio 10:1000, how will you compute the resistance of the galvanometer ? Why are there only three resistances in each of the ratio arms of a P. O. box ? How can the resistance of the galvanometer be best determined ? Why is the former method

called Kelvin's method ? Describe Kelvin's galvanometer. What part does the control magnet play in the determination of resistance of such a galvanometer by Kelvin's method ? Does the control magnet affect the deflection for given want of balance in the bridge ?

How can you determine the internal resistance of a cell with a P. O. box by Mance's method ? What is the underlying principle of the method ? When is bridge said to be balanced in this case ? On pressing the key short-circuiting the usual battery arm, why does the deflection in the galvanometer not change when the bridge is balanced ? What is the principal difference underlying Mance's and Kelvin's methods ? Why is a thick copper wire used to short circuit the usual battery arm ? What is the advantage of using a plug key in series with the cell ? Can you get a true null point in this case ? How will you get exact value of resistance even if you do not get balance point with the ratio 100/1000 ? What is the range between which resistances of primary cells lie ? What is the relation between internal resistance and efficiency ? Which galvanometer will serve better in Mance's method : a moving coil or Kelvin's galvanometer ? What part does the control magnet play in Mance's method when a Kelvin's galvanometer is used? How does it differ from that in Kelvin's method ? What is Lodge's modification of Mance's method ? Explain the action of the condenser here. What is the capacitance of the condenser used ?

What type of mirror is attached to the galvanometer coil ? Which arrangement is better : a lens and plane mirror or a concave mirror alone ? If you are using a concave mirror, how can you roughly estimate its radius of curvature ? How is the image of the filament of the lamp formed on the scale ? What type of image does a plane mirror form ? How is it then that you get a real image on the scale with the help of the plane mirror ?

CAREY FOSTER'S BRIDGE

On what factors do the sensitiveness and accuracy of a metre bridge depend ? How can these be increased ? What is Carey Foster's bridge ? In what respects is it an improvement over a metre bridge ? How can you determine an unknown resistance with a Carey Foster's bridge ? How will you determine σ, the resistance per unit length of the bridge wire ? How does the accuracy in the determination of σ depend upon the difference between the known resistances in the outer gaps ? What can be the maximum value of this difference which you can take ? If the wire be not uniform, will this value of σ give the correct result ? If not, how will you modify your method of determining σ ? Should the resistances in the inner gaps be fixed and known ? Should these two resistances be exactly equal ? If they are unequal, what is the harm when their difference is (*a*) very small (*b*) very large ? In the latter case where may the balance points lie ? What is the advantage in making the resistances in the inner gaps equal ? Can you not use a rheostat for this purpose ? If so, what advantage does it possess over fixed known resistances in the inner gaps ? What should be the order of the resistances in the inner gaps of the bridge and why ? When is the bridge said to be most sensitive ?

What do you understand by calibration of the bridge wire and how is it done ? What is the function of the rheostat introduced between the inner gaps of the bridge in this experiment ? What should be the order of the resistance of the rheostat ? What should be the order of the difference of resistances in outer gaps and why ? What is the resistance of the bridge wire between two successive balance points ? What is the resistance of the bridge wire ? How will you test the uniformity of the bridge wire ? How will you draw the calibration curve for the bridge wire ? If you also know the values of end corrections, how can you get the value of the total correction at any point from the calibration curve ?

PLATINUM RESISTANCE THERMOMETER

Describe a platinum resistance thermometer. What is the function of the compensating leads ? Define temperature-coefficient of resistance. Explain how it can be determined for platinum by Carey Foster's bridge ? What are the sources of error in your experiment and what precautions do you take to eliminate them ? Why have the usual positions of the galvanometer and the battery been interchanged ? Why should the four arms be non-inductive ? Why should the null point be determined by observing the immediate deflection in galvanometer on pressing the jockey?

Discuss the accuracy in the result obtainable by this method. How can you determine resistance of the platinum resistance thermometer graphically ?

POTENTIOMETER

What is a potentiometer ? What is it used for ? What do you understand by e. m. f. of a source ? In what units is it measured ? What is the difference between P. D. and e. m. f. ? What is the principle of a potentiometer ? Is it used for the comparison of e. m. fs or P. Ds. of two cells ? What are the requirements of a potentiometer wire ? Why must it be of high specific resistance ? Why should the wire be of uniform diameter ? What would the P. D. be equal to if the wire were not uniform throughout ? What do you mean by potential gradient of a potentiometer and how will you determine it ? What do you understand by sensitiveness of a potentiometer ? How can you increase it ? What do you understand by the accuracy of a potentiometer and upon what factors does it depend ? Without sacrficing ancusracy how can you increase its sensitiveness ? How does the accuracy of obscrvaions differ from that of the instrument ? What sort of galvanometer ismost suitacle in a potentiometer circuit ? Can you use a tangent galvanometer instead ? If it is not very sensitive, how does increasing the sensitiveness of the potentiometer affect the determination of the exact null point ? What are the requirements of the ell used in the main circuit of the potentiometer and why ? Why is it that a Leclanche cell can be used in Wheatstone bridge experiments and not with a potentiometer ? What is the function of the key in the main circuit of a potentiometer ? What is the use of the rheostat in the main circuit ? It is sometimes found that the deflection is in the same direction all over the potentiometer wire, what is it due to ? In case the p. d. across the potentiometer wire is less than the p. d. to be measured, how can you increase the former ? Is it necessary that positive terminals of the cells should be connected to one and the same end of the potentiometer wire ? Can all the negative terminals be connected at one end of the potentiometer wire instead of the positive ones ? If so, how ? What is the effect on the null point if (*a*) the number of cells in the main circuit is increased, (*b*) the resistance in the main circuit is altered, (*c*) a resistance is introduced, in series with the galvanometer ? Trace the current (*a*) when the null point has been obtained. (*b*) when the deflection is towards the left, (*c*) when the deflection is towards the right. What are the uses of a potentiometer ?

Why is a potentiometer used for comparison of e. m. fs. of two cells ? What other methods are there for the comparison of e. m. fs. of cells ? Compare their relative merits. How can you measure the internal resistance of a cell with a potentiometer ? Why should the cell, whose internal resistance is to be determined, be not ditsturbed during the course of the experiment ? Why should a resistance of about a housand ohms be kept connected in series with a galvanometer until the approximate position of the null point has been found ? Does this resistance affect the position of the null point ? Do you get a consistent value for the internal resistance of a cell by this method ? If not why ? Compare this method of measuring internal resistance of a cell with Mance's method ? Can you measure internal resistance of an accumulator by this method ? How can potentiometer be used for the comparison of two resistances ? Why should the supply batteries in the main and auxiliary circuits be of constant e. m. fs. and ample capacities ? Why should the p. d. between the ends of the potentiometer wire be always greater than the p. d. across each of the two resistances to be compared ? Describe the working of the special key used for changing over from one resistance to the other in the auxiliary circuit. Compare this method of measuring resistances with Wheatstone's bridge method.

How can you use a potentiometer for measurement of current ? What do you understand by calibration of an ammeter and that of a voltmeter ? Why is it done at all ? How do you calibrate an ammeter and a voltmeter with a potentiometer ? How do you find out the potential gradient of the potentiometer wire ? Why should a standard cell be used for the purpose ? Will a Daniell cell serve the purpose ? What is the use of a known standard resistance in the auxiliary circuit ? Why should it be preferably of one ohm ? What is the function of the keys in the main and the auxiliary circuits ? What is the range of the voltmeter or the ammeter you are calibrating ? What should be the p. d. across the ends of the potentiometer wire in order to calibrate the entire range of the instrument and why ? How will you adjust this to be so ? Discuss the accuracy of the result obtainable by this method.

CHAPTER XXVII

CHEMICAL EFFECTS OF CURRENTS

27·1. Faraday's Laws of Electrolysis. When an electric current is allowed to pass through aqueous solutions of inorganic acids and their salts or fused salts which contain charged atoms or groups of atoms called *ions*, the current directs the ions towards the plates by which the current enters or leaves the liquid. The ions give up their charges to the plates and are liberated there. The plates are termed the *electrodes*, that by which the current enters being called *anode* and the other by which the current leaves, the *cathode*. The conducting liquid is called the *electrolyte* and the whole process *electrolysis*. The —vely charged ions which travel towards the anode and are liberated there are called *anions* while the +vely charged ions which travel towards the cathode and are liberated there are called *cations*. The products obtained by electrolysis depend upon (*a*) the nature of the solute, (*b*) the nature of the solvent, (*c*) the nature of the electrodes and (*d*) the current density. Faraday carried out extensive researches on electrolysis and based on the results of his experiments gave the following two *laws of electrolysis*.

(1) *The mass of an ion liberated at either electrode during electrolysis is directly proportional to the quantity of electricity passing through the solution*, *i.e.*, to the product of the current strength and the time during which the current flows. Thus, if m be the mass of an ion liberated by a current of strength i flowing for a time t

$$m \propto it$$

or

$$m = Zit \qquad \dots \qquad (27·1)$$

where Z is a constant for the ion and is called its *electro-chemical equivalent*. *The* **electro-chemical equivalent of** *an ion is equal to the mass liberated by a current of one ampere flowing for one second or by one coulomb of electricity.* The unit in which the electro-chemical equivalent is expressed is *gram per coulomb.*

(2) *The mass of an ion liberated by a given quantity of electricity is directly proportional to the chemical equivalent of the ion*, *i.e.*, when the same quantity of electricity is passed through different electrolytes, the masses of the ions liberated are directly proportional to their respective chemical equivalents. Thus, if m_1 and m_2 be the masses of two ions (of chemical equivalents C_1 and C_2 respectively) liberated by the *same* quantity of electricity Q

$$\frac{m_1}{m_2} = \frac{C_1}{C_2} \qquad \dots \qquad (27·2)$$

Now let the electro-chemical equivalent of the two ions be respectively equal to Z_1 and Z_2. Then from first law

$$m_1 = Z_1 Q \quad \text{and} \quad m_2 = Z_2 Q$$

or
$$\frac{m_1}{m_2} = \frac{Z_1}{Z_2} \tag{27·3}$$

Comparing equations (27·2) and (27·3), we get

$$\frac{Z_1}{Z_2} = \frac{C_1}{C_2} \tag{27·4}$$

Thus *the electro-chemical equivalent of an ion is directly proportional to its chemical equivalent* and since the chemical equivalent of hydrogen is unity it follows from equ. (27·4) that the *electro-chemical equivalent of an ion is equal to the chemical equivalent of the ion multiplied by the electro-chemical equivalent of hydrogen.*

From the second law it is evident that the same quantity of electricity is required to liberate *one gram-equivalent* of any ion. This quantity is termed a *faraday* and its value is equal to 96,500 coulombs or 26·8 ampere-hours.

27·2. International ampere, coulomb and volt. When a current of one ampere is passed for one second through a silver nitrate solution it follows from equ. (27·1) that the amount of silver deposited on the cathode will be numerically equal to its electro-chemical equivalent, *i.e.*, 0·001118 gm. This gives us a definition of international ampere. Thus *the* **international ampere** *is that steady current which when allowed to pass through an aqueous solution of silver nitrate deposits silver on the cathode at the rate of* 0·001118 *gm. per second.* The international ampere was intended to be an exact practical realisation of the true ampere, but it is slightly *smaller* than the true ampere.

1 international ampere=0·99997 true ampere

The quantity of charge conveyed through a circuit by a current of one international ampere in one second will be equal to one international coulomb. Thus **international coulomb** *is that quantity of electricity which liberates* 0·001118 *gms. of silver from an aqueous solution of silver nitrate.*

We have already defined international ohm in § 26·2 and hence having defined the international ampere as above, we can now give a definition of international volt from Ohms' law. *The* **international volt** *is that potential difference which will cause a current of one international ampere to flow through a resistance of one international ohm.* It is just a little *bigger* than true volt.

1 international volt=1·00049 true volt

27.3. Copper Voltameter. It consists of a glass jar containing copper sulphate solution into which is immersed a cathode copper

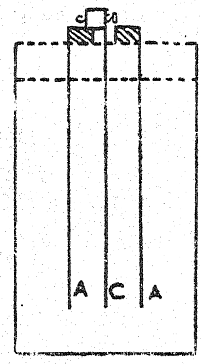


Fig. 27·1

plate C (Fig. 27·1) midway between two parallel copper plates A, A which act as anodes. The two anode plates are supported by a bridge of ebonite (Fig. 27·2) resting on the top of the glass jar and connected to one terminal fixed to the ebonite bridge. The cathode copper plate is attached to the brass block B on the ebonite bridge by a single binding screw S and therefore can be easily removed when required. The electrolyte is a solution of copper sulphate containing about 150 gm. of crystallized CuSO about 50 gms. of pure H_2SO_4 and about 30 c. c. of C_2H_5OH per litre of solution. The sulphuric acid increases the conductivity while the ethyl alcohol prevents the oxidation of the deposited copper.

The solution is dissociated into +ve copper ions and —ve sulphions. On passing the current the copper ions move towards the cathode and the sulphions towards the anode. At the cathode the copper ions are discharged and the ordinary copper is deposited. At the anode the sulphions are discharged and are converted into uncharged group SO_4 which being incapable of separate existence reacts with copper of the anode forming copper sulphate. The strength of the solution is thus *unaltered*. The reactions are :

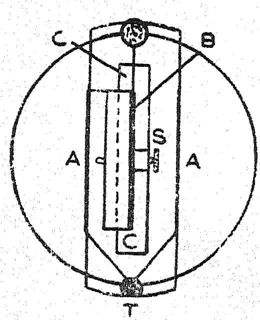


Fig. .27

$$CuSO_4 = \overset{++}{Cu} + \overset{--}{SO^4}$$

$$\text{Cathode} \longleftarrow \qquad\qquad \longrightarrow \text{Anode}$$

$$\overset{++}{Cu} + e\,(\text{—ve}) = Cu \quad ; \quad \overset{--}{SO_4} - 2e\,(\text{—ve}) = SO_4$$

$$SO_4 + Cu = CuSO_4$$

If the anode is not in a condition to be readily attacked by the SO_4, some SO_4 may attack the anode forming $CuSO_4$; some may react with water liberating oxygen which may combine with copper forming CuO. Some of the CuO may dissolve in H_2 SO_4 forming $CuSO_4$. The main result is the *blackening* of the plate with CuO. Note that there is no back E. M. F. due to polarisation as the electrodes are made of copper which is present in the electrolyte in the ionic state.

Experiment 27·1

Object. To determine the *electro-chemical equivalent* of copper using a tangent galvanometer for the measurement of current.

Theory. Let the plane of the coil of the tangent galvanometer be set in the *magnetic meridian*, and let a current of strength i amperes be allowed to pass through it and a copper voltameter connected *in series* with it. Then, if θ be deflection of the needle from the magnetic meridian and m gm. the mass of the copper deposited on the cathode plate, we have from equ. (25·9)

$$i = K \tan\theta \quad \ldots \quad \ldots \quad (7{\cdot}5)$$

where K is the *reduction factor* of the tangent galvanometer and from equ. (27·1)

$$m = Zit \quad \ldots \quad \ldots \quad (7{\cdot}1)$$

where Z is the *electro-chemical equivalent* of (cupric) copper.

Eliminating i between equations (27·5) and (27·1), we get

$$Z = \frac{m}{K\,t.\,\tan\theta}$$

This equation can be used to calculate the electro-chemical equivalent Z of copper if m, t and $\tan\theta$ are determined, the value of K being known.

Note.—For method, etc., see expt. 5.2 chapter XXV.

27·4. The Conductivity of Electrolytes. We have already defined the specific conductivity k of an electrolyte in § 26·7, as the reciprocal of resistivity. Since the ions in the solution act as carrier of electricity, the specific conductivity depends upon the concentration. Hence whenever it is desired to compare different electrolytes their conductivities must be dealt with at *equivalent* or *equi-molecular* concentrations. It is usual therefore to deal with equivalent and molecular conductivities. *The* **equivalent conductivity** Δ *is defined as the specific conductivity k multiplied by the volume V in c. c. containing one gram-equivalent of the electrolyte.* Thus $\Delta = kV$. *The* **molecular conductivity** μ *is the specific conductivity k multiplied by the volume v in c. c. containing one-gram-molecule of the electrolyte.* Thus $\mu = kv$. Both the equivalent and the molecular conductivities increase with dilution until maximum limiting values are reached. *The limiting value of equivalent conductivity is called the* **equivalent conductivity at infinite dilution** Δ_∞.

In contrast to the case of metals the conductivity of electrolytes increases with rise of temperature, the increase being about 2·5% per degree C at about 18°C.

27·5. Measurement of Resistance of an Electrolyte. The usual methods of measuring resistance cannot be used to measure resistance of an electrolyte for when a direct current passes through an electrolyte between two electrodes, a *polarisation* E. M. F. is set up due to the deposition on electrodes of the products of the electrolysis and to changes of concentration near the electrodes. The polarisation E. M. F. is *opposite* in direction to the applied E. M. F. and hence causes an apparent increase in the resistance of the electrolyte. Besides, by the continued passage of current through the electrolyte its concentration changes and so its resistance is altered.

In some cases the polarisation E. M. F. may be eliminated by using electrodes of the material which is present in the ionic state in the electrolyte, *e. g.*, in the case of copper sulphate solution, copper electrodes may be used. Since the polarisation E. M. F. is proportional to the surface density of the deposits on the electrodes, it can be greatly reduced firstly by using electrodes of *large* area, for with a given amount of deposition the larger the area of the electrodes, thinner will be the layer of deposit, and secondly by passing *alternating* current instead of direct current, through the electrolyte when the deposition due to the passage of the current in one direction will be removed by the passage of the current in the opposite direction. If the frequency of the alternating current is sufficiently high, the chemical action at the electrodes are reversed so rapidly as to make the maximum value of the polarisation E. M. F. *inappreciable.* With alternating current the ordinary galvanometer cannot be used as a detector, for the deflection in it is proportional to the first power of the current and is reversed with the reversal of the current. In this case the balance point can be detected either by a vibration galvanometer or, if the frequency of the alternating current is within the range of audible frequencies, by a telephone.

Experiment 7.2

Object. To determine the *specific conductivity* of a given electrolyte with metre bridge.

Apparatus. Electrolytic cell, a metre bridge, a small induction coil, a battery, a plug key, resistance box preferably dial type, a telephone, an interchanging commutator, thermostat and connecting wires.

Theory. Let the electrolytic cell be connected in the *right* gap and a known resistance S in the left gap of the metre bridge. Further, let an induction coil be connected in the usual position of the battery and a telephone in the usual position of the galvanometer. Then, if l be the reading on the scale of the position of the null point on the bridge wire at which the sound in the telephone is *minimum*, we have, from usual Wheatstone bridge relationship,

$$\frac{S}{R}=\frac{l}{100-l}$$

whence
$$R=\frac{100-l}{l}S \tag{27·6}$$

where R is the resistance of the *electrolyte* between the two electrodes.

Now let a be the cross-sectional area of the column of electrolyte and l its length, then the specific conductivity of the electrolyte is given by

$$k=l/aR \tag{27·7}$$

If the distance between the electrodes remains unaltered, l/a is constant for the given electrolytic cell and is called the resistance capacity of the cell or the *cell constant.* Thus, if C be the cell constant, the above equation is modified to

$$k=C/R \tag{27·8}$$

This equation can be used to calculate the specific conductivity of the given electrolyte, if the cell constant C is known. The value of the constant can be determined from the same equation, if the cell is filled with an electrolyte of *known* specific conductivity and the corresponding resistance measured. The electrolyte commonly used for this purpose is potassium chloride.

Description of the Electrolytic cell. As shown in fig. 27·3, the *electrolytic cell* consists of a cylindrical glass vessel of *uniform* diameter, fitted inside with electrodes consisting of circular platinum plates A and B. The plates are sealed into glass tubes which pass vertically through an ebonite cover, the relative positions of the tubes being fixed either by means of a glass tie or by cementing the tubes to the cover. The electrical connections with the electrodes are made by means of mercury. To increase the surface of the electrodes they are coated with platinum black, by immersing them in a platinizing solution (3 gm. of platinum chloride, 0·02—0·03 gm. of lead acetate), passing the current backwards and forwards through the solution a number of times and finally washing the electrodes with warm distilled water several imes until all soluble matter has been removed. The ebonite cover is furnished with two holes one for the insertion of a thermometer and the other for a pipette. When the cell is not being used, these holes are closed with small corks or rubber plugs.

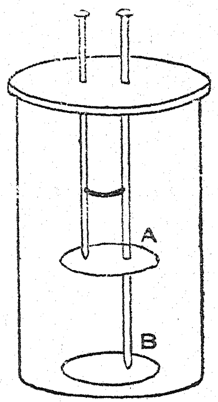


Fig. 27·3

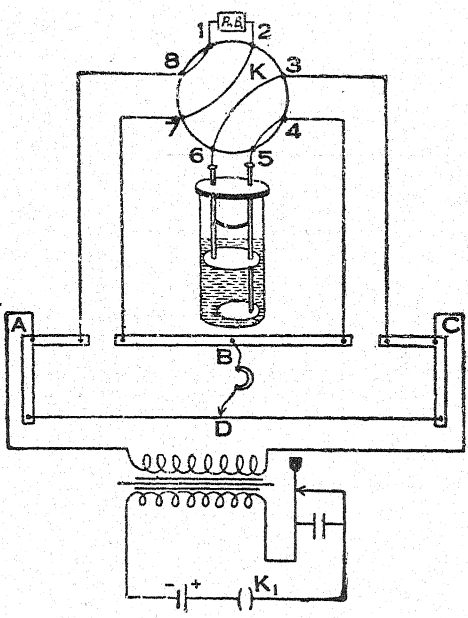


Fig. 27·4

Method. Prepare N/10 or N/100 potassium chloride solution in *conductivity water*. Into the *clean* and *dry* electrolytic cell, introduce a quantity of KC*l* solution sufficient to cover the electrodes. Place the electrolytic cell in the thermostat and regulate its temperature to 25°C. Connect

as shown in fig. 27·4, the electrolytic cell in the right gap of the metre bidge and a resistance box in the left gap of the bridge. Next connect the terminals of the secondary of thei nduction coil to the terminals A and C of the bridge and connect a battery through a plug key K to its primary. Finally connect a low-resistance telephone between the terminal B and the jockey.

Adjust the resistance box to a suitable value S. Close the key K and find by trial the position of the jockey on the bridge wire where the sound in the telephone is almost minimum. Note down the reading on the scale of this approximate position of the null point on the bridge wire and calculate from equation (27·6) the approximate resistance of the KC*l* solution between the electrodes in the cell. Now adjust the resistance box to a value slightly less or greater than this calculated value of resistance of the KC*l* solution. Shift the jockey to the middle on the bridge wire and adjust its position on the bridge wire where the sound in the telephone is *minimum*. Note down the reading *l* on the scale of the exact position of the null point on the bridge wire and calculate the value of resistance of the KC*l* solution from the equation (27.6). Take at least two more sets of observations by slightly decreasing and increasing the value of the known resistance S and calculate the value of the resistance of the KC*l* solution *separately* for each set of observation.

Next the interchange the positions of R and S and again determine the value of the resistance of the KCl solution with the same values of known resistance Then find the mean value of the resistance of the KC*l* solution and taking the value of specific conductivity of the KC*l* solution as given in the following table, calculate the value of the cell constant C from equation (27·8).

Temperature	Specification conductivity of KCl solution	
	N/10	N/100
0°C	0·007138	0·0007736
18°	0·011167	0·0012205
25°	0·012856	0·0014088

Now remove the KCl solution from the electrolytic cell. Rinse the cell and the electrodes several times first with distilled water and then with the electrolyte whose specific conductivity is to be determined. Place a quantity of the electrolyte in the cell and allow it to take the temperature of the thermostat. When the electrolyte has taken the temperature of the thermostat, say after 7—10 minutes, determine its resistance as that of KCl solution above and using the value of cell constant as determined with the help

of KCl solution, calculate the value of the specific conductivity of the electrolyte from equation (27·8). Finally note down the temperature of the thermostat.

Sources of error and precautions. (1) The electrolytic cell should be *clean* and while filling it with KC*l* solution or the given electrolyte, care should be taken that no air bubble is enclosed between the electrodes.

(2) The electrodes should be platinized, *i.e.*, coated with platinum black. This reduces the polarisation E. M. F. as well as increases the sharpness with which the sound minimum in the telephone can be determined. If the electrodes have been freshly platinized, they should be well washed several times with conductivity water until all soluble matter has been removed as tested by determining the resistance of conductivity water.

(3) For accurate work solutions should be made up with especially purified water known as 'conductivity water.' Suitable conductivity water can be readily obtained by distilling the ordinary distilled water with potassium permanganate in a retort made of Jena or 'resistance glass' and closed by tin foil instead of a cork.

(4) The resistances should be non-inductive and of small self-capacitances as possible, otherwise there will not be a perfect silence obtained in the telephone even if the resistances in the four arms of the bridge satisfy the conjugate relation. The higher the frequency of the alternating current the greater is disturbance due to this cause. The connecting wires should never be coiled.

(5) The alternating current should be small and should be passed through the apparatus only when observations are being made, thus preventing the heating of various conductors and consequent change in their resistances. The frequency of the current should not be less than 100 cycles per second. The alternating current should not be derived from the supply mains, firstly because abrupt changes of current give more distinct sound in the telephone and secondly because it is almost impossible to obtain an exact null point with currents of frequency 50 cycles per second and the point of contact of the jockey with the bridge wire can be varied considerably without perceptibly affecting the telephone.

(6) For minimum error in the result due to small error of reading l, the null point should be as close to the middle of the bridge wire as possible, while for maximum sharpness of sound minimum in the telephone the null point should be near the end of the bridge wire. To balance the errors of setting and of reading, therefore, a compromise should be affected and the null point adjusted to lie between 20 and 40, or 60 and 80 cm. of the bridge wire. The value of S should be adjusted accordingly.

(7) The actual silence in the telephone at the null point is seldom obtained and hence the null point should be determined by adjusting the position of the point of contact of the jockey with

the bridge wire so that *minimum* sound is produced in the telephone. If no definite point of sound minimum is obtained, the centre of the range over which the sound is minimum should be taken as the true null point.

(8) To eliminate error due to non-coincidence of the pointer of the jockey with the metallic edge on underside of it which comes in contact with the bridge wire, two separate measurements of unknown resistance should be made, one by introducing the unknown resistance in the left gap and the other by transferring it to the right gap. Note that when the unknown resistance R is in the left gap, $R = \frac{l'}{100-l'}$, S, where l' is the reading of the position of the null point on the scale.

(9) The jockey should always be pressed gently and its contact with the bridge wire should not be made while it is being moved along.

(10) As the conductivity of electrolytes increases with temperature, the variation being about 2·5% per degree C at about 18 °C, the temperature of the electrolytic cell should be kept constant while measurements are being made. Accordingly the cell should be placed in a thermostat which must be so regulated that the fluctuation of temperature does not exceed 0·05°—0·1 °C. The temperature should be carefully observed and stated when the results are given.

Observations.

Name of liquid in the electrolytic cell	S. No.	Known resistance S ohms	Position of balance point with the electrolytic cell in the						Mean R ohms
			Right gap			Left gap			
			l cm.	$100-l$ cm.	R ohms	l' cm.	$100-l'$ cm.	R ohms	
N/ KC*l* solution	1.								
	2.								
	3.								
	1.								
	2.								
	3.								

Temperature of the electrolytic cell= °C

Specific conductivity of the N/ KCl solution at °C=
Calculations. Resistance of KCl solution

Cell in the left gap $R = \frac{l'}{100-l'} S$

$=$ ohms

Cell in the right gap $R = \frac{100-l}{l} S$

$=$ ohms

Similarly calculate resistance of the given electrolyte

Cell constant $C = k R$

$=$

Specific conductivity of the given electrolyte

$$k = \frac{C}{R}$$

$=$ mhos per cm.

Result. The specific conductivity of the given electrolyte (............) at °C = ohms per cm.

Criticism of the method. By platinizing the electrodes and using alternating current, the polarisation E. M. F. has been *greatly* reduced. The value of cell constant has been determined by measuring the resistance of KCl solution of known specific conductivity and hence all uncertainties in its value which creep in when it is calculated by measuring the area of the electrodes and their distance apart have been eliminated. For small distances between the electrodes the lines of flow between them are curved and hence the area of flow between them is greater than the area of the electrodes.

The main difficulty in the method is that perfect silence in the telephone is never obtained as a result of which the exact position of the null point cannot be correctly determined. The sharpness with which the null point can be determined by noting sound minimum depends upon (*i*) the frequency of the alternating current, (*ii*) the self-inductance and the self capacitance of the resistance coils, *iii*) the resistance of the electrolyte, and the size of the electrodes, their distance apart and the nature of their surface, and can be increased by increasing the frequency of the current, by decreasing the self-inductance and capacitance in the resistances and by platinizing the electrodes. For very accurate results alternating current of suitable frequency, say 1000 cycles per second, generated by means of a valve oscillator, should be used and the capacitance of the two sides of the bridge should be balanced by connecting a variable air condenser in parallel with the resistance box S. The accuracy of result also depends on the uniformity of the bridge wire. Consequently if there is any doubt about it, the bridge wire should be accurately calibrated.

Cells

27·6. Simple cell. Referring to fig. 27·5, if two plates, one A of copper and the other B of zinc, are immersed in dilute sulphuric acid contained in a glass vessel, they at once acquire a P. D., the copper plate being at *higher* potential. Consequently, if the two plates are connected by means of a metal wire outside, a conventional electric current flows immediately from copper along the wire to zinc in an attempt to equalize the potential of the two plates. The conventional current from copper to zinc through the wire is equivalent to flow of electrons from zinc along the wire to copper. The departure of electrons from zinc makes it less negative, *i.e.*, *raises* its potential, while their arrival at the copper plate makes the latter less positive, *i.e.*, *lowers* its potential. Consequently, if no other action took place inside the cell the P. D. between the two plates would soon disappear and the current would stop. But actually, as electrons leave zinc the latter dissolves as positively charged zinc ions giving out negative charges until equilibrium is established, *i.e.*, the *original* potential of zinc is restored.

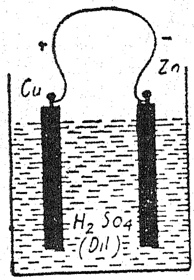


Fig. 27·5

$$Zn = Zn^{++} + 2 \text{ elementary } \textit{negative} \text{ charges}$$

The zinc ions combine with sulphions already present in the dissociated sulphuric acid forming zinc sulphate.

$$Zn^{++} + H_2So_4 = ZnSO_4 + 2H^+$$

The energy evolved during the formation of zinc sulphate forces the positively charged hydrogen ions towards the copper plate where they are discharged giving out their elementary positive charges to copper

$$2H^+ = H_2 + 2 \text{ elementary } \textit{positive} \text{ charges}$$

These positive charges on copper neutralise the elementary negative charges coming from zinc along the wire. This maintains the original P. D. between copper and zinc and hence the current in the circuit.

The above arrangement, *i.e.*, a copper plate and a zinc plate placed in dilute sulphuric acid and joined together by means of wire, is known as a *simple*, galvanic or voltaic cell. The liquid inside it is called the *electrolyte*. The part of the copper plate outside the liquid is called the *positive pole* and the part of the zinc plate outside the liquid is called the *negative pole*. The E. M. F. of the cell, *i.e.*, the P. D. between the two poles of the cell when on open circuit is 1·1 volts.

It is clear from above that *the energy required to maintain the E. M. F of the cell and hence the current in the circuit is*

furnished by the chemical action which takes place inside the cell, for it is this energy which forces the positively charged hydrogen ions towards copper, *i.e.*, drives the current from zinc to copper inside the cell. In fact chemical action which takes place inside the cell is similar to that of a pump lifting water from a lower level to a higher one from where the water will naturally run down again in virtue of the energy conferred upon it. It should be noted that if the formation of zinc sulphate from zinc and sulphuric acid takes place in the ordinary fashion, the chemical energy appears as heat of reaction instead of as electrical energy.

In practice a simple cell gives a steady current only if the latter is *very small* or if the cell is used *intermittently*. A simple cell suffers from two defects : (*a*) local action and (*b*) polarisation.

27·7. Local action. When the negative pole of the simple cell is made of *pure zinc*, it is not acted upon by dilute sulphuric acid when in contact with it until the two poles are connected together externally by means of a metal wire. Consequently there is no wastage of zinc, it being consumed only when the cell is in action. But when *commercial* zinc is used as the negative pole, the moment it comes in contact with the acid it is dissolved and hydrogen is evolved *even when the two poles are not connected externally by metallic conductor*. This is due to the presence of impurities like carbon, iron, arsenic, lead, etc., in the commercial zinc. These impurities together with zinc, when in contact with the acid, form small cells with the result that *local currents* are produced. These local currents are of no use absolutely. Consequently much zinc is consumed without any advantage being gained therefrom. This effect with the commercial zinc is called **local action.**

This can be avoided by covering the zinc plate with an *amalgam of mercury*. The zinc plate is first dipped in dilute sulphuric acid to remove zinc oxide. Then mercury is rubbed over its surface with a rag soaked in dilute sulphuric acid. Mercury combines with zinc forming a uniformly soft amalgam which covers up the impurities. This amalgam is not attacked by the acid so long as the cell circuit is not closed. When the cell is in action, zinc is eaten up by the acid and impurities fall to the bottom. Amalgamation of zinc also prevents local currents between portions of the plate differing in hardness.

27·8. Polarisation. When a current is drawn from a cell, its E. M. F. falls off steadily owing to an effect known as **polarisation.** The polarisation in a cell is really the resultant of a number of effects, namely, (*a*) the accumulation of hydrogen on the positive pole, (*b*) temporary changes in concentrations of ions near the poles, and (*c*) permanent change of concentration of ions in the electrolyte.

(a) Accumulation of hydrogen on the positive pole. When hydrogen is evolved at the positive pole, it does not escape freely. Some bubbles adhere to the positive plate and form a layer on it. This

layer of hydrogen, owing to the presence of electric field, forms a double layer of positive and negative ions which develops a back E. M. F. inside the cell as a result of which the E. M. F. of the cell decreases. In addition, hydrogen being an excellent insulator decreases the effective area of the plate and thereby increases the internal resistance of the cell.

There are three methods of preventing polarisation due to accumulation of hydrogen on the positive pole of a cell :—

(i) *Mechanical.* If the positive plate is brushed now and then, hydrogen is not deposited permanently on it and hence the cell will not be polarised on this account. This method is obviously very tedious. Smee suggested the deposition of finely divided platinum on the positive plate so that the surface may become pointed and burst the bubbles of hydrogen as soon as they appear on it. This method is also not very successful.

(ii) *Chemical.* If strong oxidizing agent is present in the cell, hydrogen may be oxidized to water as swiftly as it is formed. This method of removing hydrogen is fairly successful and is used in Leclanche cell, Bicromate cell, Grove cell, etc. The substances used as oxidizing agents are known as *depolarisers.*

(iii) *Electro-chemical.* If the metals forming the two poles of the cell be dipped into solution of their *own* salts respectively, the solutions being prevented from mixing by means of a porous pot which allows free passage only to ions under the influence of electric field, then the positive plate will receive, instead of hydrogen, an additional amount of the material of which it is made. This method of preventing the deposition of hydrogen on the positive plate is the best and is used in Daniell cell, Clark cell and Cadmium cell.

(b) Temporary changes in the concentration of ions near the poles. Unless the current drawn from a cell is very small, the rate of diffusion of positively charged ions from the negative pole towards the positive pole is less than the rate of supply of ions from the negative pole. Hence there is a net increase in the concentration of ions near the negative pole which results in lowering of its electrode potential. Similarly, if there is a decrease in the concentration of ions near the positive pole, its electrode potential is also lowered. This reduction in the electrode potentials due to change in concentration of ions near the electrodes can be recovered to some extent by allowing the cell to stand without supplying a current.

(c) Permanent change of concentration of ions in the electrolyte. When the metal from the negative pole goes into solution as ions or the ions from the electrolyte after having been discharged are deposited on the positive pole as metal, there is a net permanent change in the concentration of ions in the electrolyte resulting in polarisation of the cell due to consequent change in electrode potentials. This can be eliminated by immersing the poles in the saturated solutions of their respective salts. This method, however, reduces the maximum E. M. F. of the cell. This method is used in

Daniell cell, Clark cell and Cadmium cell. Another method which eliminates this type of polarisation and at the same time keeps the E. M. F. of the cell quite high is to introduce a substance which forms complex ions and thus keeps the maximum concentration of free ions of metals forming the poles very low. This method is adopted in Leclanche cell where in the ammonium chloride solution, complex ions $(ZnNH_3)_2^{++}$ are readily formed.

27·9. Daniell cell. As illustrated in fig. 27·6, it cons sts of a copper plate (+) dipping in saturated solution of copper sulphate which acts as a *depolariser* and is contained in a glass vessel or earthenware, and amalgamated zinc rod (—) dipping in dilute sulphuric acid or a semi-saturated solution of zinc sulphate contained in a porous pot which is placed in the copper sulphate solution. When the copper plate and the zinc rod are connected together by means of a wire outside, an electric current flows from copper through the wire to zinc and the following reactions take place inside the cell:—

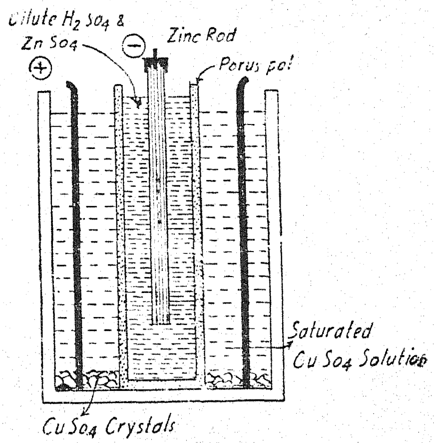


Fig. 27·6

$$Zn = Zn^{++} + 2 \text{ elementary } \textit{negative} \text{ charges}$$

$$Zn^{++} + H_2SO_4 = ZnSO_4 + 2H^+$$

$$2H^+ + CuSO_4 = H_2SO_4 + Cu^{++}$$

$$Cu^{++} = Cu + 2 \text{ elementary } \textit{positive} \text{ charges}$$

Thus, when the cell is in action, zinc is dissolved from the zinc rod while copper is deposited on the copper plate. Consequently the strength of zinc sulphate solution increases while that of copper sulphate solution decreases. This change can be represented by the equation

$$Zn + CuSO_4 = ZnSO_4 + Cu$$

The strength of the copper sulphate solution is, however, kept constant by keeping it in contact with crystals of copper sulphate. The energy liberated during the above reaction maintains the current in the circuit. The E. M. F. of the cell is 1·08 volts and its value remains fairly constant. The internal resistance of the cell is high. The cell is capable of giving steady but *small* current.

27·10. Leclanche cell. As depicted in fig. 27·7, in a Leclanche cell a carbon rod (+) is placed in a porous pot and is packed

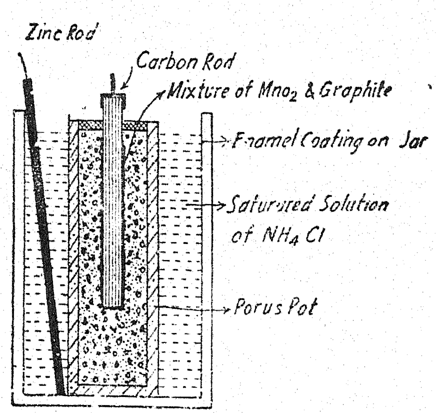


Fig. 27·7

round with a mixture of powdered manganese dioxide and graphite which acts as a *depolariser*, the graphite being added to MnO_2 to increase the conductivity. The porous pot is placed in an outer glass jar containing a saturated acqueous solution of ammonium chloride into which is dipping an amalgamated zinc rod (—). The NH_4Cl solution diffuses through the porous pot and impregnates the mixture of $Mn\,O_2$ and graphite round the carbon rod. The creeping of the NH_4Cl solution to outside of the jar is prevented by coating its upper parts with suitable enamel (black compound). When the cell is being used the following reactions take place inside the cell :—

$$Zn = Zn^{++} + \text{ elementary } \textit{negative} \text{ charges}$$

$$Zn^{++} + NH_4Cl = Zn\,(NH_3)_2\,Cl_2 + H^+$$

$$H^+ = H_2 + 2 \text{ elementary } \textit{positive} \text{ charges}$$

$$H_2 + 2MnO_2 = Mn_2O_3 + H_2O$$

Thus the net result is the consumption of zinc and the reduction of MnO_2 to Mn_2O_3. The Mn_2O_3 is, however, slowly oxidized back to MnO_2 by atmospheric oxygen. The E. M. F. of the cell is 1·46 volts. The action of the depolariser is very slow and the internal resistance is high. Consequently the cell is suitable only for *small* and *intermittent* currents.

27·11. Standard cells. The E. M. Fs. of the cells described above do not remain constant when they are being used and also vary with changes in temperature and changes in concentrations of ions in solution. Consequently these cells cannot be used as standards. It is desirable that a standard cell should have a small temperature coefficient of E. M. F. and materials used in it should be readily obtainable and easily purified. On the other hand as a standard cell is not required to give an appreciable current, its size may be very small, and hence its internal resistance very high. The two cells which are used as standards are the Latimer Clark cell and the Weston Cadmium cell.

(a) Latimer Clark cell. As shown in fig. 27·8, it consists of a positive pole of mercury in contact with a layer of crystals of mercurous sulphate which acts as *depolariser* and a negative pole of zinc mercury amalgam (Zn 10%) dipping in a saturated

solution of zinc sulphate in equilibrium with crystals of $ZnSO_4$ which are in contact with the zinc-mercury amalgam. The mercurous sulphate should be *exceptionally pure* for any trace of impurity in it has much greater effect on the permanence of E. M. F. of the cell than that produced by slight impurity in any of the other chemicals employed. The containing vessel is a small test tube of about 2 cm. in diameter and 4 to 5 cm. in length. Contact with mercury is made either by means of a

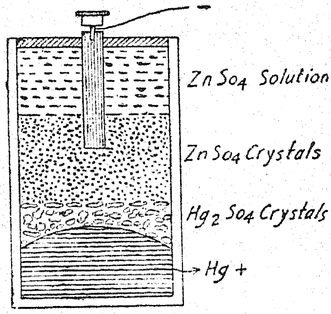


Fig. 27·8

platinum wire sealed through the glass as shown in the figure or by means of a platinum wire protected by a glass tube. The reactions which take place when the cell supplies current are :

$$Zn = Zn^{++} + 2 \text{ elementary } \textit{negative} \text{ charges}$$
$$Zn^{++} + ZnSO_4 + Hg_2SO_4 = 2ZnSO_4 + 2Hg^+$$
$$2Hg^+ = 2Hg + 2 \text{ elementary } \textit{positive} \text{ charges}$$

The E. M. F. of the cell at $15°C$ is 1·4328 international volts and its value at any temperature $t°C$ is given by

$$E = 1{\cdot}4328 - 0{\cdot}00119\,(t-15) - 0{\cdot}000007\,(t-15)^2 \text{ international volts}$$

The disadvantages of the cell are:—(*a*) High temperature coefficient of E. M. F., (*b*) difficulty of ascertaining the exact temperature of the cell and of keeping it constant, (*c*) large hysteresis effects attending temperature, (*d*) cracking tendency at the point of introduction of negative terminal wire and (*e*) interruption of the circuit by a layer of gas which is often formed in the cell.

(b) **Weston Cadmium cell.** It is used as a concrete standard of E. M. F. As illustrated in fig. 27·9, the positive pole of the cell is of mercury which is in contact with an almost solid paste of mercurous sulphate and mercury which serves as the *depolariser.* The mercurous sulphate should be *exceptionally pure* if instability of E.M.F. is to be avoided. The negative pole is of cadmium-mercury amalgam (one part of cadmium in 7 parts of mercury). The electrolyte is a saturated solution of cadmium sulphate in equilibrium with crystals of cadmium sulphate.. The container is generally in the form of H, the vertical sides being about 3''

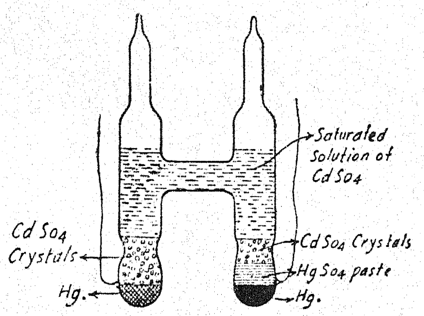


Fig. 27·9

long and $\frac{1}{2}''$ in diameter. There is a *constriction* in lower parts of the tubes which forms a taper plug of crystals. This holds everything in place and also makes the cell much more portable and safe during transit. Contact of electrodes to the external circuit is made by means of platinum wires sealed through the glass. The reactions which take place in the cell when being used are :

$$Cd = Cd^{++} + 2 \text{ elementary } \textit{negative} \text{ charges}$$
$$Cd^{++} + CdSO_4 + Hg_2SO_4 = 2Cd\,SO_4 + 2Hg^{+}$$
$$2Hg^{+} = 2Hg + 2 \text{ elementary } \textit{positive} \text{ charges}$$

The E. M. F. of the cell at 20°C is 1·0183 international volts and at any temperature t°C, its value is given by the formula

$$E = 1{\cdot}0183 - 0{\cdot}0000406\,(t-20) - 0{\cdot}00000095\,(t-20)^2 + 0{\cdot}00000001\,(t-20)^3 \text{ international volts}$$

The main advantages of the cell are:—(*a*) its long life and (*b*) low temperature coefficient of E. M. F.

The E. M. Fs. of standard cells are constant for the passage of *very small* current only. Hence standard cells are used only in *null* methods of measurement, *e. g.*, in potentiometer. *A high resistance should be connected in series with a standard cell to protect it during initial stages of manipulations* and may be removed when approximately balanced conditions are obtained. Proper care should also be taken in handling the cell, for any appreciable shaking up of the material in the cell tends to produce variations in the E. M. F. of the cell.

7·1. Characteristics of various primary cells. In all the cells described so far, the chemical reactions which take place are *irreversible*. The negative pole is used up irreversibly as the cells supply current. Such cells are known as *primary cells*. They require the replacement of the negative pole, the electrolyte and the depolariser. The table on the next page gives the characteristics of important primary cells.

27·13. Principle of a Secondary cell. Referring to fig. 27·10, let a voltameter V consisting of two large sheets of platinum dipping in dilute sulphuric acid be placed in series with a galvanometer G and let the combination be connected on one side, through a two-way key, to a battery B and on the other side to a copper connecting wire. When the battery circuit is completed by connecting a to b, a current flows through the voltameter which brings about electrolysis liberating oxygen at the anode A and hydrogen at the cathode K. These gases do not escape freely, a certain amount remaining clinging to the respective electrodes. This, on account of high electrolytic solution pressure of the gases at the respective electrodes, tends to cause the gas molecules to enter as ions into the electrolyte, there—

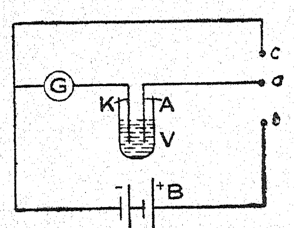


Fig. 27·10

Name of cell	Positive pole	Negative pole	Electrolyte	Depolariser	E. M. F.	Internal resistance	Remarks
Simple	Copper	Zinc	dil H_2SO_4	...	1·1 volts	High and unsteady	Suffers from polarisation, E. M. F. not constant
Daniell	Copper	Zinc	dil H_2SO_4	$CuSO_4$	1·08 volts	About 3 to 4 ohms and fairly constant	E. M. F. fairly constant, useful in telegraphy, etc.
Leclanche	Carbon	Zinc	Sol. of NH_4Cl	MnO_2	1·46 volts	Fairly high and increases with use	Action of depolariser very low, E. M. F. unsteady, suitable for intermittent work, testing bells, etc.
Bichromate	Carbon	Zinc	dil H_2SO_4	H_2CrO_4	2 volts	Very low	Inconsistent current; strength of current considerable for a short time, recovers when not used
Bunsen Grove	Carbon platinum	Zinc	dil H_2SO_4	Strong HNO_3	1·9 volts	Fairly low	E. M. F. constant, current considerable for some time
Clark cell	Mercury	Zinc mercury amalgam.	Saturated sol of $ZnSO_4$	Mercurous sulphate	1·4328 int. volts 15°C	Very high 500 to 1000 ohms depending upon size.	E. M. F. constant, high temp. coefficient, used as standard of E. M. F.
Cadmium cell	Mercury	Cadmium mercury amalgam	Saturated sol. of $CdSO_4$	Mercurous sulphate	1·0183 int. volts at 20°C	Very high depends upon size usually 900 ohms.	E. M. F. constant and almost independent of temp., used as standard of E. M. F.

by decreasing the cathode potential and increasing the anode potential. The result is that an E. M. F. is set up between the electrodes which tend to *oppose* the current sent through the voltameter by the battery. This E. M. F. is called *back* E. M. F. or polarization E. M. F. If now after some time the battery circuit be broken and *a* connected to *c*, a current flows through the galvanometer in the *reversed* direction. This current is due to the polarization E. M. F. and is called *polarization current*. It is of very short duration and *lasts only as long as there are still gas molecules adhering to the electrodes*.

If the above experiment is repeated by passing current through a lead voltameter consisting of two lead plates in dilute sulphuric acid, then as *a* is connected to *c*, the galvanometer needle will again be deflected in the reversed direction due to the polarization current. But, if the experiment is performed with a copper voltameter consisting of two copper plates dipping in copper sulphate solution, no polarization current is produced. In fact whenever chemical action occurs in a circuit due to the passage of a current, *the formation of a compound produces a forward E. M. F. while the breaking up of a compound produces a back E. M. F.* When current is passed through a copper voltameter, copper goes into solution at the anode with the formation of copper sulphate and thus producing a *forward* E. M. F. and goes out of solution at the cathode with the breaking up of copper sulphate and thus producing an equal *back* E. M. F. The result is that there is no polarization E. M. F. in a copper voltameter.

The above experiments show that the passage of a current through a voltameter, in certain cases, produces a *back* E. M. F. in it which makes the voltameter able to act *temporarily* as a voltaic cell, the anode and the cathode of the voltameter becoming respectively the positive and the negative pole of the cell. This is the principle of a *secondary cell*.

It should be noted that in a primary cell chemical energy is converted into the electrical energy and that the chemical reactions which take place inside the cell are *irreversible*, while in secondary cell the electrical energy is *stored* during electrolysis as potential chemical energy which is reconverted into the electrical energy when the current is drawn from the cell and that the chemical reactions which take place inside the cell are *reversible*. The secondary cells behave as though they are reservoirs of electricity and are, therefore, also called *storage cells* or *accumulators*.

27·14. Accumulators. There are two types of accumulators which are of practical importance, namely, the *acid* accumulators and the *alkali* accumulators.

(a) **The acid or lead accumulator.** It consists of (Fig. 27·11) a series of parallel lead plates constructed in the form of grid (Fig. 27·12) and placed in a glass vessel containing a 20% solution of sulphuric acid. Into the interstices of the grids is pressed

mechanically a paste of lead sulphate formed by mixing lead oxide PbO (litharge) with conc. sulphuric acid. The paste sets itself hard after a short while. The hardness can be increased by the addition of a little glycerine and other substances. It should be noted that the grids are merely supports for the active material and secure the paste more effectively than the ordinary lead plates. The cast lead of which they are made is an alloy of antimony and lead which is much stiffer and stronger than pure lead. The alternate plates are soldered to one common lead rod and form the anode while the remaining plates are soldered to another common lead rod and form the cathode. The anodeset has always one plate *less* than the cathode set, because active material in the positive plate expands when the cell is being charged and if all the expansion took place on one side the plate would be distorted out of shape.

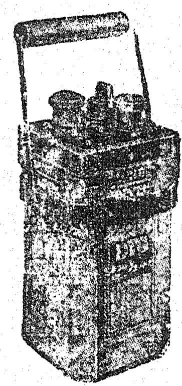


Fig. 27·11

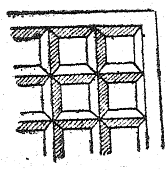


Fig. 27·12

When the current is passed through the cell, the acid solution is electrolysed. The sulphions SO_4^{--} move towards the anode where lead sulphate is converted into dark brown lead peroxide and two elementary negative charges are set free.

$$PbSO_4+SO_4^{--}+2H_2O=PbO_2+2H_2SO_4+2 \text{ elementary negative charges}$$

The hydrogen ions H^+ travel towards the cathode where lead sulphate is reduced to porous spongy lead and two elementary positive charges are set free.

$$PbSO_4+2H^+=Pb+H_2SO_4+2 \text{ elementary } \textit{positive} \text{ charges}$$

The above process is called *forming the plates.* During the formation of plates a *polarization* E. M. F. *develops* which acts within the cell from Pb to PbO_2. When the formation of plates is complete, the cell is said to be charged and is then ready for use. The E. M. F. of the cell when fully charged is about 2·1 volts and its temperature coefficient is about 0·0003 volts per degree. During charging of the cell the concentration of sulphuric acid increases. Indeed the state of charge of the accumulator is estimated from the density of the acid which is about 1·24 when it is *fully* charged.

When the charged accumulator is used to supply current through an external circuit, the PbO_2 plates form the positive pole and the Pb plates the negative pole of the cell. During discharging of the cell, the hydrogen ions H^+ travel towards the positive pole where the lead peroxideis converted into lead sulphate and two elementary positive charges are liberated.

$PbO_2 + 2H^+ + H_2SO_4 = PbSO_4 + 2H_2O + 2$ elementary *positive* charges.

The sulphions move towards the negative pole where the lead is converted into lead sulphate and two elementary negative charges are set free.

$Pb + SO_4^{--} = PbSO_4 + 2$ elementary *negative* charges

During discharge the concentration of the acid decreases and the E. M. F. of the cell falls. When the density of the acid falls to about 1·15, it is regarded as discharged and requires recharging before it can be further used to supply current. The E. M. F. of the cell when discharged is about 1·8 volts.

As the effective area of the plates is very large and the distance between them is very small, the internal resistance of the cell is *negligible*, being only a small fraction of an ohm. The cell should therefore, *be never short circuited* otherwise a heavy current will flow and cause sulphating, disintegration of active material and buckling of the plates.

The **efficiency** of an accumulator is given by the expression

$$\text{Efficiency} = \frac{\text{Watt-hours given out at discharge}}{\text{Watt-hours put in at charge}}$$

It is of the order of about 70%.

The **capacity** of an accumulator is measured in *amp-hours*, which indicates *the quantity of electricity which the cell can supply before it needs recharging.* For example, an accumulator having a capacity of 100 amp-hours can supply $100 \times 60 \times 60 = 36 \times 10^4$ coulombs of charge before recharging, *i.e.*, a steady current of 5 amps. for 20 hours. The capacity of a cell depends on the design of the plates of the cell as well as its rate of discharge, being much greater for weaker currents than that for large currents.

The main disadvantages of an acid accumulator are: (i) short life due to gradual falling of active material from the positive plate resulting in lowering of its capacity, (ii) high initial cost, (iii) low efficiency and (iv) considerable weight which renders it not very portable.

(b) The alkali accumulator. There are two types of alkali accumulators, namely, the nickel-iron or Edison cell and the nickel-cadmium or the NIFE, cell. Both the cells have the same electrolyte, about 20% solution of potassium hydroxide to which a small amount of lithium hydroxide is added to increase the capacity of the cell by about 10%. They also use the same type of positive plate the active material of which is nickel-hydroxide mixed with finely divided nickel to render the former conducting. In the Edison cell (nicrel iron), the negative plates contain finely divided iron mixed with yellow mercuric oxide, the function of the latter being to increase the couduc-

tivity as well as the capacity of the cell. In the NIFE (nickel-cadmium) cell, the negative plates contain cadmium. The plates are constructed in the form of flat steel tubes, perforated by a large number of small holes for entry of the electrolyte and the active material is pressed into the tubes. This results in very strong plates practically free from buckling and sediment formation. The container is usually steel plated on its outer surface to prevent it from atmospheric corrosion. To prevent the absorption of air into the electrolyte which lowers the capacity of the cells, the container is provided with an air-tight valve.

The reactions which take place inside the Edison cell during charging and discharging are as follows :

Discharge

At the positive plate—

$2\ Ni\ (OH)_3+2K^+=2Ni\ (OH)^-+2KOH+$ 2 elementary positive charges

At the negative plate—

$Fe+2OH^-=Fe\ (OH)_2+$ 2 elementary *negative* charges

Adding the two equations, we get

$$2Ni\ (OH)_3+Fe+2KOH=Ni\ (OH)_2+Fe\ (OH)_2+2\ KOH$$

or $$2Ni\ (OH)_3+Fe=2Ni\ (OH)_2+Fe\ (OH)_2$$

Charge

At the positive plate—

$2\ Ni\ (OH)_2+2OH^-=2Ni\ (OH)_3+$ 2 elementary *negative* charges

At the negative plate—

$Fe\ (OH)_2+2K^+=Fe+2KOH+$ 2 elementary *positive* charges

Adding the two equations, we get

$$2\ Ni\ (OH)_2+Fe(OH)_2+2KOH=2Ni(OH)_3+Fe+2KOH$$

or $$2\ Ni\ (OH)_2+Fe\ (OH)_2=2\ Ni\ (OH)_3+Fe$$

From the above set of equations it is evident that the concentration of the electrolyte remains constant during charge or discharge, OH radical being simply transferred between nickel hydroxide and iron.

When *fully* charged the E. M. F. of the alkali cell is about 1·35 volts. Its internal resistance is also low. Its efficiency is comparatively less than that of the acid accumulator, being about 50%. Its main advantages are :—(i) its insensitivity to mechanical vibrations and heavy discharge, (ii) its long life compared to that of an acid accumulator, (iii) low wt. capacity ratio, (iv) its practically constant E. M. F. even when left idle, and (v) little attention for its care.

27·15. Internal Resistance of a Secondary cell. The usual voltmeter-ammeter method or the potentiometer method for determining the internal resistance of a cell is not suitable in the case of

a secondary cell or any other cell of very low resistance for, to produce a measurable fall of potential difference across the terminals of the cell, the current drawn from the cell would be too great which would damage the cell as well as burn the resistance in the external circuit. The following *differential method* is applicable in the case of any cell of low internal resistance.

Experiment 27·3

Object. To determine the *internal resistance* of an accumulator.

Apparatus. Two *similar* accumulators, a millivoltmeter, an ammeter, rheostat of a small value, say about 20 ohms, a tapping key and connecting wires.

Theory. Let the *positive* terminals of two *similar* accumulators be connected to a sensitive voltmeter, and let their negative terminals be joined together. Then, if the E. M. Fs. of the two accumulators are equal the voltmeter will record no potential difference. Now let a resistance R be connected across one of the accumulators. A current i will flow through the resistance R, and since the resistance of the voltmeter is *very high,* this current i will be drawn only from the the accumulator across which R has been connected. Consequently potential difference across the terminals of *this* accumulator will fall by an amount ir, where r is its *internal resistance.* This fall in P. D. will be recorded by the voltmeter as v. Hence

$$r=v/i$$

From this expression the internal resistance r of the accumulator can be calculated, if v and i are determined.

Alternatively, a graph may be plotted between v and i and the *slope* of the *straight line* thus obtained will give *internal resistance* r of the cell.

Method. Connect two *similar* accumulators *in parallel* as shown in fig. 27·13 with a millivoltmeter across their +*ve* terminals, the +*ve* terminal of the accumulator of the *higher* E. M. F., if the E. M. Fs. differ slightly, being connected to the + marked terminal of the voltmeter. Connect a rheostat of about 20 ohms and an ammeter A in *series* with one of the accumulators if their E. M. Fs. are exactly equal or with the accumulator of *lower* E. M. F. if they differ slightly, including a tapping key in the circuit.

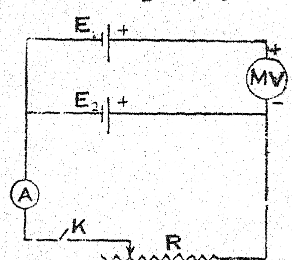


Fig. 27·13

With key K *open* note down the reading of the millivoltmeter. This will be zero only when the E. M. Fs. of the two accumulators are *exactly* equal. Next adjust the rheostat to a high resistance R, say about 15 ohms and depress the key K. A current will flow through the circuit and the

millivoltmeter reading will change. Note down the reading of the millivoltmeter. The difference between the two readings of the voltmeter gives the value of v, the *fall* in P. D. across the accumulator E_2 supplying the current i. Then calculate the internal resistance of the accumulator from the formula $r=v/i$. Repeat the experiment ten times with different values of R and find out the *mean* value of r.

Next plot a graph between v and i. This will come out to be a *straight line* as shown in fig. 27·14. Find the *slope* of this line which will give r.

Note that r gives the internal resistance of the accumulator E_2.

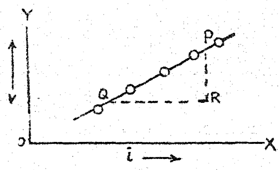


Fig. 27·14

Sources of error and precautions. The E. M. Fs. of the two accumulators should be equal. If they differ slightly, *this difference should not exceed a few millivolts.*

(2) The two accumulators should be *fully* charged otherwise the intial reading of the millivoltmeter will change after each observation.

(3) The *fall* in potential difference v across the terminals of the accumulator E_2 when current is drawn from it is *very small.* Hence to measure it accurately a *sensitive* voltmeter, *i.e.*, a millivoltmeter should be used. Similarly a sensitive ammeter should be used for measurement of current i.

(4) Special care should be taken in connecting the *positive* terminals of the accumulators to the millivoltmeter for, *if by chance two dissimilar terminals are connected to it the millivoltmeter will be burnt out.* In case the E. M. Fs. of the accumulators differ slightly, the *positive* terminal of the accumulator of higher E. M. F. should be connected to the +marked terminal of the millivoltmeter.

(5) The +vely marked terminal of the ammeter should be connected to the *higher* potential point of the circuit.

(6) The series combination of rheostat and the ammeter should be connected across the cell of *lower* E. M. F., E_2. If this is done with the cell of higher E. M. F., then, on depressing the key K, the needle of the millivoltmeter will move in the *wrong* direction.

(7) Before depressing the key K, it should be carefully ascertained that the rheostat has been adjusted to give an *appreciable* resistance, otherwise a heavy current will flow through the circuit which may damage the rheostat, the ammeter and even the cell. *On no account the rheostat should be removed or its resistance reduced to zero.*

(8) For determination of fall of potential v of the cell E_2 under test, the key K should be depressed *momentarily* and the millivoltmeter read *immediately* for thereafter the needle of the millivoltmeter often *creeps* forward.

(9) The graph between v and i should be *straight line* and should be smoothly drawn.

Observations.

Least count of millivoltmeter = millivolts

Least count of ammeter = amp.

S. No.	Millivoltmeter reading		v volt	i amp.	r ohm
	Key open millivolt	Key closed millivolt			
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
				Mean	

Calculations. Set I $r=\frac{v}{i}$

$=$ ohms.

(N. B.—Make similar calculations for other sets)

From graph (27·14)

P R= Q R =

$\therefore$ $r=$ PR/QR = = ohms.

Result. The *internal resistance* of the accumulator

(*i*) by calculations = ohms

(*ii*) by graph = ohms

Criticism of the method. It is the only practical method of determining the internal resistance of an accumulator. Using a sensitive millivoltmeter and freshly charged cells, the method yields accurate results. The method is equally suitable for cells of higher internal resistance and is more accurate than the usual voltmeter-ammeter and potentiometer methods. In the case of primary cells, however, it suffers from all the defects due to polarization.

27·16. Charging of Accumulators. The acacumulators to be charged are connected to the D. C. mains *in series* with a bulb or other suitable resistance and an ammeter as shown in fig. 27·15, care being taken that the current enters the cell by the *positive* terminal. By adjusting the variable resistance, the strength of the current is adjusted to lie within the limits of the *charging* current specified by the manufacturer. The accumulators are *fully* charged when they are gassing *freely* and *evenly*. The density of the acid in the case of lead accumulator on full charge rises to about 1·25 gm./c.c. which can be tested with a hydrometer.

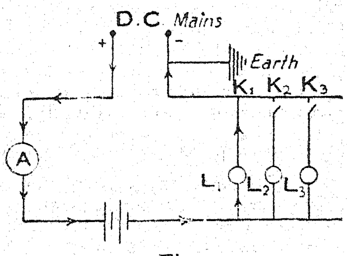


Fig. 27·15

Oral questions

ELECTROLYSIS

What is electrolysis ? How is it brought about ? What is the function of the current ? Does it decompose the electrolyte or simply direct the ions present in it towards electrodes ? What chemical changes take place at the electrodes ? What are the factors on which the products obtained by electrolysis depend ? What are Faraday's laws of electrolysis ? What is elctro-chemical equivalent and how is it related to chemical equivalet ? Given the electro-chemical equivalent of hydrogen, how can you calculate the electro-chemical equivalent for any other substance ? What is Faraday and what its value ? How do you define international ampere ? Describe a copper voltameter and explain the reactions taking place in it when a current is passed through it. How do you determine electro-chemical equivalent of copper ? (Also see questions on expt. 27.1)

CONDUCTIVITY OF ELECTROLYTES

How do you define specific conductivity, equivalent conductivity and molecular conductivity, of an electrolyte ? How do they vary with (*a*) concentration and (*b*) temperature ? How can you determine specific conductivity of an electrolyte ? Why is the Wheatstone bridge method of measuring the resistances as ordinarily employed not suitable in the case of electrolytes ? What are the factors which affect the conductivity of an electrolyte when a current is passed through it ? How does polarisation E. M. F. increase the resistance of the electrolyte ? How can you reduce the polarisation E. M. F. to minimum ? How can you increase the surface of the electrodes ? How can electrodes be platinised ? How does the use of alternating current reduce polarisation E. M. F. ? What are the sources of error in the Wheatstone bridge method of measuring the specific conductivity of an electrolyte employing an alternating current ? Describe the precautions taken by you to eliminate them. Why should the solutions be made with conductivity water ? Why should the resistances be non-inductive and of negligible self-capacitance ? Do you get a perfect balance of the bridge in this experiment ? On what factors does the sound minimum in the telephone depend ? Why don't you determine the cell constant by measuring the area of the electrode and the distance between them instead of determining it experimentally by means of an electrolyte of known specific conductivity ? How can you get more accurate result ?

CELLS

What is a cell ? What are its essential parts ? What is an electrolyte ? How many kinds of cells do you know ? Distinguish between primary and secondary cells. Name the different types of primary cells you know. What is a simple

cell ? Do you get constant current from it ? If not, why ? What is local action ? What is it due to ? How is it overcome in practice ? What is polarisation ? What harm does it produce ? How does H_2 produce back E. M. F. ? What are the methods used for removal of H_2 ? What is a depolariser ? What chemical properties must it possess ? Name some substances which are used as depolarisers. What do you understand by E. M. F. of a cell ? How is it produced ? On what factors does it depend ? Is the E.M.F. of a cell the same as P. D. between its terminals ? What is meant by open and closed circuits ? How does the circuit remain open when E. M. F. of a cell is measured with a voltmeter ?

Describe the construction and working of (*a*) a Leclanche cell and (*b*) a Daniell cell. What methods are employed to reduce local action and polarisation in these cells ? Give the values of E. M. Fs. and internal resistances of these cells. What are their uses ? Why is MnO_2 mixed with granular carbon in a Leclanche cell ? Why is the jar of the Leclanche cell blackened ? What type of cells do you use in electric torches ? Describe a dry cell.

What is a standard cell ? Why is ordinarily a Daniell cell used as a standard cell ? Describe tne Clark cell and the Weston cadmium cell. Which is better and why ? Why should the mercurous sulphate be exceptionally pure ? Why is there a constriction in lower part of the tubes in a cadmium cell ? What special precautions should be taken in using a standard cell ?

What is the principle of an accumulator ? Why is it called a storage cell ? How many types of accumulator do you know ? Describe the acid or lead accumulator explaining the reactions during charging and discharging. When is the lead accumulator said to be discharged ? What are its E. M. F. and density of the liquid when fully charged ? What is its internal resistance ? Name the two types of alkali accumulators. What is the difference between Edison and NIFE cells. Explain the reactions which take place during charging and discharging of Edison cells. What is its E. M. F. when fully charged. Which is better—the acid accumulator or the alkali one ? What do you understand by capacity and efficiency of an accumulator ? How do you charge an accumulator ?

INTERNAL RESISTANCE OF AN ACCUMULATOR

What do you understand by internal resistance of an accumulator ? On what factors does it depend ? What is its value ? Why is it so low ? How can you determine internal resistance of an accumulator ? Can you not determine it by the usual voltmeter-ammeter method or with a potentiometer ? What precautions do you take in your experiment ? Why should the cells be fully charged and of ample capacity ? Why should their E. M. Fs. be almost equal ? Why do you use a millivoltmeter instead of a voltmeter ? Does it measure the E. M. F. of the cells or their difference ? Why should the positive terminal of the cell of higher E. M. F. be connected to the + marked terminal of the millivoltmeter ? What will happen if by mistake the positive of one cell and the negative of the other cell are connected to the millivoltmeter ? Of which of the two cells do you find the internal resistance ? Give reasons. Can you use this method to find the internal resistance of a primary cell ? What is the accuracy of the results obtainable by this method ?

CHAPTER XXVIII

HEATING EFFECTS OF CURRENTS

28·1. Laws of Heating Effects of Currents. We have seen in § 26·1 that when a current is allowed to pass through a conductor, work is done at the expense of part of electrical energy of the circuit which appears as heat in the conductor and that, if the potential difference between the ends of the conductor be V e. m. units, the work done or the energy transformed into heat when one e. m. unit current flows through it for one second is V ergs. Hence, if the conductor carries a current of I e. m. units, the energy transformed into heat in one second will be VI ergs and in t seconds VIt ergs. If this energy is equivalent to H calories of heat, we have

$$H = \frac{VIt}{J} \tag{28·1}$$

where J is the *mechanical equivalent of heat*. Now $J = 4·18 \times 10^7$ ergs per cal. and from *Ohm's law* $V = IR$, where R is the resistance of the conductor, hence the heat produced in calories is given by the expressions

$$H = \frac{VIt}{4·18 \times 10^7} = \frac{I^2Rt}{4·18 \times 10^7} = \frac{V^2t}{4·18 \times 10^7 R} \tag{28·2}$$

If V, I and R are expressed in *practical units*, then since one volt$=10^8$ e. m. units of P. D. and one, ampere$=10^{-1}$ e. m. units of current, the energy transformed in t secs. will be $VIt \times 10^7$ ergs and the heat produced in calories will then be given by

$$H = \frac{VIt}{J} \times 10^7 = \frac{VIt}{4·18} = 0·24 VIt$$

and the expressions in (28·2) will be modified to

$$H = 0·24\ VIt = 0·24\ I^2Rt = 0·24\ \frac{V^2t}{R} \tag{28·3}$$

From these equations it follows that

(1) The quantity of heat developed in a conductor of given resistance by the passage of a current of given strength is directly proportional to the time for which the current flows.

(2) The quantity of heat developed in a conductor of given resistance in a given time is directly proportional to the square of the current strength or to the square of the potential difference at the ends of the conductor or the product of the current strength and the potential difference.

(3) For a given current strength, the quantity of heat produced in a given time is directly proportional to the resistance of the conductor.

(4) For a given potential difference, the quantity of heat produced in a given time is inversely proportional to the resistance of the conductor.

These relationships are known as *laws of heating effects of currents.* The relationship $H=0\cdot24\ I^2Rt$ was first experimentally established by Joule and is known as *Joule's law.* The other expressions for H in equation (28·3) are *alternative* forms of Joule's law.

Experiment 28·1

Object. To determine the value of *mechanical equivalent of heat* with *Joule's calorimeter* using a copper voltameter.

Apparatus. Joule's calorimeter, a copper voltameter, two jars, a battery of accumulators, a sliding rheostat of about 50 ohms, a voltameter, a weight box, a stop-watch, a thermometer, a physical balance, a chemical balance and connecting wires.

Description of Apparatus. The Joule's calorimeter consists of a copper calorimeter of about half-a-litre capacity silvered on its outer surface, and fitted with an ebonite cover which supports the heating coil suspended inside the calorimeter. The heating coil is made of a high resistance wire, *e.g.*, eureka and is connected to two terminals fixed to the ebonite cover by thick copper leads. The cover is provided with three holes, the central one for the thermometer and the two outer ones for the stirrer. The calorimeter is placed in a copper container which is silvered on its inner surface, the calorimeter being thermally insulated from the container by a thick layer of felt.

Theory. Let a resistance coil be immersed in water contained in a calorimeter and let a current of I amperes be allowed to pass through the resistance coil. Then, if V volts be the potential difference across the coil, the energy transformed into heat in t seconds will be equal to $VIt\times10^7$ ergs. If this energy transformed be equivalent to H calories of heat

$$H=\frac{VIt}{J}\times10^7 \tag{28·4}$$

where J is the mechanical equivalent of heat. Now, if the current I flowing through the resistance coil also passes through a copper voltameter placed in series with the coil and if it deposits m gm. of copper on the cathode plate in t seconds, we have

$$m=ZIt \tag{28·5}$$

where Z gm/coulomb is the electro-chemical equivalent of (cupric) copper. Eliminating I between equations (28·4) and (28·5) we have

$$H=\frac{Vm}{JZ}\times10^7$$

Now let M be the mass of water contained in the calorimeter and let W be the water equivalent of the calorimeter and the stirrer. Then, if θ be the rise in temperature of the calorimeter and its contents, the heat taken up by them is $(M+W)\,\theta$ calories. Assuming that all the heat produced in the heating coil is used up in raising the temperature of the calorimeter and its contents, we have, by equating the two expressions for H,

$$(M+W)\,\theta = \frac{Vm}{JZ} \times 10^7$$

whence

$$J = \frac{Vm}{Z(M+W)\theta} \times 10^7 \qquad \ldots \qquad (28{\cdot}6)$$

This equation enables us to calculate J, if other quantities are determined.

Method. Weigh the empty calorimeter with stirrer in a physical balance. Fill it with water sufficient to cover the heating coil and again weigh it and thus determine the mass of the water contained in the calorimeter. Then cover the calorimeter with its ebonite lid immersing the heating coil in water and place the calorimeter in its metal container lined with felt.

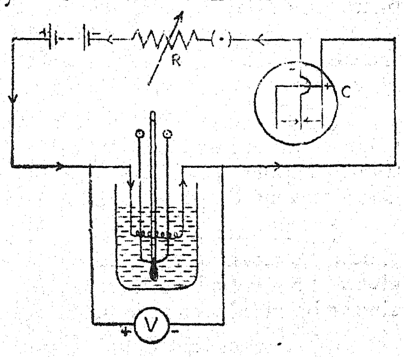


Fig. 28·1

Now clean the copper cathode plate of the copper voltameter thoroughly on its both sides with sand paper and a rug or emery cloth until its surface is quite bright and weigh it in an accurate chemical balance correct up to 1/10 of a milligramme. Then make connections as shown in fig. 28·1, connecting the heating coil to a storage battery in series with a suitable sliding rheostat R and a copper voltameter C, and placing a voltmeter in parallel with the heating coil. Note down the temperature of water contained in the calorimeter correct up to 1/5 or preferably 1/10 of a degree C. Then start the current and simultaneously with it the stop-watch. The resistance coil in the calorimeter will get heated and the temperature of the calorimeter and its contents will gradually rise. Stir the water constantly and efficiently. Note down the potential difference across the heating coil with the voltmeter and keep it constant by adjusting the rheostat, if necessary. When the temperature of the calorimeter and its contents has risen by about 10°C, switch off the current. Note down the exact time for which the current has been passed. Also note down the final temperature of water. Now allow the water to cool for the *same* time for which it was heated and determine the fall in the final temperature during this interval. Add half the fall in temperature to the final temperature of the calorimeter and its contents which will give the final temperature corrected for heat losses by radiation.

Now remove the cathode plate from the voltameter and immediately immerse it in a jar of tap water already placed near the voltameter. This will remove the copper sulphate solution left on the plate. Then transfer the plate to another jar containing distilled water to which two or three drops of sulphuric acid per litre have been added. Next press the plate without rubbing between sheets of filter paper or clean blotting paper to remove the moisture as far as possible. Then dry the plate finally in warm air coming out of a hot air blower. Weigh the cool dry plate in the chemical balance to the nearest tenth of a milligramme with the help of a rider and thus find out the mass m of the copper deposited. Finally calculate the value of J from equation (28·6) above, taking $Z = 0 \cdot 0003295$ gms per coulomb.

Sources of error and precautions. (1) The copper sulphate solution in the copper voltameter should have a density of about 1·18 gm/c.c., and should be made slightly more acid than the aqueous solution of the salt by the addition of 0·1% by volume of concentrated sulphuric acid.

(2) The total area of the cathode surface immersed in the solution should be about 50 sq. cm. per ampere passing. The deposition of copper should be made on both sides of the cathode plate by using two anode plates one on each side of the cathode plate.

(3) The cathode plate over which deposit of copper is to be made should be clean and should never be touched with fingers. It should always be handled by interposing a double strip of paper and should always be lifted by the attached wire or the hook at the top.

(4) The current must leave the copper voltameter by the cathode plate, *i.e.*, the plate over which the deposit of copper is to be made should be connected to the negative of the battery or to a point at a potential lower than that of the point to which the anode plate is connected.

(5) The rise in temperature of the calorimeter and its contents should be about 10°C.

(6) The strength of the current should be between 1·5 and 2 amperes so that during the interval the temperature of the calorimeter and its contents rises by about 10°C, a weighable amount of copper must be deposited on the cathode plate.

(7) The potential difference across the heating coil should be small, say about 6 volts. If the potential difference exceeds 8 volts the electrolysis of water which takes place when the current passes through the heating coil will affect the result seriously.

(8) The strength of current should be kept constant while deposition of copper is being made on the cathode plate. The voltmeter reading should be kept constant by adjusting the sliding rheostat, if necessary.

(9) The water in the calorimeter should be constantly and efficiently stirred during the heating of the coil in order to equalise the temperature throughout.

(10) The final temperature of the calorimeter and its contents should be corrected for loss of heat by radiation.

(11) When the cathode plate is taken out of the voltameter after the deposition of copper, it should be immediately immersed in a jar of tap water. The plate should then be transferred to a jar containing distilled water to which a few drops of sulphuric acid per litre have been added.

(12) While pressing the cathode plate between sheets of blotting or filter paper, it should never be rubbed.

Observations. Mass of the calorimeter and stirrer ... = gm.
Mass of the calorimeter, stirrer and water ... ... = gm.
Least count of thermometer ... ... = °C
Initial temperature of water ... ... = °C
Final temperature of water after the current has passed for t seconds ... = °C
Fall in final temperature of water in t seconds ... ... = °C
Mass of cathode plate before deposition of copper ... ... = gm.
Mass of cathode plate after deposition of copper ... ... = gm.
Potential difference across the heating coil ... ... = volts

Calculations. Mass of water in the calorimeter ... = gm.
Water equivalent of calorimeter and stirrer ... ... = gm.
Final temperature of water corrected for loss of heat by radiation ... ... = °C
Rise in temperature of calorimeter and its contents ... ... = °C
Mass of copper deposited on the cathode plate ... ... = gm.

$$J=\frac{Vm}{Z(M+W)\theta}\times 10^7$$

$=$

$= \quad \times \quad 10^7$ ergs/cal.

Result. The value of mechanical equivalent of heat

$= \quad \times \quad 10^7$ ergs/cal.

Criticism of the method. The value of mechanical equivalent of heat obtained by this method is always slightly higher than its correct value. The total amount of heat produced in the resistance coil cannot be accurately determined for (*i*) heat losses to the surroundings cannot be completely checked, (*ii*) the change in temperature of the calorimeter and its contents cannot be followed as accurately as we wish and (*iii*) some heat is also taken

up by the thermometer and the heating coil and it is not easy to determine the true value ot the water equivalent of the calorimeter, the stirrer, the heating coil and thermometer. Loss of heat to the surroundings can be eliminated if the calorimeter is placed in a double-walled vessel, the space between the walls being filled with water at a constant temperature equal to the mean of the initial and final temperatures of calorimeter, for then the calorimeter gains heat from the surroundings during the first half of the experiment and loses to them an equal amount during the second half. Loss of heat by evaporation can be avoided by using a liquid of low vapour pressure, *e.g.*, paraffin oil or aniline. These liquids have a small specific heat and hence with them the rise in temperature will also be quick and comparatively large. With these liquids higher voltages can be applied across the heating coil as there is no fear of electrolysis.

Now unless the heating coil is made of a material of negligible temperature coefficient, its resistance changes appreciably with temperature and it is difficult to keep both the current and the potential difference across the coil constant. A part of the main current flows through the voltmeter and unless its resistance is very high the current taken by it will not be negligible and in that case the current through the heating coil will be $(I-V/R)$ where R is the resistance of the voltmeter. This difficulty can, however, be overcome by measuring the potential difference with a potentiometer with which the accuracy will also be increased.

Most of the above defects are removed in Callendar and Barne's continuous flow calorimeter in which the heat produced in the coil is removed by means of a constant flow of water as rapidly as it is produced.

28·2. Measurement of Electrical Energy. Electrical energy for lighting and power is measured by some form of integrating meter, *i.e.*, a meter which gives *not the rate* at which energy is supplied to the circuit but the total amount of energy supplied during a given time. Such a meter evaluates $\int_{t_1}^{t_2} V\,i\,dt$, where V and i are the instantaneous values of the voltage and current and (t_2-t_1) is the time during which energy is supplied. The meter records the energy consumption directly in commercial units. A commercial or Board of Trade (B. O. T.) unit of energy is the *kilowatt-hour* (K. W. H.), which is defined as the energy used in one hour when the power is one kilowatt.

An energy meter usually consists of a small motor which is provided with a magnetic brake. The motor drives a counter whose indications on a system of dials are proportional to the total number of revolutions made by the armature As illustrated on

fig. 28·2, the field coils consist of two coils F, F of low resistance connected in series, and the armature A consists of many turns of fine wire placed in series with a suitable resistance R. The armature is carried by an upright spindle which, at its upper end gears by means of a worm into the very light train of wheels W which moves the pointers of the counter over the dials. The current is carried to the armature through silver-tipped brushes, which rest with a very slight pressure on a silver commutator C of small diameter. The lower end of the shaft is provided with a removable steel point which rests in a sapphire or diamond jewel carried by a spring support in the end of the jewel screw. The magnetic brake disc D is made of aluminium and moves through the fields of the permanent magnets M.

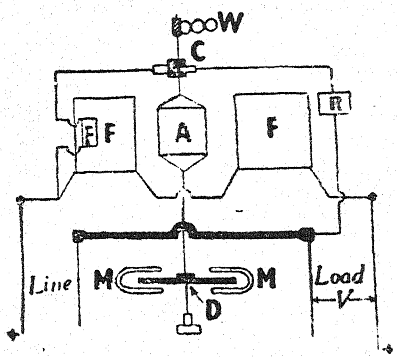


Fig. 28·2

To measure consumption of energy in a circuit, the field coils F, F are placed in the main circuit in series with the load, and the armature together with the series resistance R is connected across the supply mains. The driving torque of the motor is proportional to the current in the armature and to the field intensity due to F, F. The current in the armature is proportional to the voltage V applied to the load and the field intensity due to F, F is proportional to the current i in the main circuit. Thus the driving torque of the motor is proportional to Vi. The retarding torque due to the magnetic brake is proportional to the angular velocity $\frac{d\theta}{dt}$ of the disc D. Hence the energy supplied to the circuit during a given time is given by

$$E = \int_{t_1}^{t_2} V\,i.dt = k\int_{t_1}^{t_2} \frac{d\theta}{dt}\,.\,dt$$

But $$\int_{t_1}^{t_2} \frac{d\theta}{dt}\,.\,dt = \int_{\theta_1}^{\theta_2} d\theta = 2\pi\,(n_2-n_1) = 2\pi N$$

where N is the number of revolutions made by the armature during the time (t_2-t_1).

$$\therefore \qquad E = KN$$

i.e., the energy consumed in the circuit in a given time is proportionl to the number of revolutions of the armature during that time. *i.e.*, to the reading on the dials of the counter.

In order to compensate at small loads for the effects of mechanical friction, a field coil F′ of fine wire is put in series with the armature.

Experiment 28·2

Object. To calibrate a given electrical energy-meter by means of a Joule's calorimeter.

Apparatus. Energy-meter, Joule's calorimeter with a bulb, voltmeter, ammeter, thermometer and a balance.

Theory. The energy consumed in an electrical circuit in a given time can be measured in the following three ways :—

(1) By directly noting the change E in the reading of the energy-meter in B. O. T. units or kilowatt-hours in the time t.

(2) By determining the current i in amperes flowing through the circuit for t seconds and the voltage V in volts applied to the circuit. The energy spent is then given by

$$E_1 = Vit \text{ Joules}$$

or

$$E_1 = Vit/3{\cdot}6\times10^6 \text{ kilowatt-hours} \qquad (28{\cdot}10)$$

(3) By determining the heat H produced in the circuit. If the whole of the energy spent in the circuit is used to heat a calorimeter of water-equivalent W and water of mass M contained in it, and the consequent rise in temperature is θ,

$$H = (W+M)\ \theta \text{ calories}$$

and the energy consumed is given by

$$E_2 = JH = J\,(W+M)\,\theta \text{ ergs}$$

or

$$E_2 = J\,(W+M)\ \theta/3{\cdot}6\times10^{13} \text{ kilowatt-hours} \qquad (28{\cdot}11)$$

The *mean* of the two values E_1 and E_2 may be taken as the correct value of the energy E′ consumed in the circuit, *i.e.*,

$$E' = (E_1+E_2)/2$$

and then the percentage error in the reading of the energy-meter is given by

$$\text{Percentage error} = \frac{E-E'}{E}\times100 \qquad (28{\cdot}12)$$

Method. Weigh the empty calorimeter with stirrer in a physical balance. Fill it 3/4th with water and again weigh it thus determining the mass of the water taken in it. Place the thermometer and the heating bulb in the calorimeter covering the calorimeter with its ebonite lid, and then place the calorimeter in its metal container lined with felt.

Next make electrical connections as shown in the fig. 28·3, and note down the initial temperature of water contained in the calorimeter correct upto 1/5 °C. Then switch on the current in the circuit, starting simultaneously a stop-watch. Begin counting the number of

rotations of the disc of the energy-meter. Keep the water in the calorimeter thoroughly stirred and note down the readings of the ammeter and voltmeter after every two minutes. When 15 minutes have elapsed, switch off the current stopping simultaneously the stop-watch. Determine the total number of rotations made by the disc of the energy-meter, the final temperature of water and the time for which the current has been passed through the circuit. Allow the calorimeter and its contents to cool for the *same* time for which they were heated and determine the fall in final temperature during this interval. Add half of this fall in temperature to the final temperature of water to correct it for loss of heat by radiation. Next note down the number of rotations of the disc which correspond to an energy consumption of one kilowatt-hour. This is usually written on the meter for the given operating voltage.

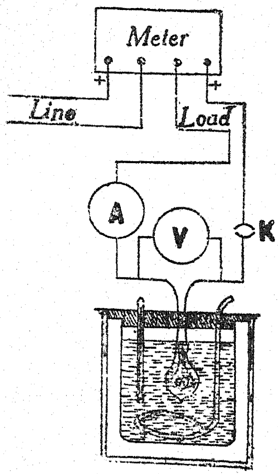


Fig. 28·3

Calculate the values of E_1 and E_2, the energy consumed in the circuit from equations (28·10) and (28·11) and finally calculate the percentage error in the reading of the meter from equation (28·12).

Sources of error and precautions. (1) In D. C. circuits care should be taken to see that the current enters all instruments by the +ve marked terminals.

(2) The readings of ammeter and voltmeter should be recorded after every 2 min. and the mean of these readings, if consistent, should represent i and V respectively.

(3) The water in the calorimeter should be constantly and efficiently stirred.

(4) The temperatures should be measured accurately with a thermometer correct upto 1/5 °C or preferably 1/10 °C.

(5) The final temperature of the calorimeter and its contents should be corrected for loss of heat by radiation.

Observations. [A] Mass of calorimeter and stirrer M_1 = gm.

Mass of the calorimeter, stirrer and water	=	gm.
Least count of thermometer	=	°C
Initial temperature of water	=	°C
Final temperature of water	=	°C
Time for which current has been passed t	=	sec.
Final temperature after cooling the calorimeter and its contents for time t	=	°C

[B] *Measurement of v and i*

S. No	Time in min.	Ammeter readings amp.	Voltmeter readings volts
1.			
2.			
...			
...			
	Mean		

[C] Number of rotations of the disc of the meter corresponding to a consumption of 1 kilowatt-hour of energy N=
Number of rotations of the disc actually observed during the expt. in time t $n=$

Calculations. (A) Mass of water contained in the calorimeter $M =$ gm.

Sp. heat of the material of calorimeter S (given) =
Water-equivalent of the calorimeter $W=M_1 \times S =$ gm.
Radiation correction = °C
Corrected final temperature = °C
Rise in temperature θ = °C
Heat produced $H= (W+M)\ \theta=$ = cal.
Energy consumed in the bulb

$$E_2=JH/3{\cdot}6\times 10^{13} = \quad \text{kilowatt-hours}$$

[B] Mean ammeter reading $i =$ amp.
Mean voltmeter reading $V=$ volts
Energy spent in the circuit

$$E_1 = Vi\ t/3{\cdot}6\times 10^6 = \quad = \quad \text{kilowatt-hours.}$$

[C] N rotations of the disc at 220 volts correspond to a consumption of energy of 1 K. W. H.

$\therefore$ n ,, ,, ,, $\frac{n}{N}$ K. W. H.

Since energy consumption is directly proportional to the operating voltage, n rotations of the disc at V volts correspond to an energy consumption of $\frac{nV}{N\times 220}$ K. W. H.

$$\therefore \qquad E= \frac{nV}{N\times 220} = \quad = \quad \text{kilowatt-hours}$$

Average energy consumption in the circuit

$$E' = (E_1+E_2)/2 = \quad = \text{kilowatt-hours}$$

$\therefore$ Percentage error in the reading of the meter

$$= \frac{E-E'}{E} \times 100 =$$

Result. The percentage error in the reading of the meter =

Oral question

JOULE'S CALORIMETER

State Joule's law for heating effects of a current. Why is heat at all produced when a current is allowed to pass through a conductor? How does the amount of heat developed in a conductor by the passage of the current in it vary with (*a*) the resistance of the conductor (*b*) the strength of the current and (*c*) the time? Explain how you can determine mechanical equivalent of heat by means of a Joule's calorimeter? Define mechanical equivalent of heat. How is work done in this experiment. Describe Joule's calorimeter. Of what material is the resistance coil made? What is its approximate resistance? Why is the calorimeter placed in a copper vessel silvered on its inner side and lined with felt? What are the sources of error in this experiment? What precautions do you take to eliminate them? Why should the P.D. across the resistance coil be small? How can you increase the rise in temperature without producing any electrolysis Name any liquid of low specific heat. How do you prevent the loss of heat to the surroundings? How can loss of heat by radiation be prevented? What are the unavoidable sources of error in this experiment?

Chapter XXIX

THERMO-ELECTRICITY

29·1. Seebeck Effect. Seebeck discovered in 1826 that, if in a circuit formed by joining two dissimilar metals, the two junctions are kept at *different* temperatures, a current flows round the circuit. Such currents are termed *thermo-electric currents* and the E. M. F. producing them is called *thermo-electric force.* The energy required to maintain the current is derived from the surroundings and the direction of current in the circuit depends upon the metals constituting the thermo-couple. A thermo-couple constructed of antimony and bismuth is one of the most sensitive couples available and in this case the direction of the current is from antimony to bismuth through the cold junction. In the case of copper-iron couple the current flows from copper to iron through the hot junction. If the temperature of the hot junction be continually increased, thermo-electric current increases until maximum value is reached when it begins to decrease. The temperature of the hot junction at which maximum current flows is constant for a given couple and is known as the *neutral temperature* for that couple. On further raising the temperature of the hot junction the current decreases to zero and is then reversed. The temperature at which the current is zero and its direction is reversed depends upon the temperature of the cold junction and is always as much above the neutral temperature as the cold junction is below it. This is illustrated in fig. 29·1.

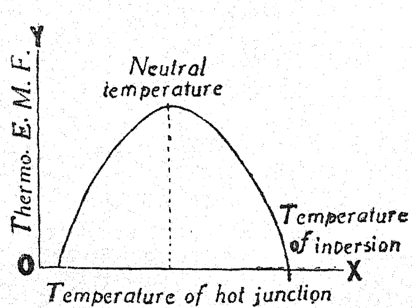


Fig. 29·1

29·2. Peltier Effect. In 1834 Peltier discovered that if a current be allowed to pass through a circuit consisting of two dissimilar metals, there is either an evolution or absorption of heat at the junctions, *i.e.*, the junctions are either heated or cooled, the condition whether a junction will be heated or cooled being determined by the direction of the current. This effect, known as *Peltier effect*, is quite distinct from Joule's heating effect which takes place at all points of the circuit and arises from its resistance. Joule's effect is independent of the direction of the current, *i.e.*, it is irreversible, while the Peltier effect is reversible for if the current be reversed, the junction which previously evolved heat now absorbs it and *vice versa.*

The Peltier effect arises on account of the existence of ontact potential difference at the junction of two dissimilar metals. When the current traverses a junction where there is a slight up-gradient of potential due to contact, the current gains energy. Consequently there is an absorption of heat at that junction as a result of which it is cooled. On the other hand, if the current traverses a junction in the direction of the down-gradient of potential due to contact, work is done and the current gives out some energy producing a heating effect at the junction.

29·3. Thomson Effect. In 1856 Thomson showed that whenever a current is allowed to pass through an unequally heated conductor, *i.e.*, whose different parts are at different temperatures, there is an evolution or absorption of heat in the conductor, the condition whether there will be an evolution or absorption of heat being determined by the direction of the current. This effect is known as *Thomson effect* and is reversible. In an unequally heated conductor different parts are at different potentials and hence there will be a potential gradient along the conductor. In the case of iron the colder parts are at higher potentials, while in the case of copper the hotter parts are at higher potentials. When the current flows in the direction of up-gradient of potential there is an absorption of heat while if it flows in the direction of down-gradient of potential there is an evolution of heat.

29·4. Measurement of Thermo-electromotive Force. The potentiometer method of measuring E. M. Fs. as ordinarily employed cannot be used to measure thermo E. M. Fs. on account of the *very small* value of such E. M. Fs. The magnitude of the thermo E. M. Fs. is of the order of a few millivolts *e.g.*, the E. M. F. of a copper-iron couple, when the junctions are kept at 0°C and 100°C, is only about 0·0013 volts. The thermo E. M. Fs. can be measured by a suitable modification of the ordinary potentiometer method so as to produce a P. D. of about a micro-volt across each cm. length of the potentiometer wire which is small enough to admit of sufficiently accurate measurement.

Experiment 29·1

Object. To study the variation of thermo E. M. F. of a copper-iron circuit with temperature.

Apparatus. A potentiometer, an accumulator, a standard cadmium cell, a resistance box, a high resistance of 15,000 ohms, a rheostat, a sensitive galvanometer, two thin glass tubes, two small test tubes, iron wire, a two-way key, two single way plug keys, connecting wires and heating apparatus.

Theory. Let a high resistance R of about 1000 ohms be connected in series with the potentiometer wire AB (Fig. 29·2) and let the combination be connected to an accumulator in series with a rheostat R′, the current entering the combination of R and the potentiomete

wire at M and leaving at B. Further, let the + ve pole of a standard cell S be joined to the higher potential terminal M of the resistance R, and the hot junction H of the copper - iron thermo-couple to the jockey ; and let the—vepole of the standard cell and the cold junction C of the thermo-couple be connected through a two-way key to one terminal of the galvanometer, the other terminal of which is joined to the lower potential terminal N of, the resistance R. Now if the standard cell circuit be closed by means of the two-way key and the rheostat R′ so adjusted that there is no deflection in the galvanometer, the P. D. across the resistance R will be balanced by the E. M. F. of the cell. Hence, if E be the E. M. F. of the standard cell, and I the steady current through the resistance R or the potentiometer wire, we have

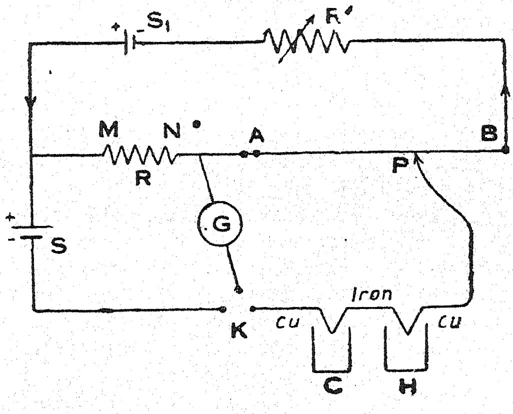


Fig. 29·2

$$E = IR \qquad (29{\cdot}1)$$

Now let the standard cell circuit be broken and the cold junction of the thermo-couple connected to the galvanometer. Then, if P be the position of the null point on the potentiometer wire, the thermo E. M. F. of the copper-iron couple is equal to the P. D. between A and P. Hence, if e be the thermo-electric E. M. F., and r the resistance of the portion of the wire between A and P.

$$e = Ir \qquad (29{\cdot}2)$$

Dividing equ. (29·2), by equ. (29·1), we get

$$\frac{e}{E} = \frac{r}{R} \qquad (29{\cdot}3)$$

Let ρ be the resistance per unit length of the potentiometer wire and let l be the length of the wire between A and P. Then as the wire is uniform $r = \sigma l$. Putting this value of r in the above equation we get,

$$e = \sigma \frac{E.\, l}{R} = Kl \qquad (29{\cdot}4)$$

where $K = \sigma E/R$

This equation enables us to calculate the thermo E. M. Fs

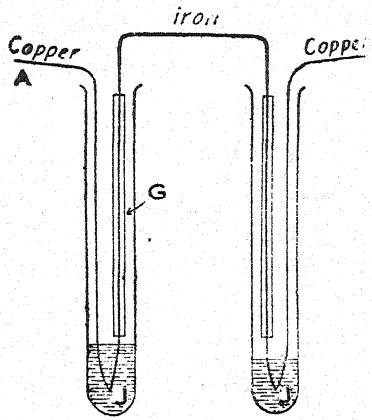


Fig. 29·3

e, if E, σ and R are known and the value of t determined.

Method. Pass one end of the iron wire down a thin glass tube G (Fig. 29·3) and join to it at J a copper wire AJ. This ensures that the wires are in contact at the junction only. Then to ensure good contact at J, place the combination in a test tube containing a small amount of mercury and dip the junction J into the mercury. Next pass the other end of the iron wire down another thin glass tube G and prepare in a similar manner another junction J′ of iron and copper. Place one of the junctions in air and the other in an oil bath which may be kept at different temperature.

Now connect a resistance box R in series with the potentiometer at the end A (Fig. 29·4). Then connect the combination to an accumulator S′ in series with a rheostat R′ of high value,

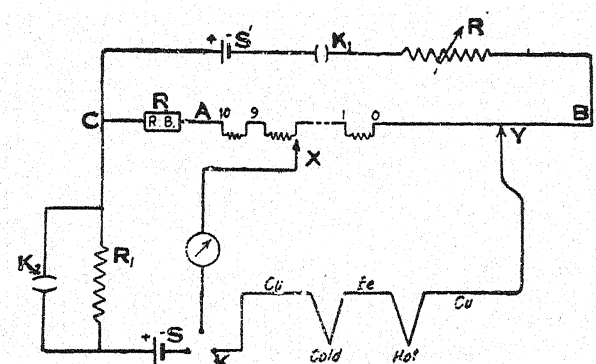


Fig. 29·4

including a plug key K_1 in the circuit, the +ve pole of the accumulator being joined to the end C of the combination of R and the potentiometer. Next connect the +ve pole of the standard cadmium cell S through a high resistance R_1 of about 15000 ohms to the higher potential terminal C of the resistance box R and connect the hot junction of the copper iron thermo-couple to the jockey Y. Then connect the −ve pole of the standard cell S and the cold junction of the thermo-couple through a two-way key K to one terminal of the galvanometer, the second terminal of which is connected to the sliding contact maker X. Finally connect a plug key K_2 across R_1.

Adjust the resistance box R to about a thousand ohms, preferably to 1018·3 ohms. Shunt the galvanometer and place the slidiing contact maker X at the end A of the last coil of the potentiometer. Close K_1 and with K_2 open, connect the —ve pole of the standard cell by means of the two-way key to the galvanometer. Adjust the rheostat R′ until there is practically no deflection in the galvanometer. Then remove the shunt from the galvanometer and closing K_2 adjust the rheostat R′ finally until there is no deflection in the galvanometer. This exactly balances the P. D. across R by the E. M. F. of the standard cell.

Next open K_2, shunt the galvanometer again and connect the cold junction of the thermo-couple by means of the two-way key K to the galvanometer. Then find by trial by placing the sliding contact maker X on various studs corresponding to the different coils and pressing the jockey over the stretched wire, an approximate posiition of the null point on the wire. Now remove the shunt from the galvanometer and determine the exact position of the null point on the potentiometer wire. Note down the number of coils and the length of the potentiometer wire between the sliding contact maker X and the jockey Y and find the equivalent length l of the potentiometer wire corresponding to the thermo-electric E. M. F. of the copper-iron couple. Then calculate the value of thermo E. M. F. from equation (29·4), taking the value of σ as given.

Heat the oil containing the hot junction to a high temperature and then allow it to cool. As it cools measure the thermo E. M. F. after an interval of temperature of about 5°C until the temperature of the hot junction has fallen to about 30°C. Plot a graph taking temperatures along X—axis and the corresponding thermo E. M. Fs. along the Y—axis. The graph will form part of a parabola shown in fig. 29·1 and will give the variation of thermo E. M. F. with temperature.

Sources of error and precautions. (1) The ends of the connecting wires should be clean and the connections should he firmly made. As the thermo E. M. F. to be measured is small, bad contacts will lead to troublesome difficulties.

(2) The accumulator should be fully charged and its E. M. F. should remain constant while observations are being made.

(3) A key should be included in the main circuit of the potentiometer which should be closed only when observations are to be made.

(4) The P. D. between the ends of the stretched wire of the potentiometer should not be greater than a millivolt otherwise the length l of the potentiometer wire corresponding to the thermo E. M. F. of the copper-iron couple will be very small and the value of l cannot be measured sufficiently accurately. Accordingly the high resistance R and the rheostat R′ should each be about a thousand ohms. If the stretched wire of the potentiometer

is of exactly one ohm and there are 1000 divisions on the measuring scale, then with R=1018·3 ohms, the potential difference across each division will be exactly equal to *1* microvolt and the calculations will then be also simplified.

(5) To protect the standard cell a high resistance of about 15000 ohms should be connected in series with it. This will prevent large currents from being taken from the cell. This resistance should be removed from the cell circuit when its E. M. F, has been nearly balanced, so that the final adjustment of the rheostat R′ may be made more accurately. Note that this resistance does not affect in any way the position of balance.

(6) While balancing the P. D. across the resistance R by the E. M. F. of the standard cell, the sliding contact maker X must lie at the end A of the *last* coil of the potentiometer.

(7) In the thermo-couple the wires of the two metals should be in contact at the junctions only and there should be good contact at the junctions. To ensure this, the junctions should be made especially in the manner described under method.

(8) The copper wires connecting the cold and hot junctions of the thermo-couple to the two-way key and the jockey should be long enough to ensure that their junctions with the key and the jockey do not differ appreciably in temperature.

(9) The galvanometer should be a very sensitive one. A shunt should be connected across it to avoid damage to the instrument by excessive deflections in it during determination of the approximate position of the null point. The exact position of the null point should be determined with full galvanometer sensitivity by removing the shunt from it.

(10) The contact between the jockey and the wire should always be momentary. The contact between the jockey and the wire should not be made while the former is being moved along.

Observations.

(*i*) Room temp. = °C
(*ii*) E. M. F. of the standard cell = volts.
(*iii*) High resistance R = ohms
(*iv*) Resistance per unit length σ of the potentiometer wire=ohms

S- No.	Temperature of hot junction in °C	Length of potentiometer wire corresponding to the thermo E. M. F.			Thermo E. M. F.
		No. of coils	Length of stretched wire div.	Equivalent length l div.	microvolts
1.					
2.					
3.					
...					
...					

Calculations. $K = \sigma E/R$

$=$

$=$ microvolts/div.

I reading $e = Kl_1$

$=$

$=$ microvolts

[Make similar calculations for other readings].

Result. The graph between thermo E. M. F. and temperature for copper-iron couple is a parabola as shown in fig.......

Criticism of the method. The method yields sufficiently accurate result. The accuracy of the result depends upon the constancy of the E. M. F. of the accumulator maintaining a steady current through the potentiometer wire, the uniformity of the wire and the accuracy of knowledge about the value of E. M. F. of the standard cell at the time of experiment and the resistance per unit length of the wire. For very accurate results, the thermo E. M. F. should be measured with a Crompton potentiometer.

Exercise. *To determine the melting point of paraffin wax by measuring thermo E. M. Fs. of copper-iron circuit.*

Place one junction of copper-iron couple in melting ice and the other in boiling water. Determine the thermo E. M. F. of the circuit by means of the potentiometer and note down the temperature of boiling water. Allow the water to cool and determine the thermo E. M. Fs. after an interval of temperature of about 5°C until the temperature of the junction has fallen to about 30°C noting down the temperature of the hot junction at which the thermo E. M. F. is measured. Then place the hot junction in melting paraffin wax and when the wax solidifies, measure the thermo E. M. F. of the circuit. Next plot a graph taking the temperatures of the hot junction as abscissae and the corresponding thermo E. M. Fs. expressed in microvolts as ordinates. The thermo-electric curve so obtained will be a small portion of a parabola (Fig. 29·1). From the curve find out the temperature corresponding to the thermo E. M. F. when the hot junction was placed in melting wax. This will be the melting point of wax.

Oral questions

What is a thermo-electric effect ? Who discovered it ? What is Seebeck effect ? Where from does the energy come to maintain thermo-electric current ? On what does the direction of thermo-electric current depend ? What is the direction of current in the case of (*a*) antimony bismuth couple and (*b*) a copper-iron couple ? What happens to the current when the temperature of the hot junction is continuously increased ? What is neutral temperature ? What is thermo-electric inversion ? What is Peltier effect ? How does it differ from Joule's heating effect ? What determines whether there would be the evolutior

of heat or its absorption at a junction ? What is contact potential ? How do you explain Peltier effect ? What is Thomson effect ? How does it differ from Peltier effect ? How do you explain Thomson effect by the existence of a potential gradient along an unequally heated conductor ?

What is thermo E. M. F. ? How do you measure it ? Why does the potentiometer method of measuring E. M. Fs. ordinarily employed cannot be used to measure thermo E. M. F. ? What is the order of magnitude of a thermo E. M. F. ? What is the actual magnitude in the case of a copper-iron couple when the junctions are at 0°C and 100°C ? Describe the method of measuring thermo E. M. F. with a potentiometer. What is the function of the high resistance connected in series with the potentiometer wire ? What should be its approximate value and why ? Why do you require a potential gradient of about a micro-volt per div. in this case ? What precaution do you take in this experiment ? Why has a high resistance been connected in series with the standard cell ? Why should it be removed from the standard cell circuit when the E. M. F. of the cell nearly balances the P. D. across the high resistance ? How do you prepare the thermo-couple ? How do you ensure good contact at the junctions only ? Which of the two junctions hot or cold you will connect to the jockey and why ? Discuss the accuracy of result obtainable by this method.

CHAPTER XXX

MEASUREMENT OF CAPACITANCE

30·1. Capacitance of a conductor and its unit. When an insulated conductor is charged with a certain quantity of electricity the potential of the conductor is raised, but the extent to which the potential is raised depends not only upon the magnitude of the charge but also upon the size of the conductor and the nature of its surroundings. If these conditions remain unchanged the potential acquired by the conductor is directly proportional to the quantity of charge given to it. Similarly the charge given to a conductor of given size is proportional to the potential acquired by it. Thus we may write

$$Q = CV$$

where Q is the charge given to the conductor, V its potential and C is a constant for the conductor under the specified conditions. From the above expression it is evident that the greater the value of C greater will be the charge required to produce a given potential. For this reason the factor C is called electrical capacity or capacitance of the conductor. *The* **capacitance** *of a conductor is measured by the ratio of the charge given to the conductor to the potential which that charge produces*, or alternately *by the quantity of charge necessary to change the potential of the conductor by one unit.*

The capacitance of a conductor depends upon its size, being greater the greater the size. Further if a conductor carrying a charge Q is immersed in a medium of dielectric constant K, its potential will be $1/K$ of its value in air and hence to bring it to the same potential, the charge on the conductor must be made K Q. This shows that the capacitance of the conductor has now become K times its value in air. Thus the capacitance of a conductor depends on the dielectric medium, being greater the greater the value of K. The capacitance of a conductor is also affected by neighbouring conductors as dealt with in § 30·2.

Now when $Q=1$ and $V=1$, C is also equal to unity. This gives us the definition of unit of capacitance. A conductor has a capacitance of one absolute C. G. S. electrostatic unit if one e. s. unit of charge raises its potential by one e. s. unit. Similarly a conductor has a capacitance of one absolute or C. G. S. electro-magnetic unit if one e. m. unit of charge raises its potential by one e. m. unit. The practical unit of capacitance is called the **farad** (named after Faraday) and it *is the capacitance of a body whose potential is raised one volt by a charge of one coulomb.* For practical purposes farad is a very large unit, and hence a submultiple of it, called the microfarad μf which is one-millionth of the farad is often employed. A still smaller unit called the micro-micro farad $\mu\mu f$ which is one-millionth of a microfarad, is also used.

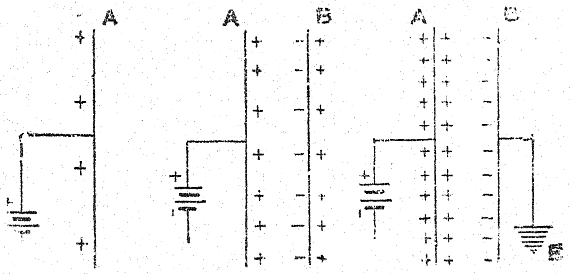
one e. m. unit of capacitance $=9\times10^{20}$ e. s. units

one farad $f=10^{-9}$ e. m. units $=9\times10^{11}$ e. s. units

one microfarad $\mu f=10^{-6}$ farads$=10^{-15}$ e. m. units$=9\times10^{5}$ e. s. units

one micro-microfarad $\mu\mu f=10^{-6}$ $\mu f=10^{-12} f=10^{-21}$ e. m. units$=0{\cdot}9$ e. s. unit.

30·2. Principle of a Condenser. Let a metal plate A be connected to positive pole of a battery [fig. 30·1 (*a*)] and let it acquire the potential of the pole of the battery. Then the plate will be fully charged and no more charge can be accumulated



(*a*) (*b*) (*c*)

Fig. 30·1

on it. Now let a second insulated plate B be brought near to A [fig. 30·1 (*b*)]. It will be charged by induction, the —ve charge being induced on the near side of B and the +ve charge on its further side. The —ve charge on B will tend to lower the potential of A while the +ve cahrge on B will tend to raise the potential A, and since the —ve charge is nearer to A its effect on the potential of A will be greater than that of +ve charge even though they are equal in amount. The result is that the potential of A is lowered a little and more charge flows from the battery to A in order to raise its potential to its original value, *i.e.*, the capacitance of A increases.

Now let the plate B be earthed [fig. 30·1 (*c*)]. Its +ve charge will go to earth and the remaining —ve charge will slightly increase as the nearby +ve charge which tended to decrease it has been removed. This increased —ve charge on B lowers the potential of **A** considerably so that still more charge can be accumulated on it in order to bring its potential to its original value, *i.e.*, the capacitance of A has further increased. Thus the capacitance of a charged conductor increases when an insulated conductor is brought near it and more so when the latter is earthed. Such an arrangement of increasing the capacitance of a conductor is called a condenser. It consists of two conductors separated by an insulated substance. The conductors

are called the coatings and the insulating medium between them the dielectric of the condenser.

In laboratories the condenser is usually charged by connecting one coating A to the positive pole and the other coating B to the negative pole of a battery. The coating A takes the positive potential of the positive pole and a big positive charge accumulates on it and the other coating B takes the negative potential of the negative pole of the battery and a big negative charge accumulates on it. The action of the condenser is, however, the same as above.

30·3. Capacitance of a condenser. *The capacitance of a condenser is measured by the quantity of electricity which must be given to the condenser to establish unit potential difference between the coatings.* If one coating be earthed, the capacitance of the condenser is equal to the quantity of electricity required to raise the other coating to unit potential. The capacitance of a condenser is measured in the same units as the capacitance of a conductor. While defining these units phrase "potential difference between the coatings of the condenser" is now substituted in place of "potential of the conductor." Thus farad is the capacitance of a condenser which requires one coulomb of electricity to establish a potential difference of one volt between its coatings.

The capacitance of a condenser depends upon three factors :—

(*a*) The size of the coatings. The capacitance of a condenser is directly proportional to the size of coatings—the greater the size the greater the capacitance.

(*b*) The distance between the coatings. The capacitance of a condenser is inversely proportional to the distance between the coatings—the greater the distance the less the capacitance.

(*c*) The dielectric medium between the coatings. The capacitance of a condenser is directly proportional to the dielectric constant or permitivity K of the dielectric medium between its coatings—the greater the dielectric constant K, the greater the capacitance. The dielectric constant K is sometimes known as the specific inductive capacity of the medium.

Below are given the values of capacitance of a few types of condenser :—

(i) Spherical condenser. Inner sphere of radius a insulated and outer sphere of radius b earthed. $K\dfrac{ab}{b-a}$

(ii) Spherical condenser. Inner sphere of radius a earthed and outer sphere of radius b insulated $K\dfrac{b^2}{b-a}$

(iii) Parallel plate condenser. Area of each plate A and distance between the plates t $K\dfrac{A}{4\pi t}$

(iv) Cylindrical condenser. Length l, radii of inner and outer surfaces a and b respectively $\frac{Kl}{2 \cdot 3026 \times 2 \log_{10}(b/a)}$

Experiment 30.1

Object. To compare the capacitances of two condensers by means of a ballistic galvanometer.

Apparatus. Two condensers whose capacitances are to be compared, a ballistic galvanometer, an accumulator, a rheostat, a Morse key, a two-way key, a tapping key and connecting wires.

Theory. Let the two condensers of capacitances C_1 and C_2 respectively be charged to the same potential difference V between the coatings, then, if Q_1 and Q_2 be the respective charges on the coatings of the two condensers, we have

$$Q_1 = VC_1 \quad \text{and} \quad Q_2 = VC_2$$

or $$Q_1/Q_2 = C_1/C_2 \tag{30·1}$$

Now let the condensers be discharged separately through a ballistic galvanometer and let θ_1 and θ_2 be the values of the first throw of the galvanometer in the two cases corresponding to movement of the spot of light through d_1 and d_2 cm. Then, we have

$$Q_1 = K\,\theta_1\,(1+\lambda/2) \quad \text{and} \quad Q_2 = K\,\theta_2\,(1+\lambda/2)$$

or $$Q_1/Q_2 = \theta_1/\theta_2$$

where λ is the logarithmic decrement.

But $$\frac{\tan 2\theta_1}{\tan 2\theta_2} = \frac{d_1}{d_2}$$

or if throws are small

$$\theta_1/\theta_2 = d_1/d_2$$

and hence $$Q_1/Q_2 = \theta_1/\theta_2 = d_1/d_2 \tag{30·2}$$

Comparing with equ. (14·1), we get

$$C_1/C_2 = d_1/d_2 \tag{30·3}$$

Alternatively , a graph may be plotted taking d_2 along X-axis and d_1 along Y-axis. This will come out to be a straight line as shown in (fig. 30·2) and its slope PR/QR will give the ratio C_1/C_2 of the two capacitances.

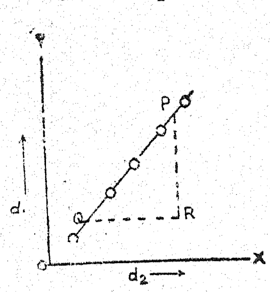


Fig. 30·2

Morse key. Morse key makes a good charge and discharge key and at the same time does not permit the condenser to be

insulated from the circuit. It consists of a stiff metal lever AK, one end A of which is held by a spring in contact with a metal stud connected with '*a*'. When the other end K of the lever is depressed, the contact of A with the stud below it is broken and the contact is made between K and the other stud connected with *b*. When the lever is released the spring breaks contact at *b* and restores at *a*. If the condenser is connected to the fulcrum L and the galvanometer and the battery at *a* and *b* respectively, then, when K is depressed the condenser-battery circuit is completed and the condenser is charged, and when K is released the condenser-battery circuit is broken while the condenser-galvanometer circuit is made and the condenser is discharged through the galvanometer.

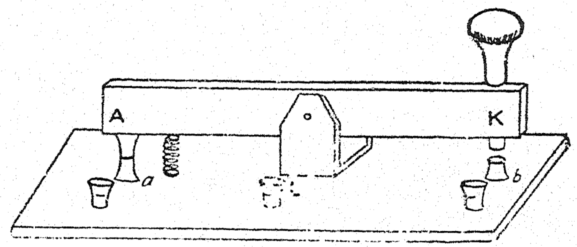


Fig. 30·3

Method. Level the base of the galvanometer by means of the levelling screws and release its coil. Throw light on the mirror of the galvanometer and get the spot of reflected light on the scale. Next make connections as shown in fig. 30·4. Adjust the sliding contact O of the rheostat XY to get a suitable voltage between XO. Adjust the position of spot of light on the scale at zero. Connect P to M. This connects the condenser C_1 to the galvanometer. Depress the end K of the lever of the Morse key, the condenser-battery circuit is completed and the condenser is charged by the battery Note that while the condenser is being charged, one terminal of the galvanometer is kept insulated from it and hence the charge does not pass through the galvanometer. When the condenser has been charged for about 20 seconds, release the lever of the Morse key. The condenser-battery circuit will be opened while the condenser galvanometer circuit will be closed. The condenser will be discharged and a quantity of electricity equal to the charge one of its coatings will pass through the galvanometer. Note down

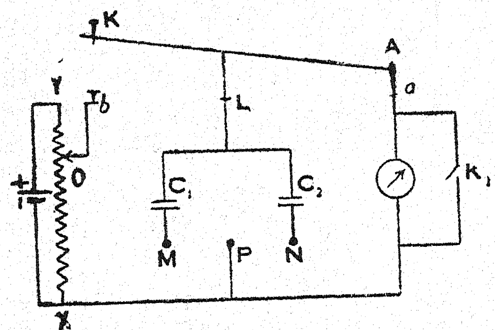


Fig. 30·4

the movement of the spot of light on the scale corresponding to the first throw of the galvanometer.

Bring the galvanometer coil to rest by tapping the key K. When the spot of light on the scale is stationary substitute the second condenser for the first by connecting P to N and determine the movement of spot of light corresponding to the first throw of the galvanometer as above by first charging the condenser for about 20 seconds and then discharging it through the galvanometer. The ratio of the two throws gives the ratio of the capacitances of the two condensers. Repeat the experiment at least six times by applying different voltages to the condensers with the help of the rheostat XY and determine the mean ratio of the capacitances of the two condensers.

Finally plot a graph taking d_2 along X-axis and the corresponding values of d_1 along Y-axis. This will come out to be a straight line as shown in fig. 30·2. Measure its slope PR/QR which will give C_1/C_2.

Sources of error and precautions. (1) The base of the galvanometer should be carefully levelled by means of the levelling screws and the coil should be unclamped. This ensures the free movement of the galvanomoter coil in the space between the magnet and the soft iron piece.

(2) A tapping key should be connected across the galvanometer coil in order to bring the coil to rest in a short time by damping its motion by tapping the key. When the key is tapped induced currents are produced in the coil due to its movement in the magnetic field. A force exists between these currents and the magnetic field, which is always in the direction opposing the motion of the coil.

(3) A Morse key should be used in charging and discharging the condensers in order to avoid the risk of connecting the battery and the galvanometer directly. The use of two separate tapping keys is dangerous for they are liable to be pressed down simultaneously and thus connect the battery to the galvanometer.

(4) The E. M. F. of the battery used for charging the two condensers should be constant. An accumulator may be used for the purpose for its E. M. F. remains sensibly constant. If with the accumulator the throws of the galvanometer are very large, a fraction of E may be used by potentiometer arrangement.

(5) In absorptive condensers the charging current dies away gradually over a long period of time after the condenser has acquired the applied potential difference, and the electricity continues to flow into the condenser for some time after the charge necessary to produce the required potential difference has been taken up. In such condensers, therefore, on account of electric absorption the capacitance is rather indefinite and depends upon the time of charging. The shorter the charging time, the nearer the measured capacity approaches its true value, provided of course the time is long enough to charge the condenser to the potential difference applied. Thus the condensets

should be charged for short intervals only, especially if the dielectric be oil or paraffined paper, and their charging time should be nearly equal.

(6) The graph between d_1 and d_2 will come out to be a straight line and it should be smoothly drawn.

Observations

Rest position of spot of light=zero

So. No.	Deflection d_1 corresponding to throw θ_1 mm.	Deflection d_2 corresponding to throw θ_2 mm.	d_1/d_2
1.			
2.			
3.			
...			
...			
...			
		Mean	

Calculations. 1 set. $\frac{C_1}{C_2}=\frac{d_1}{d_2}$

$=$

(Make similar calculations for other sets).

$\therefore$ Mean value of C_1/C_2 $=$

From graph (30·2) PR $=$, QR=

$$\therefore \quad \frac{C_1}{C_2}=\frac{PR}{QR}=$$

Result. The capacitances of the two condensers are in the ratio of.........

Criticism of the method. Since the ratio of capacitances is equal to the ratio of first throws of the galvanometer, quite accurate results can be obtained by this method by using a sensitive galvanometer. When the deflections, however, are large the approximate formula (30·3) will not hold good and then the exact equation $C_1/C_2=\theta_1/\theta_2$ should be used for calculating the ratio of the capacitances. The values of θ_1 and θ_2 can be easily obtained from the relationships $\tan 2\theta_1=d_1/L$ and $\tan 2\theta_2=d_2/L$, where L is the distance of the scale from the mirror. For more accurate results, methods which depend upon adjusting for no deflection of the galvanometer should be used.

Exercise 1. *To compare the E. M. Fs. of two cells by means of a ballistic galvanometer.*

Take a standard condenser. First charge it fully with the cell of lower E. M. F., E_1 and then discharge it through the ballistic galvanometer noting down the first throw θ_1 of the galvanometer (Fig. 30·5). Next charge the condenser fully with the cell of higher E. M. F. E_2 and then discharge it through the galvanometer noting down the first throw θ_2 of the galvanometer. Then the ratio of the E. M. Fs. of the two cells is given by

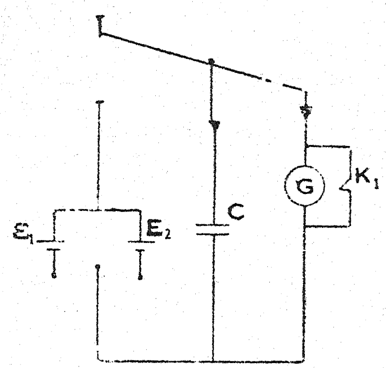


Fig. 30·5

$$E_1/E_2=\theta_1/\theta_2$$

It is advisable to obtain the throw with the cell of lower E. M. F. first otherwise if the condenser is not dielectric loss-free, the residual charge left over after experimenting with the cell of higher E. M. F. may appreciably affect the throw with the cell of lower E. M. F. This method of comparing E. M. Fs. is absolutely free from errors due to polarisation. Its only disadvantage is that it is not a null method and hence its possible accuracy is less than that obtainable with a potentiometer.

Exercise 2. *To determine the internal resistance of a cell by means of a ballistic galvanometer.*

Connect a standard condenser through a tapping key to the cell of E. M. F. E in series with a ballistic galvanometer as shown in fig. 30·6. Place a resistance box across the cell. Remove the infinity plug of the resistance box. Tap the key and observe the first throw θ_1 of the galvanometer due to the charging of the condenser. Then $E \propto \theta_1$. Next insert the infinity plug in the resistance box and remove plug of resistance R equal to 2 or 3 ohms. Tap the key and again note the first throw θ_2 of the galvanometer due to charging of the condenser to the P. D., V between the poles of the cell. Then $V \propto \theta_2$. Hence

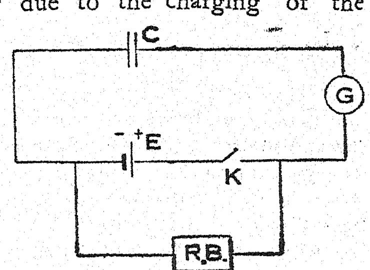


Fig. 30·6

$$\frac{E}{V}=\frac{\theta_1}{\theta_2}$$

But from equation (26·36)

$$V=ER/(R+r)$$

Thus $$\frac{E}{V}=\frac{R+r}{R}=\frac{\theta_1}{\theta_2}$$

whence $$r=\frac{\theta_1-\theta_2}{\theta_2}.\ R$$

Note that by closing the key K, the closing of the cell circuit and the charging of the condenser is affected simultaneoulsy. Consequently, the throw of the galvanometer corresponding to V can be obtained at the instant the circuit is completed through the resistance R, *i.e.*, before the cell gets time to polarise. This method is especially suited to cells which polarise quickly.

Experiment 30.2

Object. To determine the ballistic constant K of moving coil ballistic galvanometer with a standard condenser of known capacitance.

Apparatus. The moving coil ballistic galvanometer of suspended type, a capacity box, an accumulator, a voltmeter, a Morse key and a tapping key.

Theory. Let a standard condenser of known capacitance C be charged fully to a known potential difference E and let it be then discharged through the ballistic galvanometer whose constant K is to be determined. Then, if θ_1 be the first observed throw of the galvanometer, the charge passing through it is given by

$$Q=CE=K\theta_1\,(1+\lambda/2)$$

whence $$K=\frac{CE}{\theta_1\,(1+\lambda/2)} \tag{30·4}$$

where λ is the logarithmic decrement of the circuit and can be obtained from the expression

$$\lambda=\frac{1}{10}\times 2{\cdot}3026\ \log_{10}\frac{\theta_1}{\theta_{11}}$$

where θ_n stands for the nth throw of the galvanometer.

Method. Level the base of the galvanometer by means of the levelling screws and release its coil. **Set up a scale in front of the galvanometer distant one metre from it.** Throw light on the mirror of the galvanometer and get a bright spot of reflected light on the scale. Adjust the spot of light on the scale at zero. Connect one terminal of the capacity box through a Morse key to one terminal of an accumulator and one terminal of the galvanometer. Connect the second terminal of the capacity box directly to the second terminal of the accumulator and the galvanometer as shown in fig. 30·7. Connect a tapping key across the galvanometer.

Adjust the capacity box to a *suitable* capacitance. Press the knob K. The cell circuit will be closed and the condenser will be

charged. During the charging of the condenser one terminal of the galvanometer remains insulated from the condenser and hence the charge does not pass through it. When the condenser has been charged for about 20 sec., release K ; the cell circuit will be broken while that of the galvanometer closed. The condenser will discharge through the galvanometer. Note down the deflection of spot of light on the scale corresponding to first and the eleventh throws of the galvanometer. Measure the E. M. F. of the accumulator with a voltmeter. Calculate the constant K of the galvanometer from equation (30·4). If even with the least capacitance available in the capacity box, the spot of light goes off the scale, the condenser shall be charged with a fraction of E. M. F. of the cell by the potential divider arrangement as depicted in fig. 30·4. Repeat the experiment several times with different values of C and find the mean value of K.

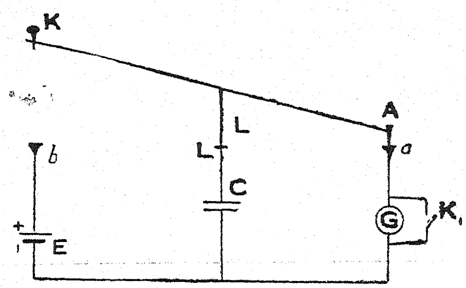


Fig. 30·7

Sources of error and precautions. (1) The base of the galvanometer should be carefully levelled by means of the levelling screws and its coil unclamped before taking any observations with it.

(2) A tapping key should be connected across the galvanometer.

(3) A Morse key should be used in charging and discharging the condenser.

(4) The E. M. F. of the cell used to charge the condenser should be constant. An accumulator may be used for the purpose.

(5) The capacitance of the condensers chosen should be of such a magnitude as to give suitable throw of light-spot on the scale. In case this is not possible and the spot of light moves off the scale, the potential divider arrangement as depicted in fig. 30·4 should be used to charge the condenser by a fraction of the E. M. F. of the cell.

(6) The condensers used in the experiment must be dielectric loss-free and their capacitances must be accurately known. They should be charged only for a short time which should be less than the period of the galvanometer but sufficient to charge the condensers to the potential difference applied.

(7) The observations for logarithmic decrement should be taken with the galvanometer circuit closed, for the damping on open circuit is less than that on closed circuit.

Observations. Determination of θ_1 and λ

Rest position of spot of light=zero

S. No.	Capacitance C μf	P. D. across the condenser E in volts	Deflection of spot of light corresponding to throws of galvanometer		λ	K Coulomb per mm
			θ_1	θ_{11}		
1.						
2.						
3.						
					Mean	

Calculations. Set I. $\lambda = \frac{1}{10} \times 2{\cdot}3026 \log_{10} \times \frac{\theta_1}{\theta_{11}}$

$$=$$

$$K = \frac{CE}{\theta_1 (1 + \lambda/2)}$$

$$=$$

(*N. B.* Make similar calculations for other sets).

Result. The constant K of the ballistic galvanometer

= coulombs/mm.

Criticism of the method. The method gives a fairly satisfactory value of K. If the deflection d_1 of the spot of reflected light on the scale is large, then instead of taking θ_1 equal to d_1, its value should be calculated from the expression $\tan 2\theta_1 = d/L$, where L is the distance of the scale from the mirror. The accuracy of result is affected by the dielectric loss and the inaccuracy in the knowledge of true value of the capacitance of the condensers. The value of K depends upon the nature of the working conditions and upon the distance of the scale from the mirror if θ_1 is taken to be equal to d_1. Hence while experimenting with a ballistic galvanometer either K is eliminated from the formula used to determine the unknown quantity or it is evaluated in the course of the work.

Experiment 30.3

Object. To compare the capacitances of two condensers by *de Sauty's method.*

Apparatus. Two condensers, two high-resistance boxes, a battery of accumulators of e. m. f., 6 to 9 volts, Morse key, a high resistance galvanometer and connecting wires.

Theory. As depicted in fig. 30·8, the two condensers whose capacitances c and c' are to be compared form the two *adjacent* arms AB and AD of a Wheatstone bridge. The remaining two arms BC and CD of the bridge consist of *non-inductive* high resistances r and r' respectively. A high-resistance galvanometer G is inserted between B and D; and a battery of rather high E. M. F., E is connected between A and C through a Morse key K.

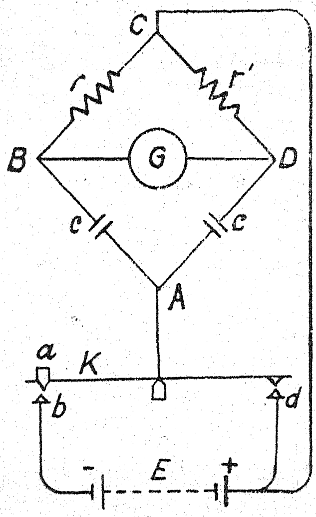


Fig. 30·8

When the knob a of the key K is pressed, it makes contact at b and the condensers are charged. When the knob a is released, contact is made at d and the condensers discharge. The values of the resistances r and r' are so adjusted that the galvanometer coil remains *undeflected* whether the key K is pressed or released. Then obviously the points B and D remain at the *same* potential during charging as well as discharging of the condensers.

Let at any instant t, reckoned from the instant the key K is depressed, the charge on the condenser c is q and that on c' is q'. Then from the theory of charging of a condenser with a resistance in series, we have

$$q=q_0e^{-t/Cr}$$

and $$q'=q'_0e^{-t/C'r'}$$

where q_0 and q_0' are the charges on the two condensers respectively when they are *fully* charged.

Now let V and V′ be the P. Ds. across the condensers c and c' respectively at the instant t. Then obviously

$$V=\frac{q}{C}=\frac{q_0}{C}e^{-t/Cr}$$

and $$V'=\frac{q'}{C'}=\frac{q_0'}{C'}e^{-t/C'r'}$$

But, if B and D are to be at the *same* potential at any instant t during the charging of the condensers,

$$V=V'$$

or $$\frac{q_0}{C}e^{-t/Cr}=\frac{q_0'}{C'}e^{-t/C'r'}$$

Now $q_0/C=q_0'/C'=E$, the E. M. F. of the battery. Hence the above equation gives

$$\frac{t}{Cr}=\frac{t}{C'r'} \tag{30.5}$$

i.e., the time-constants for the two condenser-circuits are equal when the bridge is balanced.

The *same* condition for the balancing of the bridge would also result if the process of discharging of the two condensers is considered.

Thus, if the galvanometer-coil remains stationary during both charging and discharging of the two condensers, equ. (30·5) gives

$$\frac{C}{C'}=\frac{r'}{r} \tag{30.6}$$

This equation can be used to compare the two capacitances.

Alternatively, a graph may be plotted between the various values of r' taken along the X-axis and the corresponding values of r' taken along the Y-axis. Then the slope of the *straight line* thus obtained will give the ratio C/C' of the two capacitances.

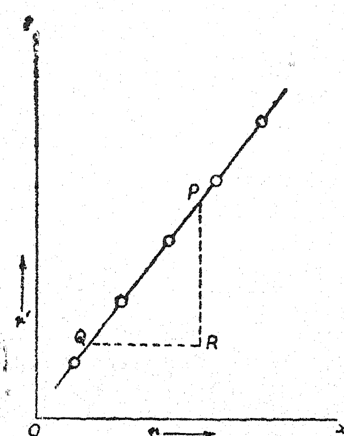


Fig. 30·9

Method. Level the base of the high-resistance galvanometer by means of the levelling screws and unclamp its coil. Throw light on the mirror of the galvanometer and get a bright spot of reflected light on the scale. Form a Wheatstone bridge as shown in fig. 30·8 by connecting the two given condensers c and c' between A and B, and A and D respectively and two high-resistance boxes, one between B and C and another between C and D. Next connect the galvanometer between B and D and a battery of 6-9 volts between A and C through a Morse key K as shown.

Adjust the resistance box in BC to a suitable value r, say 10,000 ohms and the resistance box in CD to the *same order* r'. Depress the knob a of the key K, when the condensers will be charged and, in general, a throw will be produced in the galvanometer and hence the spot of light will move on the scale. Release the key K when the condensers will get discharged and there will again be a movement of the spot of light on the scale.

Now keeping r *fixed,* adjust the value of r' such that there is no throw in the galvanometer, *i.e.,* the spot of light remains *stationary* on the scale, whether the key K is pressed or released, *i.e.,* when the condensers are charged or discharged. To effect this adjustment of r', first give r' some value and observe the direction of movement of the spot of

light on the scale during charging of the two condensers. Then give r' the maximum value available in the resistance box and again observe the direction of movement of the spot of light on the scale during charging of the condensers, when it will be found to be *opposite* to that in the previous case. Gradually narrow down the gap between these two values of r' until a value of r' is found out with which the bridge is balanced. In the initial stages it may be found convenient to alter r' in steps of 1000 ohms, then 100 ohms and finally, if necessary, in steps of 10 ohms.

Note down the values of r and r' when the bridge is *balanced* and calculate the ratio c/c' from equ. (30·6). Repeat the above steps by altering the value of r and find the mean value of c/c' after calculating its value *separately* from *each* set of observations. Finally plot a graph between r and r', taking r along the X-axis and r' along the Y-axis. The graph will come out to be a *straight line* as shown in fig. 30·9. Find the *slope* PR/QR of *this* line which will give c/c'.

Sources of error and precautions. (1) The base of the galvanometer should be carefully levelled and its coil should be unclamped before use.

(2) The galvanometer should be of *high* resistance.

(3) The resistances r and r' should be non-inductive and should have a value of the order of 10,000 ohms. Sufficiently high values of r and r' may be obtained by using a P. O. box for each.

(4) For sufficient sensitiveness of the bridge, the battery should be of rather high E.M.F., 6-9 volts but need not be of large capacity.

(5) The graph between r and r' should be a straight line and should be smoothly drawn.

Observations.

Set No.	r ohms	r' ohms	Direction of movement of spot of light	r' for balance	Point on graph	r/r
1	10,000	9000 9100 9200	Left No deflection Right	9100	A	
2					B	
...					...	
...					...	
					Mean	

Calculations. Set I. $\frac{C}{C'} = \frac{r'}{r} = \qquad =$

(*N. B.*—Make similar calculations for other sets).
from graph (30·9)

PR = and QR =

$$\therefore \quad \frac{C}{C'} = \frac{PR}{QR} = \qquad =$$

Result. The ratio of the capacitances of the given two condensers
(*i*) from calculations =
(*ii*) from graph =

Criticism of the Method. This is a simple method of comparing two capacitances. It is a *null* method and gives the best results when the resistances r and r' are considerable and the condensers have large capacitances and are dielectric loss-free. But the sensitiveness is not great for the quantity of charge which flows through the galvanometer, is only a *small* part of the difference of charges on the condensers which themselves are *small*. Consequently the deflection of the galvanometer is very small and the adjustment of the resistances r and r' can be varied usually over a *wide* range, without causing any appreciable change in the deflection of the galvanometer.

The experiment can also be performed by using an A. C. source such as an andio-frequency oscillator, instead of the battery and some A. C. detector, *e.g.* head-phones, in place of the galvanometer. In such a case the resistances r and r' are so adjusted that there is perfect silence in the head-phones.

30·4. Combination of Condensers. (1) *Law of capacitances in parallel.* When the condensers are so arranged that their similarly charged coatings are connected together (Fig. 30·10), they are said to have been connected in parallel; and the total capacitance of the system of condensers arranged in parallel is equal to the sum of the individual capacitances. Thus mathematically

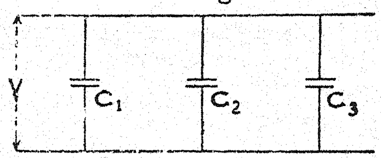


Fig. 30·10

$$C_t = C_1 + C_2 \ldots\ldots C_n$$

or

$$C_t = \Sigma C_n$$

It is evident from above that the capacitance increases when condensers are connected in parallel.

(2) *Law of capacitances in series.* When the condensers are so arranged that one coating of one condenser is connected to the oppositely charged coating of the other and so on (Fig. 30·11), they are said to have been connected in series and the reciprocal of the total capacitance of the system of condensers arranged in series is equal to the sum of the reciprocals of the individual capacitances. Thus mathematically

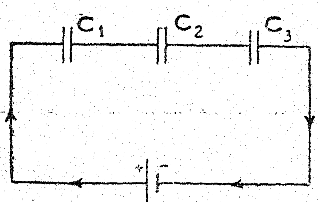


Fig. 30·11

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots\dots\dots \frac{1}{C_n}$$

or
$$\frac{1}{C_t} = \Sigma \frac{1}{C_n}$$

It is evident from above that the capacitance is decreased by connecting condensers in series. In a series arrangement note also that the total capacitance is *less* than the least of the individual capacitances.

30·5. The Dielectric Constant or Permitivity. We have stated in § 30·3 that the capacitance of a condenser increases K times when the space between its coatings is filled with a medium of dielectric constant K instead of air. Hence *the* **dielectric constant** *K of any substance can be measured by the ratio of the capacitance of a condenser with that substance filling the space between its coatings to the capacitance of the same condenser with air in the space between the coatings.* The dielectric constant for air is equal to unity for most practical purposes. When a dielectric is traversed by lines of force, there is a mechanical strain in it. When this strain exceeds a certain limit depending upon the nature of the dielectric, a spark passes across it and the dielectric is punctured. Hence while selecting a substance for the dielectric of a condenser, we must consider not only its cost and value of dielectric constant but also its dielectric strength, *i.e.*, its ability to stand the application of high potential differences without being punctured. Sce the break down potential difference is approximately proportional to the thickness of the dielectric, the dielectric strength is usually measured by the P. D. necessary to puncture one millimetre thickness and is expressed in volts or kilovolts per mm. Mica is the best dielectric for it has a high dielectric constant and offers great resistance to rupture by electric discharge and is, therefore, used in the construction of standard condensers. For relatively small P. Ds., say about 200 volts, the dielectric used is generally paraffined paper covered on both sides with tin or aluminium foil. For higher P. Ds. use is made of glass and oil. The following table contains the dielectric constant and the dielectric strengths of some important substances.

Table of dielectric constant and dielectric strength

Material	Approx. Dielectric Strength in kilo-volts per mm.	Approx. Dielectric Constant K
Asbestos	3—4·5	...
Bakelite	20—25	5—6
Cotton	3—4	...
Ebonite	10—40	2—3
Glass	5—12	3—8
Gutapercha	10—20	3—5
India rubber	10—25	2—3
Mica	40—150	3—8
Paper	4—10	2
Paraffin wax	8	2
Porcelain	9—20	4—7
Shellac	5—20	2·5—3·5
State	3	7
Oil	25—30	2—3·1
Water	...	40—90
Air	3	1·000585

Oral questions

COMPARISON OF CAPACITANCES

What do you understand by capacitance of a conductor and upon what factors does it depend? Define the absolute and practical units of capacitance and state the relation between them. How does the capacitance of a conductor vary with the nature of the neighbouring conditions? What is a condenser? Explain how the capacitance of a conductor can be increased by placing near it an earthed conductor? Upon what factors does the capacitance of a condenser depend? How can you increase the capacitance of a condenser? Is the capacitance of a condenser defined in the same way as the capacitance of a conductor? How can you determine or compare the capacitances of condensers. How do you charge a condenser? What precautions do you observe in your experiment? What is the use of Morse key? Explain the charging and discharging of a condenser with its help? Why do you use a two-way key? Why don't you use a Leclanche cell in this experiment? Why do you charge the condensers for a short time only? Is the result in any way affected by the time of discharge also? Why should the throws be small? If the throws are large, how can you reduce them?

How will the formula for comparing the capacitances be modified if the throws are large? If the throw obtained with the condenser of large capacitance is beyond the scale, how will you modify your method to get it on the scale keeping the throw with the other condenser the same? What are laws of capacitances in series and in parallel? Are they the same as the laws of resistances in series and parallel? How can you verify the laws of combination of condensers by a ballistic galvanometer? How are standard condensers constructed? Why is mica generally used as dielectric in standard condensers? What should be the qualities of a good dielectric for a standard condenser? How do you define dielectric constant and dielectric strength of a material? How are variable condensers constructed? Describe the construction of a condenser box.

CONSTANT OF A BALLISTIC GALVANOMETER

What is a ballistic galvanometer? What are its chief features? To what use is it generally put? What do you understand by the constant of a ballistic galvanometer? How can you determine it with a standard condenser of known capacitance? What precautions do you take in this experiment? Do you know any other method of calibrating the ballistic galvanometer? Compare them. Does the value of K depend upon the working conditions and the distance between the scale and the mirror? What do you understand by logarithmic decrement? Why and how do you determine it? Do you take observations for logarithmic decrement on closed or open circuit of the galvanometer and why? On what factors does the damping depend? How can you increase or decrease the damping? What is the use of the tapping key connected across the galvanometer? Explain how by tapping this key the galvanometer coil can be brought to rest in a short time? What other experiments have you performed with a ballistic galvanometer? How can you compare the E. M. Fs. of two cells by a ballistic galvanometer? Compare this method with the potentiometer method of comparing the E. M. Fs. of two cells. Can you determine the internal resistance of a cell by a ballistic galvanometer? If so, how? Compare this method with other methods of determining the internal resistance of a cell.

CHAPTER XXXI
ELECTROMAGNETIC INDUCTION

31·1. Laws of Electromagnetic Induction. In 1831 Faraday made the most important discovery that when the number of lines of magnetic induction passing through a conducting circuit is changed, a current is produced in the circuit. This phenomenon is called *electromagnetic induction* and the current produced by induction is termed the *induced current*. Faraday summed up the result of his experiments in the following laws of electromagnetic induction :—

(1) *Whenever the magnetic flux threading or linked with a conductor is changed an induced* E. M. F. *acts round the conductor.* If the conductor be a closed circuit, an induced current flows round the circuit. The induced E. M. F. or the current lasts only while the change is taking place.

(2) *The magnitude of the induced* E. M. F. *is equal to the rate of change of the flux* N *linked with the conductor*, that is

$$e = -\frac{dN}{dt} \tag{31·1}$$

Note that the magnitude of the induced E. M. F. is absolutely independent of (i) the shape or size of the conductor, (ii) the nature of its material, or even (iii) the existence of a closed circuit. If the conductor forms a part of a closed circuit, the magnitude of induced current will be equal to the induced E. M. F. divided by the resistance of the circuit. The magnitude of induced current, therefore, depends upon the shape, size and the material of the conductor as well.

The change in magnetic flux linked with a conductor may be due to motion, relative to the conductor, of magnets or of conductors carrying currents, or, without relative motion, to the starting or stopping or variation of the currents in the neighbouring conductors or in the conductor itself. When the induced E. M. F. in a circuit has been produced on account of relative motion of a neighbouring circuit or on account of change of current in it, the circuit in which the E. M. F. is induced is called the secondary and the neighbouring circuit, the primary.

The negative sign in equ. (31·1) shows that *the direction of induced* E. M. F. (*and current*) *is such that it tends to oppose the very cause which produces it*, *i.e.*, the induced current establishes a magnetic flux which tends to neutralize the change in flux which causes it. This statement is known as *Lenz's law* or third law of electromagnetic induction and is very helpful in determining the direction of the induced current. From this law it is evident that when the current in the primary circuit is increased or the primary moved nearer to

the secondary, the current induced in the secondary will be in a direction opposite to the primary current, *i.e.*, *inverse*, and when the current in the primary is decreased or the primary moved away from the secondary the current induced in the secondary will be in the same direction as the primary current, *i.e.*, *direct*. In the case of a single conductor moving in a magnetic field at right angles to itself and the field, direction of induced E. M. F. (and current) can be conveniently determined by Fleming's Right Hand Rule which runs as follows—*Hold the thumb and the first two fingers of the right hand mutually at right angles. Then if the forefinger points in the direction of the magnetic field and the thumb in the direction of motion of the conductor, the second finger will point in the direction of the induced* E. M. F. *(and current)*.

31·2. The Self-Inductance (L) of a Circuit. When the current in a closed circuit is started, stopped or varied, the magnetic flux linked with the circuit changes and hence an E. M. F. (and current) is induced in the circuit. This effect is known as *self-induction*. When the permeability of the medium is constant, the value of the effective magnetic flux linked with a circuit is proportional to the primary current in the circuit. Thus, if N be the effective magnetic flux linked with a circuit when a current I is flowing through it

$$N \propto I$$

or

$$N = LI \qquad (31·2)$$

where L is a constant for the circuit and is known as *coefficient of self-induction* of the circuit. If the circuit consists of a coil of wire with n turns, each of the magnetic lines of force threads through all of the turns. Hence the effective flux N in this case is equal to n times the flux ϕ through each turn, that is $N = n\phi = LI$. If $I=1$, $N=L$. Hence **coefficient of self-induction** *or self-inductance of a circuit is equal to the effective magnetic flux or linkages of the circuit when unit current is flowing through it.*

When the magnetic flux linked with a circuit is changing owing to the current changing, then the E. M. F. induced in the circuit is

$$e = -\frac{dN}{dt}$$

where $\frac{dN}{dt}$ is the rate of change of flux through the circuit.

Putting N=LI, the above equation is modified to

$$e = -\frac{d}{dt}(LI)$$

$$= -L\frac{dI}{dt} \qquad (31·3)$$

where $\frac{dI}{dt}$ is the rate of change of current in the circuit.

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If $\frac{dI}{dt}=1$, $e=-L$. Thus **coefficient of self-induction** *or inductance of a circuit is numerically equal to the induced E. M. F. in the circuit when the current is changing at the rate of one unit per second.* The negative sign in equ. (31·3) shows that the E. M. F. is inverse or opposing when the rate of change of current is positive, *i.e.*, current is increasing. Equations (31·2) and (31·3) can be used to define unit of self-inductance. A circuit has a self-inductance of one C. G. S. unit if the magnetic flux linked with it due to unit e. m. current in it is unity or if the E. M. F. induced is one e. m. unit when the current is changing at the rate of one e.m. unit per second. The self-inductance of an iron-free coil depends only upon the geometry of the coil and the number of turns in it.

The practical unit of self-inductance is *henry* which is equal to 10^9 C. G. S. units. A circuit possesses a self-inductance of one henry if the magnetic flux or linkages be 10^8 when one ampere passes through it or if the E. M. F. induced in the circuit is one volt when the current is changing at the rate of one ampere per second.

The self-inductance is analogous to inertia for when the current is started in a circuit, the induced E. M. F. tends to retard the growth of current. The result is that the current does not reach its steady maximum value instantaneously but takes some time which may in some cases be several minutes. For instance, when a D. C. motor is started it takes a fairly considerable time, on account of large self-inductance of field coils, before the field current attains its full strength. Similarly when the current is stopped, the induced E. M. F. on account of self-induction tends to make it keep on, and if the self-inductance is large and the break very quick, the magnitude of the induced E. M. F. is very great. For example, when a D. C. motor is shut off, the E. M. F. induced is very great so that an arc is formed at the switch which, in time, burns the contacts away.

A loop of wire and a short straight wire have negligible self-inductance and are said to be practically non-inductive. In a loop of wire the induction effect of every small portion of it is neutralised by the part diametrically opposite to it. It should be noted that no circuit is absolutely non-inductive, for even with a straight wire, some field must exist around it. If a coil of wire consists of many turns closed together, the self-inductance is very large, *i.e.*, it is highly inductive. If a coil is wound on an iron core, the self-inductance will be considerably increased, *e.g.*, in field coils of D. C. motors. In coils of resistance boxes the coils are wound almost completely non-inductively by first doubling the wire along its length and then winding it on the bobbin. This bifilar winding gives the effect of two wires side by side but carrying currents in opposite directions; the result is that the magnetic field due to current in one is neutralised by the magnetic field due to opposite current in the other and the coil has a negligible self-inductance.

Experiment 31·1

Object. To determine the self-inductance of a given coil by Rayleigh's method.

Apparatus. The coil whose self-inductance is to be determined, a Post Office box, a moving coil ballistic galvanometer of suspended type, a decimal ohm box containing resistances of the order of 0·1 and 0·01 ohms, a platinoid wire and a binding screw, an accumulator, a special double key as shown in fig. 31·2, a tapping key and connecting wires.

Theory. Let the coil whose self-inductance L is to be determined be placed in one arm of a Wheatstone bridge as shown in fig. 31·1, the resistances in the other arms being non-inductive, and let the resistances of the various arms be so adjusted that the bridge is balanced for steady currents, the galvanometer used being ballistic. Then, if the battery circuit is opened while the galvanometer circuit remains closed, during the decay of current an E. M. F. $-L\frac{dI}{dt}$ will be induced in the coil in the branch CD, where I is the current in CD at any instant. This will send an instantaneous current in the galvanometer, the result being a momentary throw in the galvanometer. Let θ_1 be the first observed throw of the galvanometer, then the quantity of electricity passing through the galvanometer due to the above cause will be given by

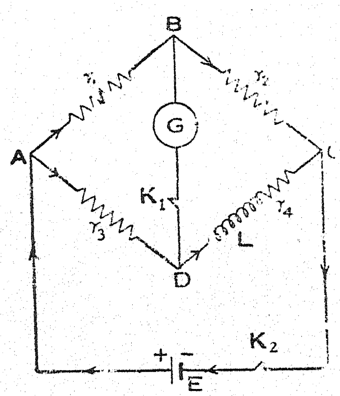


Fig. 31·1

$$Q=\frac{CT}{2\pi\, AH}\theta_1\left(1+\frac{\lambda}{2}\right) \tag{31·4}$$

where C is the torsional reaction per unit radian twist, H the field of the permanent magnet, λ the logarithmic decrement, T the period of the galvanometer and A the effective face area of the coil of the galvanometer.

The instantaneous current i in the galvanometer due to induced E. M. F. in L will be proportional to $-L\frac{dI}{dt}$ that is

$$-L\frac{dI}{dt}=\mu i \tag{31·5}$$

where μ is a constant for the circuit depending upon the relative values of resistances in the various branches. Integrating equ. (31·5), we get

$$\int_0^t -L\frac{dI}{dt}\,dt=\int_0^t \mu i\,dt$$

or

$$-L\int_{I_0}^{0} dI=\mu\int_0^t i\,dt$$

or

$$LI_0=\mu Q \qquad \ldots \qquad (31{\cdot}6)$$

where I_0 is the maximum steady current flowing through CD. From equ. (31·6) it is evident that the value of charge Q is independent of the time taken by the process of induction. Combining **equ.** (31·4) and (31·6), we get

$$LI_0=\frac{\mu CT}{2\pi AH}\theta_1\left(1+\frac{\lambda}{2}\right) \qquad (31{\cdot}7)$$

Now let a small resistance r be introduced in the branch CD which developes an additional potential difference rI_0 in the branch CD, r being so small that the current I_0 in the branch does not change appreciably. The effect of this additional P. D. in CD is to cause a current to flow through the galvanometer, which is given by

$$i'=\frac{rI_0}{\mu}$$

If the steady deflection produced in the galvanometer by this current i' be ϕ we have

$$AHi'=C\phi$$

or

$$AH\frac{rI_0}{\mu}=C\phi$$

whence

$$\mu=\frac{AHrI_0}{C\phi}$$

Substituting the value of μ in equ. (31·7), we get

$$L=\frac{T}{2\pi}.r.\frac{\theta_1}{\phi}\left(1+\frac{\lambda}{2}\right) \qquad (31{\cdot}8)$$

The value of logarithmic decrement can be obtained from the expression

$$\lambda=2{\cdot}3026\ \log_e\frac{\theta_1+\theta_2+\theta_3\ldots\theta_{n-1}}{\theta_2+\theta_3+\theta_4+\ldots\theta_n} \qquad (31{\cdot}9)$$

where θ_n is the nth observed throw of the galvanometer.

Description of Double Key. The double key shown in fig. 31·2 is very helpful in breaking the galvanometer circuit the moment the discharge has passed through it. It consists of three strips PP′, QQ′ and RR′ and a block SS′ of brass mounted horizontally in an ebonite block and connected to separate terminals T_1, T_2, T_3 and T_4 as shown. The battery is connected between T_3 and T_4 and the galvanometer between T_1 and T_2. When M is depressed R and S are connected and the battery circuit is closed

On depressing P the contact is made between P and Q and the galvanometer circuit is also closed. The strips Q and R remain insulated by the ebonite strips in between them. Thus

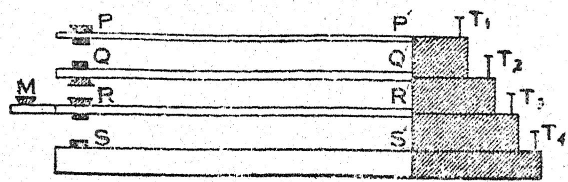


Fig. 31·2

M and P are simply battery and galvanometer key respectively, and with their help the bridge can be balanced for steady currents. Now if keeping P depressed M is released this will not break any of the two circuits but if now P is also released the battery circuit will be broken first and then after some time the galvanometer circuit, thus opening the galvanometer circuit immediately after the discharge has passed through it and the impulse given to it. In this case the throw of the galvanometer will almost be independent of electromagnetic damping.

Method. Level the base of the galvanometer by means of the levelling screws and release its coil. Throw light on the mirror of the galvanometer and get a bright spot of reflected light on the scale. Adjust the spot of light on the scale at zero. Connect the coil L whose self-inductance is to be determined in the unknown resistance arm of the Post Office box inserting in series with it a decimal ohm box containing resistances of the order of 0·1 and 0·01 ohms. In series with the rheostat arm of the P. O. box put a platinoid wire the length of which can be varied by slipping it through a binding screw. Connect the galvanometer and the battery directly to their places in the P. O. box through the special double key shown in fig. 31·2, thus avoiding the use of the P. O. box keys. Insert a rheostat of high value in the battery circuit and adjust it to a suitable value. Connect a tapping key across the galvanometer.

Adjust the decimal-ohm box in series with the coil L to zeroe ohm. Adjust the ratio arms of the P. O. box to 10 : 10 or to 1 nc: if available according as whichever order is nearer the resistance of the coil L. Close the battery circuit by depressing M and then the galvanometer circuit by depressing P. Adjust the rheostat in the battery circuit if necessary until a convenient deflectsion is obtained and the spot of light appears on the scale. Now adjust the rheostat arm of the P. O. box until by changing tqe resistance in it by one ohm the direction of deflection in the galvanometer is reversed. Then adjust the length of the platinoid wire in the circuit until a perfect balance of the bridge for steady currents is obtained, the final adjustment being done by passing the current only for a short time and thus avoiding the heating of the coil L.

Having obtained perfect balance for steady currents, release M keeping P depressed. Note that this will not break either the battery circuit or the galvanometer circuit. Next release P. R and S will separate first and then P and Q. This will first break the battery circuit and then immediately after it the galvanometer circuit and a throw will be produced in the galvanometer. Note down the deflections of the spot of light on the scale corresponding to successive throws of the galvanometer. Then determine the period of the galvanometer; and after it bring it to rest by tapping the key connected across it.

Now test the bridge again for balance for steady currents. If the balance has not remained perfect, alter the length of platinoid wire until the balance of the bridge is again perfect. Next introduce in the arm containing the coil L a suitable small resistance r of the order of 0·01 ohm with the help of decimal-ohm box connected in series with the coil L. Depress the key M and then key P. A steady deflection will be produced in the galvanometer. Note down the deflection of spot light on the scale corresponding to this steady deflection.

Now calculate the logarithmic decrement λ from the equation (31·9) and then the value of self-inductance L of the given coil from equation (31·8). Repeat the experiment with different values of r and take at least three sets of observations. Find the mean value of L.

Sources of error and precautions. (1) The connecting wires should be short and straight. No coiled wire should be used for making connections.

(2) The base of the galvanometer should be carefully levelled by means of the levelling screws and the coil should be unclamped.

(3) A tapping key should be connected across the galvanometer in order to bring the galvanometer coil to rest in a short time by tapping the key, when so desired. The motion of the coil in the magnetic field induces in it an opposing current which brings the coil to rest.

(4) The E. M. F. of the cell used should be constant while the induced E. M. F. $-L\frac{dI}{dt}$ is being developed in the coil L in CD. Accumulator may be used for its E. M. F. remains sensibly constant.

(5) To get a suitable deflection a high adjustable resistance should be connected in series with the cell.

(6) Since the resistances r_1 and r_3 or r_2 and r_4 are in a way shunting the galvanometer, throw obtained in it is small, specially when the resistance of coil L is small. A measurable and reliable throw can be obtained by opening the galvanometer circuit immediately after the discharge has passed through it with the help of a special doubl- key (Fig. 31·2).

(7) In order to have high sensitiveness of the bridge, it is advisable to insert a suitable small non-inductive resistance in series with the coil L in the same arm CD.

(8) The current should be allowed to pass through the bridge only for as short a time as possible otherwise the balance point will continually change due to the change of the resistance of the coil L in CD. The coil generally consists of copper wire which has a large temperature coefficient for resistance. The key, therefore, should be pressed only when observations are to be made.

(9) The balancing of the bridge for steady currents must be perfect.

(10) The resistance r introduced in the arm CD in series with the coil L to produce steady deflection ϕ should be very small so that it may not affect the value of the steady current I_0 in that branch appreciably. It may be preferably of such a magnitude as to produce ϕ equal to θ at the same time not affecting the value of I_0.

(11) For the determination of logarithmic decrement λ, as many successive throws of the galvanometer as it is possible to observe should be noted after the discharge has passed through it.

(12) While determining the period T of the galvanometer, the galvanometer circuit must be kept *open.*

Observations. (A) Determination of θ_1, ϕ, and λ

Rest position of spot of light=zero

S. No.	Scale deflection corresponding to successive throws of galvanometer						r ohms	ϕ in scale divisions	λ
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6			
1.									
2.									
3.									

(B) Determination of period of the galvanometer.

S. No.	No. of oscillations	Time taken		Period
		Min.	Sec.	Sec.
1				
2				
3				
			Mean	

Calculations. Set 1 $\lambda = 2\cdot3026 \log \dfrac{\theta_1+\theta_2+\theta_3+\dots+\theta_{n-1}}{\theta_2+\theta_3+\theta_4+\dots+\theta_n}$

$$L = \frac{T}{2\pi} \cdot r \; \frac{\theta_1}{\phi}\left(1+\frac{\lambda}{2}\right)$$

$$= \qquad \text{henry}$$

Result. The self-inductance of the given coil= henry.

Criticism of the method. The method gives accurate results of the self-inductance of a coil specially when the double key is used to break the galvanometer circuit immediately after the discharge has passed through it. For more accurate results null methods, *e.g.*, Anderson's method in which the throw due to self-induction is neutralised by an equal and opposite effect due to a known capacitance, should be used. Rayleigh's method is suitable for the determination of self-inductance of coils not containing any magnetic material and having a fairly large value of self-inductance. When the coil has an iron core the *make and break* methods must not be used. With iron core the magnetic flux threading the circuit is not proportional to the current.

31·3. The Mutual Inductance (M) of two circuits. When two closed circuits are in close proximity, then, whenever the current in any one of them is started, stopped or varied, there is a change in the magnetic flux threading the second circuit due to current in the first circuit and an E. M. F. (and current) is induced in the second circuit. This effect is known as *mutual induction*. Clearly any one of the two coils can be used as the primary and the other as the secondary. If the permeability of the medium be constant the magnetic flux N linked with secondary due to the current in the primary is directly proportional to the magnitude of the current in the primary. Thus

$$N \propto I$$

or

$$N = MI$$

where M is a constant of the two circuits and for iron free circuits depends only upon the geometrical shapes and the relative positions of the two circuits. It is called the coefficient of mutual induction or simply the mutual inductance of the two circuits. If $I=1$, $N=M$. Thus *the* **coefficient of mutual induction** *or mutual inductance of two circuits is equal to the effective magnetic flux or linkages of one circuit due to unit current flowing in the other.*

When the magnetic flux threading the secondary circuit is changing owing to the current in the primary changing, then the E. M. F. induced in the secondary is given by

$$e = -\frac{dN}{dt}$$

where $\dfrac{dN}{dt}$ is the rate of change of flux through the secondary owing

to the current in the primary changing. Putting N=MI in the above equation, we have

$$e = -\frac{d}{dt}(\mathrm{MI})$$
$$= -\mathrm{M}\frac{d\mathrm{I}}{dt}$$

where $\frac{d\mathrm{I}}{dt}$ is the rate of change of the current in the primary circuit. When $\frac{d\mathrm{I}}{dt} = 1$, $e = -\mathrm{M}$. Thus *the* **coefficient of mutual induction** *or mutual inductance of two circuits is numerically equal to the E. M. F. induced in one circuit when the current strength in the other varies at the rate of one unit per second.* The mutual inductance is also measured by the mutual potential energy of the two circuits when unit current is flowing in each. There is only one value of mutual inductance between two coils for any one of them can be used as the primary circuit.

The unit of mutual inductance is the same as that of self-inductance. The practical unit henry is the mutual inductance of two circuits when the effective magnetic flux or linkages of one circuit due to a current of one ampere in the other be 10^8 or when the induced E. M. F. in one due to change of current at the rate of one ampere per second in the other be one volt.

The mutual inductance of two circuits can be increased by increasing the number of turns in the coils as well as by winding them on iron cores. When iron core is used, the iron gets magnetised when the current passes through the primary. The change in magnetic flux is, therefore, much greater and hence the induction effects are considerably increased. The magnitude of the induced E. M. F. in the secondary varies directly as the number of turns in the secondary for the E. M. Fs. produced in different turns are added up. Hence the E. M. F. induced in the secondary can be made as large or as small as we please by using many or few turns. This accounts for the large number of turns used in ghe secondary of a transformer or an induction coil. Further, the reater the rate of change of current in the primary, greater will be the E. M. F. induced in the secondary. Hence, if the current in the primary is stopped suddenly, large E. M. F. can be developed in the secondary. This explains the production of very high E. M. F. in the secondary of an induction coil at break which is very sudden.

Experiment 31.2

Object. To determine the mutual inductance of two coils by means of a ballistic galvanometer.

Apparatus. The two coils whose mutual inductance is to be determined, a moving coil ballistic galvanometer of suspended

type, rheostat, a resistance box, an accumulator, a plug-type reversing commutator and two tapping keys.

Theory. Let the secondary winding of the mutual inductance be connected to a ballistic galvanometer and let a current I be allowed to pass through the primary winding. Then, as the current grows from o to I in the primary circuit, an E. M. F. equal to $-M\dfrac{di_1}{dt}$ is induced in the secondary, where M is the mutual inductance of the two coils and i_1 the current in the primary at any instant. This induced E. M. F. produces an instantaneous current in the secondary circuit which is given by

$$i_2 = \frac{e}{R} = -\frac{M}{R} \cdot \frac{di_1}{dt}$$

where R is the resistance of the secondary circuit including that of the galvanometer. The charge passing through the galvanometer during the growth of current I in the primary is given by

$$Q = \int_0^t i_2 \, . \, dt = -\frac{M}{R}\int_0^t \frac{di_1}{dt} . \, dt$$

or

$$Q = -\frac{M}{R}\int_0^I di_1 = \frac{MI}{R} \qquad (31 \cdot 10)$$

If $\theta_1, \theta_2, \ldots \theta_n$ be the consequent successive throws of the galvanometer, the charge passing through it is also given by

$$Q = K\theta_1 \, (1 + \lambda/2) \quad \ldots \quad (31 \cdot 11)$$

where K is the ballistic constant of the galvanometer and λ the logarithmic decrement.

Equating the two expressions (31·10) and (31·11) for Q, and simplifying, we get

$$M = \frac{R}{I} K\theta_1 \, (1 + \lambda/2)$$

If T is the period of oscillation of the galvanometer and k its figure of merit,

$$K = \frac{T}{2\pi} . k$$

$$\therefore \qquad M = \frac{R}{I} . k \frac{T}{2\pi} . \theta_1 \, (1 + \lambda/2) \quad \ldots \quad (31 \cdot 12)$$

Now let the secondary winding in series with the ballistic galvanometer be connected across a small resistance r already included in the primary circuit. This causes an E. M. F. rI to act in the secondary circuit which sends round current I′ producing a steady deflection φ in the galvanometer so that

$$I' = k\varphi$$

or
$$k = \frac{I'}{\varphi} = \frac{rI}{R}.\phi$$

Putting this value of k in equ. (31·12), we have

$$M = \frac{T}{2\pi} \cdot \frac{r}{\varphi} . \theta_1 (1 + \lambda/2) \tag{31·13}$$

from which M may be calculated. The value of λ can be obtained from the expression

$$\lambda = 2·3026 \log \frac{\theta_1 + \theta_2 + {}_3 + \dots + \theta_{n-1}}{\theta_2 + \theta_3 + \theta_4 + \dots + \theta_n}$$

Method. Level the base of the galvanometer by means of the levelling screws and unclamp its coil. Throw light on the mirror of the galvanometer and get a bright spot of light on the scale. Adjust the spot of light on the scale at zero. Connect the primary winding P (Fig. 31·4) to an accumulator B in series with a rheostat R_1, a resistance box R and a tapping key K_1. Next connect the terminals of the resistance box R to the two collinear terminals a and b of a plug type commutator K, to the remaining terminals c and d of which connect the secondary winding in series with a ballistic galvanometer. Connect a tapping key K_2 across the galvanometer.

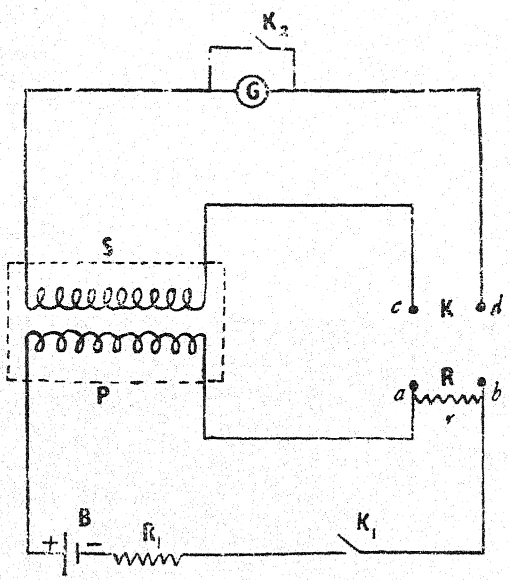


Fig. 31·4

Adjust the resistance box R to a small value of resistance, say r ohms. Close the primary circuit and adjust the current to a suitable value. Connect c to d and when the galvanometer coil is at rest, release the key K_1, and note down the deflections of the spot of reflected light on the scale corresponding to successive throws of the galvanometer. Bring the galvanometer coil to rest by tapping the key K_2.

Next connect c to a and d to b and observe the steady deflection produced in the galvanometer.

Now alter the value of r and repeat the experiment taking at least three sets of observations with different values of r. Determine the period of the galvanometer. Calculate the value of M from equation (31·13) for each set of observation separately and then take the mean value of M.

Sources of error and precautions. (1) The connecting wires should be short and straight. No coiled wire should be used for making connections.

(2) The base of the galvanometer should be carefully levelled by means of the levelling screws and the coil should be unclamped before passing current through it.

(3) A tapping key should be connected across the galvanometer.

(4) If the coils used are large, they should, if possible, be placed as far from the galvanometer as possible so that there is no appreciable effect on the galvanometer coil due to direct action of maximum current in the primary circuit.

(5) A rheostat should be included in the primary circuit to adjust the value of the current in it to a suitable value, say abou one ampere.

(6) The cell used in the primary circuit should be an accumulator.

(7) For determining the successive throws of the galvanometer, its circuit should be kept closed for the damping on open circuit is not the same as that on closed circuit. Further, as many throws of the galvanometer as it is possible to observe should be noted.

(8) The resistance r should be very small compared to the resistance of the secondary circuit.

(9) While determining the period of the galvanometer its circuit should be kept open.

Observations. [A] Determination of θ_1, ϕ and λ.

Rest position of spot of light = zero

S. No.	Scale deflection corresponding to successive throws of galvanometer						r ohms	ϕ in scale divisions	λ
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6			
1.									
2.									
3.									

(B) Determination of period of the galvanometer.

S. No.	No. of oscillations	Time taken		Period
		Min.	Sec.	Sec.
1				
2				
3				
			Mean	

Calculations. Set I $\lambda = 2{\cdot}3026 \log \dfrac{\theta_1+\theta_2+\theta_3+\ldots+\theta_{n-1}}{\theta_2+\theta_3+\theta_4+\ldots+\theta_n}$

$$=$$

$$M = \frac{T}{2\pi} r \; \frac{\theta_1}{\phi}\left(1+\frac{\lambda}{2}\right)$$

$$= \qquad \text{henry}$$

Result. The mutual inductance of the given coils = ...henry

Criticism of the method. The method gives a fairly satisfactory value of mutual inductance of two coils. The main source of error is the effect of self-inductance in the primary. The method is not applicable to coils which have an iron core, for then the magnetic flux linked with secondary due to current in the primary will not be proportional to the current in the primary. For more accurate results alternating current bridges employing vibration galvanometer as detector should be used.

Experiment 31·3

Object. To determine a high resistance by the method of leakage of a condenser.

Apparatus. Condenser, high resistance, accumulator, ballistic galvanometer, Morse key, a two-way key and two one-way keys.

Theory. If a charged condenser of capacitance C is connected across a resistance R, the charge leaks through the resistance. If the charge on the condenser at any time t is Q, the P. D. between its coatings is Q/C and that across the resistance is R $\dfrac{dQ}{dt}$. Hence, as there is no applied voltage in the circuit, the potential equation of the circuit is

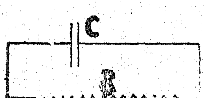


Fig. 31·5

$$\frac{Q}{C}+R\frac{dQ}{dt}=0$$

or

$$\frac{dQ}{Q}=-\frac{dt}{CR}$$

Integrating, this gives

$$\log Q=-\frac{t}{CR}+k \qquad (31\cdot14)$$

If *initially* the charge on the condenser is Q_o, we have $Q=Q_o$, when $t=0$, and therefore

$$k=\log Q_o$$

Substituting this in equation (31·14) and simplifying, we have

$$Q=Q_o e^{-t/CR} \qquad (31\cdot15)$$

Now let a condenser of capacitance C be given a charge Q_0 and then immediately discharged through a ballistic galvanometer. If θ_1 is the first throw of the galvanometer and λ the logarithmic decrement, we have

$$Q_o=K\theta_1(1+\lambda/2) \qquad (31\cdot16)$$

Next, let the same condenser be again given the same charge Q_0 and then, after having allowed the charge to leak through a resistance R for a known time t, let the condenser be discharged through the same ballistic galvanometer. If α_1 is the first throw of the galvanometer in this case, we have

$$Q=K\alpha_1(1+\lambda/2) \qquad (31\cdot17)$$

Dividing equ. (31·17) by equ. (31·16), we have

$$\frac{Q}{Q_o}=\frac{\alpha_1}{\theta_1}$$

But from equ. (31·15), we have

$$\frac{Q}{Q_o}=e^{-t/CR}$$

$$\therefore \qquad \frac{\alpha_1}{\theta_1}=e^{-t/CR}$$

or

$$\log\frac{\theta_1}{\alpha_1}=\frac{t}{CR}$$

whence

$$R=\frac{t}{C\log(\theta_1/\alpha_1)}$$

or

$$R=\frac{t}{2\cdot3026\,C\log_{10}(\theta_1/\alpha_1)} \qquad (31\cdot18)$$

This equation can be used to determine a high resistance.

Rearranging equ. (31·18), we have

$$\log_{10}\frac{\theta_1}{\alpha_1}=\frac{t}{2\cdot3026\ RC}$$

This equation shows that if the charge is allowed to leak through the resistance for different intervals of time and then a graph plotted taking the various values of t as abscissae and the corresponding values of $\log_{10}(\theta_1/\alpha_1)$ as ordinates, the slope of the graph, as illustrated in fig. 31·6, will be given by

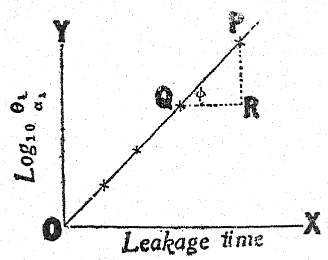


Fig. 31·6

$$\tan\varphi=\frac{1}{2\cdot3026\ RC}$$

whence $$R=\frac{1}{2\cdot3026\ C\tan\varphi}$$

If a condenser has a self-leakage resistance, the experiment should be performed in two parts :

(1) By allowing the charge to leak *without* the external resistance R connected across the condenser. This will determine the self-leakage resistance r of the condenser.

(2) By allowing the charge to leak *with* the external resistance R connected across the condenser. This will determine the equivalent resistance R′ of the parallel combination of r and R.

The external resistance R can then be calculated from the formula

$$\frac{1}{R'}=\frac{1}{r}+\frac{1}{R} \qquad (31\cdot19)$$

Method. Level the base of the galvanometer by means of the levelling screws and release its coil. Throw light on the mirror of the galvanometer and get a bright spot of light on the scale. Adjust the spot of light on the scale at zero. Make electrical connections as shown in fig. 31·7. Join a to b and then press the Morse key at P. This completes the cell-condenser circuit whereby the condenser is charged to a constant P. D. between its coatings equal to the E. M. F. of the cell. When the condenser is fully charged, release the Morse key. This breaks the cell-condenser circuit but at the same time completes the galvanometer-condenser circuit. The condenser discharges itself through the galvanometer. Note down the first throw θ_1 of the galvanometer corresponding to the charge, say Q_0 on the condenser.

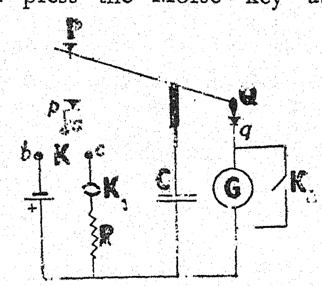


Fig. 31·7

Again charge the condenser to the same voltage by pressing the Morse key at P and then keeping the knob P depressed and

the plug key K_1 open, turn the key K from the position *ab* to the position *ac*, and immediately start a stop-watch. When the charge on the condenser has leaked, if at all, for a known time t, say 5 seconds, release the Morse key and observe the first throw α_1' of the galvanometer. Repeat this self-leakage experiment several times for different values of leakage time t, noting down the value of α_1' for each value of t.

Next close the key K_1, turn the key K into the position *ab* and press the Morsekey at P. When the condenser is again fully charged, turn key K from the position *ab* to the position *ac* keeping the Morse key depressed. When the charge on the condenser has leaked for a known time t, say 5 seconds, release the Morse key and note down the first throw α_1 of the galvanometer. Repeat this leakage experiment with the resistance R connected across the condenser, several times with different values of t observing the value of α_1 for each value of t.

Plot a graph taking the various values of t along the **X-axis** and the corresponding values of $\log_{10}\frac{\theta_1}{\alpha_1'}$ along the **Y-axis.** Find out the slope of the graph and then with the help of equ. (31·18), calculate the value of r, the self-leakage resistance of the condenser.

Next plot a graph between t and $\log_{10}\frac{\theta_1}{\alpha_1}$ and determine with the help of equ. (31·18), the equivalent resistance R' of the parallel combination of r and R.

Finally calculate the external resistance R from equ. (31·19).

Sources of error and precautions. (1) The base of the galvanometer should be carefully levelled and the coil should be unclamped before its use.

(2) A tapping key should be connected across the galvanometer.

(3) The call used should be an accumulator.

(4) The capacity of the condenser should be such as to ensure a satisfactory rate of leak.

(5) There should be no leakage of charge through the keys K and K_1.

(6) The graphs should be smoothly drawn.

Observations. [A] (*i*) Capacitance of the condenser C= f

(*ii*) Rest position of spot of light =zero.

(*iii*) First throw of the galv. corresponding to the full charge Q_0 on it θ_1 = cm.

[B] Measurement of t, α_1' and α

S. No.	Leakage-time sec.	First throw of galv. after the charge has leaked through: Self-leakage resistance of the condenser, *i.e.* with K_1 open α_1' cm.	Parallel combination of self leakage resistance and external resistance, *i.e.*, with K_1 closed α_1 cm.	$\log_{10}\frac{\theta_1}{\alpha_1'}$	$\log_{10}\frac{\theta_1}{\alpha_1}$
1.	5				
2.	10				
3.	...				

Calculations. From the graph between t and $\log_{10}(\theta_1/\alpha_1')$, we have

$$\tan\varphi' = \frac{PR}{QR} = \qquad =$$

$$\therefore \qquad R = \frac{1}{2\cdot3026\ C\tan\varphi'} = \qquad = \qquad \text{ohms}$$

From the graph between t and $\log_{10}(\theta_1/\alpha_1)$, we have

$$\tan\varphi = \frac{PR}{QR} = \qquad =$$

$$\therefore \qquad R' = \frac{1}{2\cdot3026\ C\tan\varphi} = \qquad = \qquad \text{ohms}$$

From equ. (31·19), the external resistance R is given by

$$\frac{1}{R} = \frac{1}{R'} - \frac{1}{r}$$

$$=$$

$$= \qquad \text{ohms}$$

Result. The value of the given resistance as determined by the method of leakage of a condenser = ohms.

Criticism of the Method. The experiment, if performed carefully, gives accurate results. The method is, however, suitable for determination of *high* resistances only, of the order of 20 meg-ohms. The method can be used to measure the resistance and capacity of a condenser if a suitable known high resistance is available.

31·4. Rotation of a plane Closed Coil in a uniform Magnetic Field. When a closed coil is rotated in magnetic field, the number of lines of magnetic induction cut by the coil varies from one instant to the next and hence the magnetic flux threading the coil continuously changes. An induced E. M. F. is, therefore,

produced in the coil which causes a current to flow in it. The induced E. M. F. lasts only so long as the coil is rotating and its magnitude varies from one instant to the other.

Let A be the effective face area of a plane closed coil rotating in a uniform magnetic field of strength F and let the plane of the coil at any instant make an angle θ with the direction of the field. The effective magnetic flux linked with the coil is FA sin θ and hence the instantaneous value of the induced E. M. F. is given by

$$e=-\frac{dN}{dt}=-\frac{d}{dt}(\text{FA}\sin\theta)$$
$$=-\text{FA}\cos\theta.\frac{d\theta}{dt}$$

If the coil is rotating with a uniform angular velocity $\omega=\frac{d\theta}{dt}$,

$$e=-\text{FA}\,\omega\cos\theta \qquad (31{\cdot}20)$$

From this equation it is evident that the induced E. M. F. is maximum when $\theta=0$ or π, *i.e.*, the plane of the coil is parallel to the direction of the field, and minimum when $\theta=\pi/2$, *i.e.*, the plane of the coil is perpendicular to the direction of the field. Note that for first half of one rotation the E. M. F. induced is one directior and for the second half of the same rotation it is in the opposite direction, *i.e.*, it is alternating. These effects on a coil rotating in a magnetic field are utilised in the construction of all dynamos for generating current on a large scale.

If the ends of the coil be connected to a galvanometer, the instantaneous current in it will be given by

$$i=\frac{e}{\text{R}}=-\frac{\text{FA}}{\text{R}}\cos\theta\frac{d\theta}{dt}$$

where R is the resistance of the coil and the galvanometer. The quantity of electricity passing through the galvanometer in time dt is given by

$$d\text{Q}=i dt=-\frac{\text{FA}}{\text{R}}\cos\theta\frac{d\theta}{dt}dt$$

Hence the amount of charge passing through the galvanometer when the coil is rotated through half a complete turn from a position at right angles to the direction of the field is given by

$$\text{Q}=\int_0^{\text{T}/2} i.\,dt=-\frac{\text{FA}}{\text{R}}\int_{\pi/2}^{-\pi/2}\cos\theta.\,d\theta$$

or

$$\text{Q}=2\text{FA}/\text{R} \qquad (31{\cdot}21)$$

It is evident from this equation that the charge passing through the galvanometer is independent of the time taken to rotate the coil through 180°. Note that if the coil is rotated through 180° from a position in which its face is parallel to the direction of the field the quantity of charge passing through the galvanometer will be zero.

31·5. Earth Inductor. The earth inductor (Fig. 31·8) consists of a coil of known dimensions and of about 1000 turns of insulated copper wire, mounted so that it can be rotated about an axis in its plane, the rotation being confined to exactly 180° by means of stops. The stand carrying the coil can be placed with the axis of rotation of the coil horizontal or vertical. The ends of the coil are connected by short pieces of flexible wire to binding screws on the stand by means of which the coil can be connected to an external circuit. The earth inductor can be used to determine the angle of dip, the total intensity of earth's magnetic field and its horizontal and vertical components.

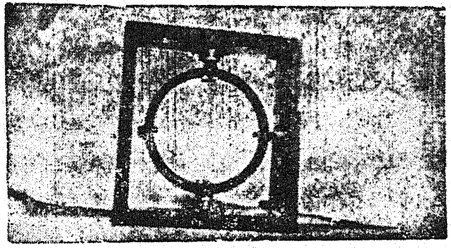


Fig. 31·8

Experiment 31 4

Object. To determine the angle of dip in the laboratory (.........) by means of an earth inductor.

Apparatus. An earth inductor, a moving coil ballistic galvanometer of suspended type, compass needle, a tapping key and connecting wires.

Theory. Let the earth inductor be placed with its axis of rotation vertical and its plane perpendicular to the magnetic meridian and let it be connected to a ballistic galvanometer. Then, if the coil be rotated rapidly through 180°, it will cut the earth's horizontal field and the quantity of charge passing through the galvanometer will from equ. (31·21) be given by

$$Q_1 = 2HA/R \qquad (31{\cdot}22)$$

where H is the horizontal component of earth's magnetic field, R the resistance of the coil, its leads and the galvanometer, and A the effective face area of the coil. Let θ_1 be the first observed throw of the galvanometer. Then

$$Q_1 = K\theta_1\ (1+\lambda/2) \qquad (31{\cdot}23)$$

where K is the constant of the galvanometer and λ the logarithmic decrement.

Combining equ. (31·22) and (31·23), we get

$$H = \frac{RK}{2A}\,\theta_1\ (1+\lambda/2) \qquad (31{\cdot}24)$$

Now let the earth inductor be placed with its plane horizontal and its axis of rotation lying in the magnetic meridian. Then, if the coil be rapidly rotated through 180° it will cut the earth's vertical field and the quantity of charge passing through the galvanometer will from equ. (31·21) be given by

$$Q_2 = 2VA/R \qquad (31{\cdot}25)$$

where V is the vertical component of the earth's magnetic field. Let θ_2 be the first observed throw of the galvanometer in this case. Then

$$Q_2 = K\theta_2 \ (1+\lambda/2) \qquad (31{\cdot}26)$$

Combining equations (31·25) and (31·26), we get

$$V = \frac{RK}{2A}\,\theta_2 \ (1+\lambda/2) \qquad (31{\cdot}27)$$

Dividing equ. (31·27) by equ. (31·24), we have

$$\frac{V}{H} = \frac{\theta_2}{\theta_1}$$

If ϕ be the angle of dip in the laboratory, we have

$$\tan\phi = \frac{V}{H} = \frac{\theta_2}{\theta_1} \qquad (31{\cdot}28)$$

Thus observing the values of θ_1 and θ_2, the angle of dip can be calculated from the above equation.

Alternatively, a graph may be plotted taking θ_1 along the X-axis and θ_2 along the Y-axis. This will come out to be a straight line as shown in fig. 31·9 and its slope PR/QR will give tan φ

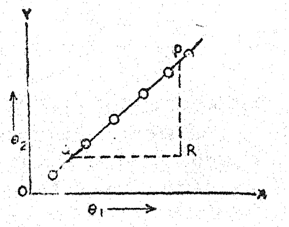


Fig. 31.9

Method. Level the base of the ballistic galvanometer by means of the levelling screws and release its coil. Throw light on the mirror of the galvanometer and get a bright spot of reflected light on the scale. Adjust the spot of light on the scale at zero. Then connect (Fig. 31·10) the earth inductor by means of a piece of twin flexible wire to the galvanometer includ g a resistance R and a key K in the circuit. Connect a tapping key K' across the galvanometer.

Next by means of a compass needle, represent the magnetic meridian on the table by a straight line. Then arrange the earth inductor, when against one of the stops, so that its axis of rotation is vertical and its plane perpendicular to the magnetic meridian. Quickly rotate the coil through 180° and note the deflection d_1 of the spot of reflected light on the scale corresponding to the first throw of the galvanometer. Then bring the galvanometer coil to rest by tapping the key K' connected across the galvanometer.

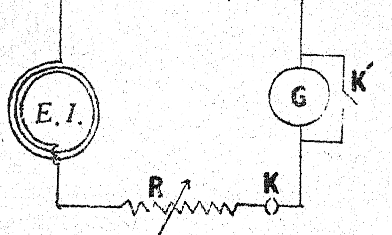


Fig. 31·10

Next set up the earth inductor with its plane horizontal, its axis of

rotation lying in the magnetic meridian. Quickly rotate the coil through 180° and note deflection d_2 of the spot of reflected light on the scale corresponding to the first throw of the galvanometer. Then calculate the angle of dip ϕ from the equation (31·28). Repeat the experiment several times with different values of R and at least three times with each value of R and find the mean value of ϕ.

Finally plot a graph taking θ_1 along X-axis and θ_2 along Y-axis. This will come out to be a straight line as shown in fig. 31·9. Find out the slope PR/QR of this line which will give tan ϕ.

Sources of error and precautions. (1) The base of the galvanometer should be carefully levelled by means of the levelling screws and its coil should be unclamped before beginning to take any observations with it.

(2) The galvanometer should be at a considerable distance from the earth inductor and the connections between the two should be made by a piece of twin flexible wires.

(3) A tapping key should be connected across the galvanometer. By tapping the key the galvanometer coil can be brought to rest in a short time, when so desired.

(4) While determining the throw of the galvanometer on account of cutting the earth's horizontal field by the earth inductor, the axis of rotation of the coil should be vertical and its plane perpendicular to the magnetic meridian.

In case only the plane of the coil is made vertical and not the axis of rotation also, the rotation of the coil *will also cut* the *vertical* component of the earth's field and, unless the setting of the coil is *perfect* and the rotation is *exactly* through 180°, the charge passing through the galvanometer on this account will not be equal to zero.

(5) While determining the throw of the galvanometer on account of cutting the earth's vertical field by the earth inductor, the plane of the coil should be horizontal with its axis of rotation lying in the magnetic meridian.

In case the axis of rotation is not set in the magnetic meridian but only the plane of the coil is made horizontal, the rotation of the coil *will also cut* the *horizontal* component of the earth's field and, unless the setting of the coil is *perfect* and the rotation is *exactly* through 180°, the charge passing through the galvanometer on this account will not be equal to zero.

(6) The graph between θ_2 and θ_1 should come out to be a straight line and should be smoothly drawn.

Observations. Rest position of spot of light=zero.

S. No.	d_1 mm.	d_2 mm.	$\tan\phi = d_2/d_1$	φ
1.				
2.				
3.				
...				
12.				
			Mean	

Calculations. Set I $\tan\phi = \frac{\theta_2}{\theta_1} = \frac{d_2}{d_1}$

$\therefore$ $\phi = \quad ^\circ$

(Make similar calculations for other sets).

From graph (31·9) $PR = \quad , QR =$

$\therefore$ $\tan\phi = PR/QR =$

$\therefore$ $\phi =$

Result. The angle of dip in the laboratory (............)= $^\circ$

Criticism of the method. This is a very quick and convenient method of determining the angle of dip at a point on the earth's surface. By the use of stops the rotation can be confined to exactly 180° and by the use of spring this rotation can be affected very quickly. The method possesses two distinct advantages:—(1) magnetic fields of any strength can be compared or measured by increasing or decreasing the effective area of the coil and (2) the magnetic fields in any direction can be measured by properly mounting the coil with its plane perpendicular to the direction of the field and hence the method is superior in this respect to the magnetometer methods by which only the horizontal component can be determined.

Exercise 1. *To determine the constant of a moving coil ballistic galvanometer (suspended type) by means of an earth inductor.*

Connect the earth inductor to the ballistic galvanometer. Arrange the earth inductor so that its axis is vertical and its plane is perpendicular to the magnetic meridian. Rotate the coil quickly through 180° and determine the successive throws of the galvanometer. Then calculate the constant K of the galvanometer from the formula

$$K = \frac{2AH}{R\,\theta_1\,(1+\lambda/2)} \qquad \ldots \qquad (31·29)$$

where H is the horizontal component of the earth's magnetic field, A the effective face area of the coil, R the total resistance of the coil, the galvanometer and the leads, θ_1 the first throw of the

galvanometer and λ the logarithmic decrement which can be obtained from the expression

$$\lambda = 2{\cdot}3026 \log \frac{\theta_1+\theta_2+\theta_2+\cdots+\theta_{n-1}}{\theta_2+\theta_3+\theta_4+\cdots+\theta_n}$$

where θ_n is the nth throw of the galvanometer.

Exercise 2. *To calibrate a ballistic galvanometer by a solenoid inductor.*

In a convenient form of solenoid inductor, the primary consists of a double layer of a silk-covered copper wire wound uniformly on a glass tube of about 100 cm. in length and of about 4 cm. in diameter, and the secondary consists of two layers of about 100 turns each of double-silk covered fine copper wire wound uniformly over the middle portion of the primary.

To calibrate a ballistic galvanometer make connections as shown in fig. 31·11 and allow a steady current of i amperes to pass through the primary coil P. This produces a magnetic field of $0{\cdot}4\pi n_1 i$ oersteds at the centre of the primary coil, where n_1 is the number of *turns per unit length* of the primary. Since the primary is long its external field in the region of the secondary is extremely small and hence the flux linked with the secondary due to current i in the primary is $0{\cdot}4\ \pi n_1 i \times n_2 A$, where n_2 is the *total number of turns* in the secondary and A the face area of each turn. Now by means of the commutator K_2, reverse the current in the primary, then the change in flux linked with the secondary is evidently given by $N = 0{\cdot}8\ \pi n_1 n_2 A i$ Maxwells. This will produce a flow of charge Q through the ballistic galvanometer. Note down the successive throws of the galvanometer. If θ_1, θ_2, θ_3 are consequent successive throws of the galvanometer, we have

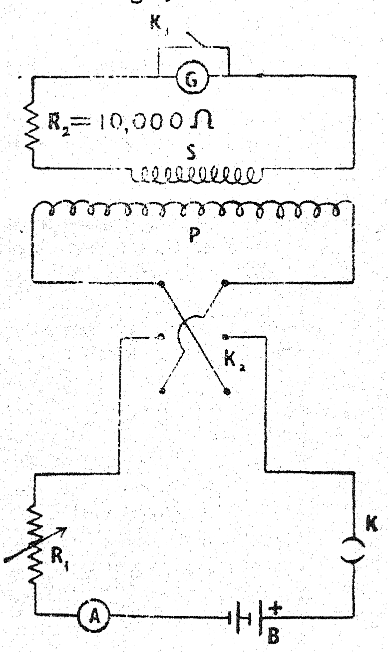


Fig. 31·11

$$Q = K\theta_1\,(1+\lambda/2) \quad \ldots \quad (31{\cdot}30)$$

where K is the ballistic constant of the galvanometer and λ the logarithmic decrement which is given by

$$\lambda = 2{\cdot}3026 \log \frac{\theta_1+\theta_2+\theta_3+\cdots\theta_{n-1}}{\theta_2+\theta_3+\theta_4+\cdots\theta_n}$$

If R is the *total* resistance of the secondary circuit (including that of the galvanometer and its leads), the charge passing through

the galvanometer due to reversal of current in the primary is also given by

$$Q = \frac{N}{R} = \frac{0{\cdot}8\pi n_1 n_2 A i}{R_1} \text{ e. m. u.}$$

$$= \frac{8\pi n_1 n_2 A i}{10^9 R} \text{ coulombs} \quad \ldots \quad (31{\cdot}31)$$

Equating the two expressions (31·30) and (31·31) for Q, we have

$$K\,\theta_1\,(1+\lambda/2) = \frac{8\pi n_1 n_2 A i}{10^9 R}$$

whence $$K = \frac{8\pi n_1 n_2 A i}{10^9 R\theta_1\,(1+\lambda/2)} \quad \ldots \quad (31{\cdot}32)$$

In order to avoid excessive kick in the galvanometer a high resistance R_2 of about 10000 ohms should be included in the secondary circuit.

31·6. Measurement of Magnetic Fields by means of a search Coil. Very intense magnetic fields such as those between the poles of an electromagnet, can be easily measured with the help of a search coil and a calibrated ballistic galvanometer. A search coil is merely a small flat circular coil consisting of a large number of turns of very fine copper wire wound on an ebonite bobbin attached to a handle. The coil is made narrow because firstly most of the fields to be measured are not uniform and secondly the coil is often to be introduced into narrow gaps for such measurements.

Experiment 31.5

Object. To determine the magnetic field between the pole-pieces of an electromagnet with the help of a search coil and a ballistic galvanometer, using an earth inductor for the calibration of the galvanometer.

Apparatus. The electromagnet with battery, ammeter, rheostat, etc., for passing steady current through the field coils, a search coil, a ballistic galvanometer, an earth inductor and keys.

Theory. Let a search coil be placed in the magnetic field between the pole-pieces of an electromagnet with the face of the coil perpendicular to the magnetic lines of force. Also let the coil be connected to a ballistic galvanometer in series with an earth inductor placed with its axis of rotation vertical and the plane of the coil perpendicular to the magnetic meridian. If n be the number of turns in the search coil and a the face area of each turn, the change of magnetic flux through the coil when it is rapidly removed from between the pole-pieces of the electromagnet is given by

$$N = naF$$

where F is the strength of the field between the pole-pieces of the electromagnet. The change of flux through the coil causes a charge Q to flow through the galvanometer. If θ_1 is the consequent first throw of the galvanometer,

$$Q = K\theta_1\,(1+\lambda/2) \quad \ldots \quad (31{\cdot}33)$$

where K is ballistic constant of the galvanometer and λ the logarithmic decrement.

If the current at any instant through the galvanometer is i and

the combined resistance of the galvanometer *circuit* including that of its leads is R, the charge passing through the galvanometer at that instant is also given by

$$Q=\int idt = \frac{1}{R}\int \frac{dN}{dt}\,dt$$
$$=N/R$$
$$=naF/R \qquad (31{\cdot}34)$$

Equating the two expressions (31·33) and (31·34) for Q and simplifying, we get

$$F=\frac{RK}{na}\,\theta_1\,(1+\lambda/2) \qquad (31{\cdot}35)$$

Now let the earth inductor be rotated rapidly through 180°. Then, if a_1 is consequent first throw of the galvanometer, we have from equation (31·24)

$$H=\frac{RK}{2\,n'a'}\,\alpha_1\,(1+\lambda/2) \qquad (31{\cdot}36)$$

where n' is the number of turns in the earth inductor and a' the face area of its each turn and H the horizontal component of earth's magnetic field.

Dividing equ. (31·35) by equ. (31·36), we have

$$F=\frac{2\,n'a'}{n\,a}\cdot\frac{\theta_1}{\alpha_1}\cdot H \qquad (31{\cdot}37)$$

This equation can be used to evaluate F if the constants of the search coil and the earth inductor are known and other quantities are determined.

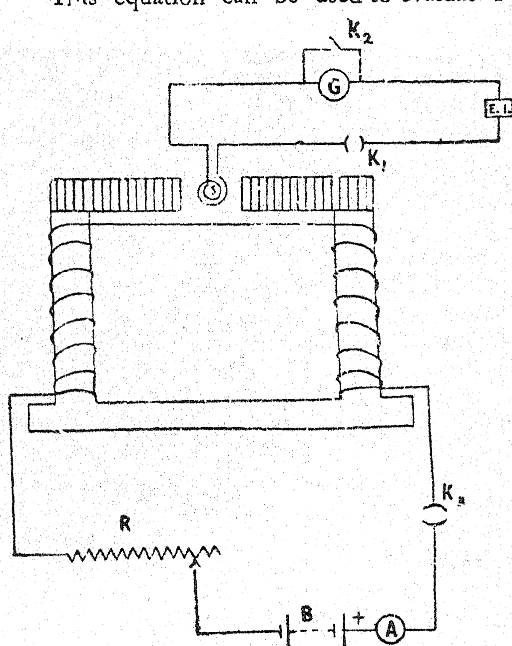


Fig. 31·12

Method. Level the base of the galvenometer and release its coil. Thow light on the mirror of the galvanometer and get a bright spot of light on the scale. Adjust the spot of light on the scale at zero. Connect the search coil S (Fig. 31·12) to a ballistic galvanometer G in series with an earth inductor, including a key K_1 in the circuit. Also connect a key K_2 across the galvanometer.

Next by means of compass needle, arrange the earth inductor, when against one of the stops, so that its axis of rotation is vertical and its plane is perpendicular to the magnetic meridian.

Now place the search coil in the gap between the pole-pieces of the electromagnet such that its plane is parallel to the plane of the pole-pieces. Then connect the field coils of the electro-magnet to the battery B in series with a rheostat R and an ammeter A, including a plug key K_2 in the circuit. By adjusting the rheostat R allow a steady current of suitable value to pass through the field coils and thus produce a steady magnetic field between the pole-pieces of the electromagnet.

Next close key K_1 and rapidly withdraw the search coil from the gap between the pole-pieces of the electromagnet and note down the deflection of the spot of light on the scale corresponding to the first throw θ_1 of the galvanometer. Then bring the galvanometer coil to rest.

Next quickly rotate the earth inductor through 180° and observe the deflection of the spot of light on the scale corresponding to the first throw a_1 of the galvanometer. Then calculate the value of F from equation (31·37).

Now alter the current i through the field coils of the electromagnet and determine as before the value of F for several values of i. Finally plot a graph showing the variation of F with i.

Sources of error and precautions. (1) The base of the galvanometer should be carefully levelled and its coil unclamped before using it.

(2) A tapping key should be connected across the galvanometer.

(3) The earth inductor and the ballistic galvanometer should be at a considerable distance from each other and from the electro-magnet.

(4) The connections in the galvanometer circuit should be made by twin flexible wires; and when the search coil is between the pole-pieces, its plane should be parallel to the plane of the pole-pieces.

(5) The search coil should be withdrawn rapidly from the gap b tween the pole-pieces of the electromagnet and to a considerable distance from it.

(6) The axis of rotation of the earth inductor should be vertical and its plane perpendicular to the magnetic meridian. (see pre. 4, expt. 31·4)

Observations. [A Constants of the search coil (given):

(i) No. of turns n =

(ii) Face area of each turn a = sq. cm.

[B] Constants of the eaath inductor (given)

(i) No. of turns n' =

(ii) Face area of each turn a' = sq. cms.

[C] Horizontal component of earth's field H = oersteds

[D] Determination of θ_1 and α_1.

Rest position at spot of light = zero.

No.	Current in field coils amp.	θ_1 mm.	α_1 mm.	θ_1/α_1	Mean θ_1/α_1	F oersteds

Calculations. (*i*) Current in the field-coils = amp.

$$F = \frac{2\,n'\,a'}{n\,a}\,\frac{\theta_1}{\alpha_1}\,.\,H$$

$$= \qquad \text{oersteds}$$

(*N. B.* Make similar calculations of F for other values of current).

Result. The strength of the magnetic field between the pole-pieces of the electromagnet for various values of current in the field coils are given in the above table.

Criticism of the method. It is a very convenient method for determination of strong magnetic fields and is especially suitable when the field is non-uniform and also when the space in which the field is to be measured is narrow, and is, therefore, in these respects superior to the earth inductor method of measuring magnetic fields. It should be noted that this method does not require a knowledge of the ballistic constant of the galvanometer and the resistance of the galvanometer circuit.

31·7. **Eddy or Foucault currents.** When a piece of metal, *e.g.*, a metal disc, is placed in a varying magnetic field or whenever there is a relative motion between the metal piece and the magnetic field, the magnetic flux threading the metal piece changes and induced currents are set up in it. These currents are called eddy or Foucault currents. The electrical energy E^2/R of these eddy currents is dissipated as heat in the metal and hence may cause the metal to become very hot. This is a source of great trouble in a metal apparatus placed in a varying magnetic field and may be avoided to a great extent by increasing the electrical resistance of the apparatus. This is done by making the apparatus

from flat metal strips insulated from one another instead of in one solid mass, *e.g.*, iron core of transformers are made of thin laminations covered with oxide and riveted together.

There are two chief applications of eddy currents :—

(1) *Damping or breaking the motion of a body.* The body whose motion is to be damped is attached to a metal disc usually of aluminium placed in a magnetic field. The motion of the disc in the field induces in it eddy currents which, by Lenz's law, are so directed as to oppose the motion of the disc.

(2) *Melting of metals.* Eddy currents of considerable magnitudes are produced in the mass of metal to be melted by placing it close to aneddy current heater which is a device for producing high frequency oscillations of great intensity. The eddy current loss in the metal is sufficient to raise it to red or white heat. This method is nowadays used in the degassing of metal parts of radio valves and in preparing alloys of metals by melting them in vacuum.

Oral questions

ELECTROMAGNETIC INDUCTION

What is electro-magnetic induction ? State the laws of electro-magnetic induction. Upon what factors does the magnitude of the induced E. M. F. depend ? If the conducting circuit is not closed will there be any induced E. M. F. produced in it when the magnetic flux threading the circuit varies ? Upon what factors does the strength of the induced current depend ? What are the various ways in which the magnetic flux linked with a circuit may be changed ? State Lenz's law of electro-magnetic induction and explain how with its help you can find the direction of induced current. What is Fleming's right hand rule ? Is it different from Lenz's law ? What are eddy currents ? Why are iron cores of transformers made of thin laminations instead of one solid mass ? What are the chief practical applications of eddy current ? Give examples. State some applications of the phenomenon of electro-magnetic induction.

SELF-INDUCTANCE BY RAYLEIGH'S METHOD

What is self-induction ? How do you define co-efficient of self-inductance of a circuit ? State the factors upon which the self-inductance of a coil-depends. Define the absolute and practical units of self-inductance. What is the relation between them ? How do you determine the self-inductance of a coil by Rayleigh's method ? What precaution do you observe in your experiment ? What is the special use of the double key ? Describe its construction. What is the harm if the connecting wires are coiled ? Why don't you use a Leclanche cell in this experiment as you do in other experiments with P. O. box ? Why should the ratio arms be adjusted to 10 : 10 or preferably to 1 : 1 if possible ? If the resistance of the coil is small how can you adjust the four arms to the same order ? Why should the balancing of bridge for steady current be perfect ? What is the use of platinoid wire connected in the rheostat arm ? If the battery key is kept depressed for a long time, why does the balance point alter ? How do you prevent the heating of the coil while the bridge is being balanced ? Why should the resistance introduced in the arm of the coil to produce steady deflection be small ? What should be its approximate value ? How and why do you determine logarithmic decrement ? While determining the period of the galvanometer why should the galvanometer circuit be kept open ? Do you know other method by which more accurate results can be obtained ? Is Rayleigh's method suitable for determining the self-inductance of a coil having an iron core ? If not, why ? State some prac-

tical applications of self-inductance. Compare self-inductance with inertia. Why does the current take some time to attain its steady maximum value when allowed to pass through a circuit containing self-inductance? Why does an arc form at the switch when a motor is switched off? How are induction effects reduced to minimum in bifilar winding of resistance coils? [See questions on ballistic galvanometer also at the end of Chapter XXXII].

DETERMINATION OF MUTUAL INDUCTANCE

What is mutual induction? Define co-efficient of mutual induction or mutual inductance of two coils. Upon what factors does it depend? Is it the same for both the coils when any one of them acts as the primary and the other as the secondary? What are absolute and practical units of mutual inductance? Describe how you find the mutual inductance of two coils. What precautions do you take in the experiment? Why should the coils be placed away from the galvanometer? Why should the connecting wires be straight? Can you use a Leclanche cell in place of accumulator? If not, why? What is logarithmic decrement? Do you determine it on closed circuit or open circuit of the galvanometer and why? What are the main sources of error in this experiment? Is the method also applicable to coils having iron cores? If not, why? How can you determine the mutual inductance of two coils accurately? State some practical applications of a mutual induction. Why are there a large number of turns in the secondary of a transformer or an induction coil? Why is the E. M. F. at break in the induction coil very high compared to that at make?

EARTH INDUCTOR

What is an earth inductor? What purposes is it used for? How can you determine the strength of a magnetic field by it? Upon what factors does the E. M. F. induced in the coil depend when it is rotated in the magnetic field? What type of E. M. F. do you get in this case? Do its direction and strength remain constant during one complete rotation? When is the magnitude of E. M. F. induced zero and when it is maximum? How do you arrange the earth inductor before turning it through 180°? If it be initially parallel to the field will you get any throw in the galvanometer. If not, why? How can you determine the angle of dip by the earth inductor? What precautions do you take in this experiment? Why should the galvanometer be at a distance from the earth inductor? Why should the connections between the galvanometer and the earth inductor be made by flexible wires? What is the use of stops and spring fixed in the earth inductor? Why should the rotation of the coil be exactly through 180°? What is the harm if the coil were rotated through a complete turn or say one quarter of it? Why should the coil be rotated quickly? Does the charge passing through the galvanometer depend upon the time of rotation through half a turn? On what factors does the charge passing through the galvanometer depend? Can you determine the field strengths of any magnitude by the earth inductor? If so, how? In what way is the earth inductor superior to magnetometer? What is the principle underlying the working of dynamo used for generation of current?

SEARCH COIL

What is a search coil and what is it used for? Why is it made so small? How can you measure strong magnetic fields with it? What is the use of earth inductor in this experiment? Why should it be placed at a considerable distance from the electro-magnet? Why should its plane be perpendicular to the magnetic meridian and its axis of rotation vertical? What are the advantages of this method of measuring fields over the earth inductor method?

CHAPTER XXXII

GALVANOMETERS, AMMETERS AND VOLTMETERS

32·1. Galvanometers. Galvanometers are instruments which are intended primarily to indicate the existence of a current and which may, under certain circumstances, be capable of measuring it. The construction of galvanometers is based on the interaction between coils carrying currents and magnets. They may be in general divided into two classes as given below according as the moving part is the magnet or the coil.

[A] Moving-magnet Galvanometer. In this type the current is passed through a fixed coil which produces a magnetic field under which the magnet moves.

[B] Moving-coil Galvanometer. In this type the current is passed through a movable coil placed between the poles of a powerful magnet. The coil moves on account of the interaction between its magnetic field and that of the permanent magnet.

Galvanometers are also classified as pointer galvanometers and reflecting or mirror galvanometers according as a pointer or a mirror is attached or fixed to the moving part to note down its movement.

32·2. Moving-magnet Galvanometers, (A) Galvanometers of ordinary sensitiveness :—(a) Tangent Galvanometer. As described in § 25·5, consists essentially of a small magnetic needle pivoted or suspended at the centre of a circular coil and provided with a long light aluminium pointer which moves over a circular scale of degrees. To use the instrument the plane of the coil is set in the magnetic meridian and the current to be measured passed through it. If then θ be the deflection of the needle from the magnetic meridian, the current is from equation (25·8) given by

$$i = \frac{r\text{H}}{2\pi n}\tan\theta$$

where r is the radius of the coil, n the number of turns in the coil and H the horizontal component of the earth's magnetic field. If the values of r and n are accurately known, the tangent galvanometer can be used to measure current absolutely, for the value of H can be evaluated at any place by a quite independent method. The instrument possesses several disadvantages :

(1) The instrument can be used only in a particular orientation, *viz.*, with the plane of its coil in the magnetic meridian.

(2) The deflection is not proportional to the current. It uses the tangent relation and hence becomes inaccurate at angles greater than 70°.

(3) The reduction factor depends upon the value of H which is affected by the presence of the magnetic substances and current bearing conductors. The evaluation of H requires a separate and highly accurate experiment.

(4) The magnetic needle has a finite size and its poles move out of the coil as θ increases. As a result the deflecting field is not proportional to the current.

(5) Unless the coil is exactly circular and has only a single layer, the value of r is also uncertain.

(6) The instrument is bulky and the system of supporting the needle is not robust.

The tangent galvanometer is not a sensitive instrument and hence is not suitable for use in null methods, *e.g.*, in Wheatstone bridge and potentiometer.

(b) Sine Galvanometer. This instrument closely resembles tangent galvanometer and consists essentially of a small magnetic needle pivoted or suspended at the centre of a coil of large radius. The coil in this case is capable of rotation about a central vertical axis, the rotation being read off on a horizontal circular scale at the base. When the current is passed through the coil, the needle is deflected. The coil is then rotated about the vertical axis until the coil overtakes the deflected needle and both the needle and the coil lie in one common plane. The deflecting field F is then perpendicular to the magnetic needle and in the equilibrium position

$$F=\frac{2\pi n i}{r}=H \sin\theta$$

whence $$i=\frac{rH}{2\pi n}\sin\theta$$

or $$i=K\sin\theta \tag{32.1}$$

where $K=\dfrac{rH}{2\pi n}$ is the reduction factor of the sine galvanometer.

Since the maximum value of $\sin\theta=1$, it is clear from equ. (32·1) that the maximum current which may be measured by means of this instrument is equal to K. For currents greater than K, the coil cannot catch up the needle when the former is rotated and hence it is then impossible to make the needle lie in the plane of the coil. The sensitiveness of sine galvanometer increases as θ increases and is maximum when $\theta=90^\circ$. The instrument possesses he advantage that deflections up to 90° may be used.

(c) Helmoholtz or Double-coil Tangent Galvanometer. As described in § 25·6, it is a modification of the ordinary tangent galvanometer in which the deflecting field has been made uniform by placing the magnetic needle midway between equal circular coils, mounted vertically at a distance apart equal to the radius of either and connected in series. When the coils are parallel to the magnetic

meridian, the current passing through them is from equation (25·16) given by

$$i = \frac{5\sqrt{5}\, r\, H}{32\,\pi\, n} \tan\theta$$

or

$$i = K \tan\theta$$

where

$$K = \frac{5\sqrt{5}\, r\, H}{32\,\pi\, n}$$

In the case of above galvanometers, the deflections are generally measured with the help of a pointer. The accuracy in reading deflections in such a case can be increased by increasing the length of the pointer. For great accuracy, however, the pointer-over-scale system of measuring deflection is dispensed with and deflections are measured with the help of a mirror and lamp-and-scale arrangement. In a mirror galvanometer, if the deflections are not large, the current is proportional to the displacement of the index produced by it. This is a very great advantage in practical work.

(B) Galvanometers of high sensitiveness. There are three methods of increasing the deflection due to a current of given strength in the case of moving-magnet galvanometer :

(1) *By increasing the deflecting field* $F = 2\pi n\, i/r$. This can be done by increasing the number of turns n in the coil and by decreasing its radius r. But number of turns cannot be increased beyond a certain limit firstly, because this increases the resistance of the coil and secondly, because the value of r then becomes uncertain as the turns are not coincident. Further it is not desirable to make r very small otherwise the magnetic needle will not lie in uniform field and the tangent relation will not hold good.

(2) *By decreasing the controlfield* H. This is done with the help of a control magnet so placed that its field at the centre of the coil is in the opposite direction to that of the earth. By adjusting the position of the control magnet with respect to the coil, the controlling field H can be reduced as much as you please. When the field due to the control magnet is made equal and opposite to the earth's field, the resultant controlling field is zero. In such a case the torsion of suspension provides the necessary restoring couple.

(3) *By the use of an astatic pair of needles.* An astatic pair consists of two light magnetic needles (Fig. 32·1), fitted in a light frame with

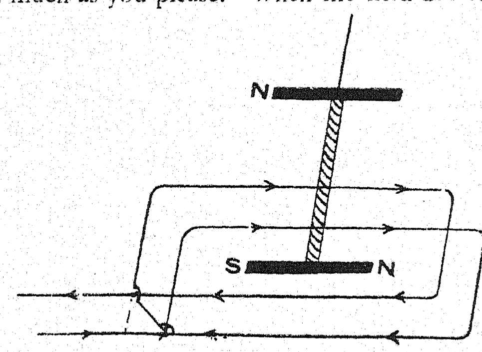


Fig. 32·1

their axes parallel to each other and with their dissimilar poley pointing in the same direction. The needles are magnetised nearls

equally and are suspended one inside and the other outside a flat coil of a large number of turns, the needle lying parallel to the plane of the coil. Flattening of the coil is equivalent to a reduction of the radius and thus increases the sensitiveness of the instrument.

When a current passes through the coil, the two parts of the coil field, one around the needle outside the coil and the other around the needle inside the coil, are in opposite directions and since dissimilar poles of the needles are also pointing in the same direction the needles are acted upon by couples tending to deflect them in the same sense. If M_1 and M_2 be the magnetic moments of the two needles, the deflecting couple exerted on the astatic pair by the coil field is proportional to (M_1+M_2) roughly. Further since the controlling field is practically uniform and similar poles of the needles are oppositely directed, the controlling couple is proportional to $H\ (M_1-M_2)$. Thus by using an astatic pair, the senstiveness can be increased by $(M_1+M_2)/(M_1-M_2)$ times roughly. If $M_1=M_2$, the restoring couple due to the controlling field is zero ; in such a case a torsion suspension must be used to provide the necessary restoring couple, otherwise the system will become unstable and all currents weak or strong would produce a deflection of 90° and there would be no return of the needle to zero. Sometimes the needles are placed in separate coils one above the other wound oppositely so as to produce deflecting couple on each of the needle in the same sense.

A galvanometer using an astatic pair is very sensitive and can be chiefly used to detect minute currents in Wheatstone bridge experiments. But such a galvanometer suffers from two disadvantages : (1) The absolute value of current cannot be measured for the value of the factor $(M_1+M_2)/(M_1-M_2)$ is unknown and (2) it cannot be relied upon to give the same deflection from day to day for the same value of current as M_1 and M_2 may change.

(a) **Kelvin's Galvanometer.** It consists of a small, light and very carefully magnetised strip of steel attached by shellac or cement to the back of a small mirror *m* (Fig. 32·2) and suspended by a single fibre of unspun silk inside a small circular coil. The coil is wound on a small circular reel R enclosed in a cylindrical brass box B with a glass window, the length of the wire wound on the reel depending upon the sensitiveness required. Above the coil is supported a permanent controlling magnet NS to vary the sensitiveness of the galvanometer. The deflections are measured by the mirror and lamp-and-scale method. The galvanometer can detect currents of the order of 10^{-6} amperes.

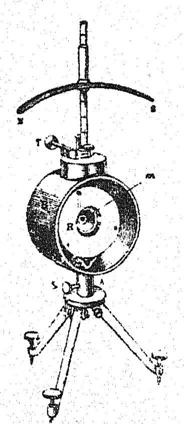


Fig. 32·2

The most modern form of this galvanometer is the Kelvin's high resistance astatic galvanometer in which the parts are doubled and

the moving system thereby made astatic. It consists of two sets of three or four very small magnetic needles (Fig. 32·3) attached to a light aluminium wire and suspended by a fine quartz or silk fibre between two pairs of coils as shown. The needles are arranged in astatic order, *i.e.*, with the N-poles of the upper set pointing in the same direction as the S-poles of the lower set. For more delicate adjustment, the instrument is generally provided with two controlling magnets which may be adjusted either to weaken or to strengthen each other's effect. The mirror is placed either between the coils or under lower coil on an extension of the aluminium support. The galvanometer is very sensitive and can detect currents of the order of 10^{-8} amperes.

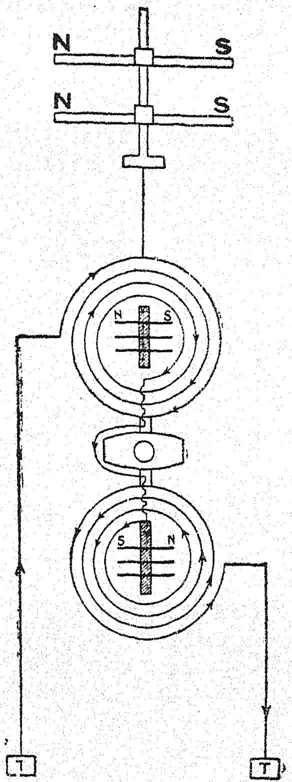


Fig. 32·3

(b) Broca Galvanometer. This is the most sensitive moving magnet galvanometer and consists of an astatic pair suspended by a fine quartz fibre inside the coil which is wound in two halves and arranged with respect to the needle as shown (Fig. 32·4). The astatic pair consists of two steel wires, carefully magnetised, placed vertically and very close together, each having consequent poles. This form makes it possible to use comparatively powerful needles, whilst keeping the moment of inertia small, and at the same time it is very astatic. The deflections are measured by means of small mirror and lamp-and-scale arrangement. The instrument can detect currents of the order of 10^{-10} amperes. The sensitiveness can be varied by a control magnet at the back.

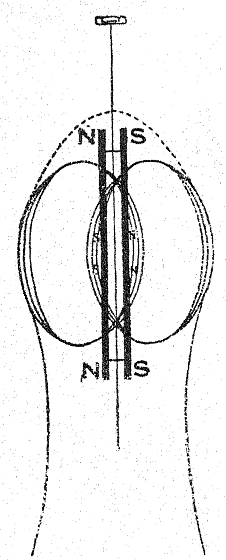


Fig. 32·4

Both the Kelvin and Broca galvanometers are very sensitive ubt they possess the disadvantage that the deflections are readily disturbed by the presence of magnetic materials or of current bearing conductors in their vicinity, which makes it very troublesome for ordinary experimental work.

32·3. D' Arsonval or Moving coil Galvanometer. A moving-coil galvanometer consists of a narrow rectangular coil of many turns of fine silk-covered copper wire suspended (Fig. 32·5a) or pivoted (Fig. 32·5b) so as to move freely in a narrow annular space between the pole-pieces of a powerful steel horse-shoe magnet. The pole-pieces of the magnet are made of soft iron and are hollowed out to make them cylindrically concave in shape. This makes the field *radial* (Fig. 32·5c). Within the coil is fixed a cylinder of soft iron which serves to concentrate the lines of force in the gap thus making the field in the gap strong and practically uniform. The coil is wound on a light frame-work which is either of metal or of non-conducting material, according as a dead-beat (§32·5) or a ballistic (§32·6) action is required.

In the suspended coil type, the suspension is a fine rectangular strip of phosphore-bronze which also serves as one of the current leads to the coil. These rectangular strips have very small torsional rigidity and hence the twist for a given couple is much greater and the stress on the material, much less than with a wire of circular cross section. They possess an additional advantage of offering a comparatively large surface area for the dissipation of heat produced by the current. Connection to the other end of the coil is made by means of a very light metal spring attached to the bottom of the coil by a wire. The spring is made of a very fine strip and consists of relatively large turns so that it shall exert only a small controlling couple on the coil.

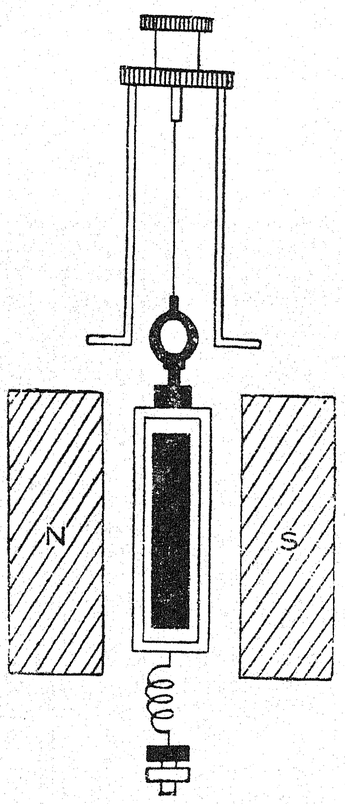


Fig. 32·5 *a*

In the pivoted coil type, the coil rests vertically on two jewelled pivots and working loosely within two brass pillars. The motion of the coil is controlled by two light metallic springs attached to the bottom and top of the coil and coiled in opposite directions to neutralise the effect of rotation of the coil due to variation of temperature in the springs. The current leaves and enters the coil through these two springs.

When a current flows through the coil, forces act in contrary directions on the opposite sides of the coil, *i.e.*, a couple acts on the coil which tends to make it set at right angles to the field so as to enclose as many lines of force as possible. If A be the face area of the coil, n the number of turns in it, H the uniform field in which it moves and i the current in the coil, the deflecting couple is equal to $HAni$. The motion of the coil is resisted by the controlling couple $C\theta$ furnished by the torsion of the suspension in the suspended coil type and by the springs in the pivoted coil type. The coil in consequence takes up an intermediate position in which the deflecting and the controlling couples balance each other, *i.e.*, where

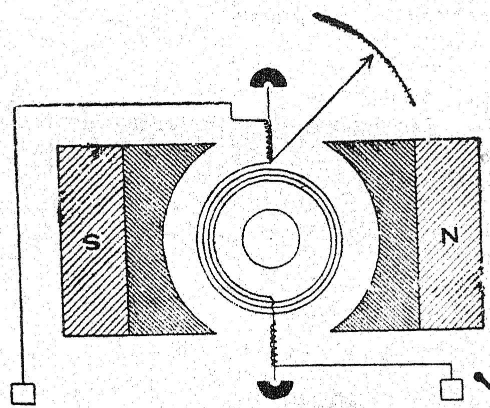


Fig. 32·5 (*b*)

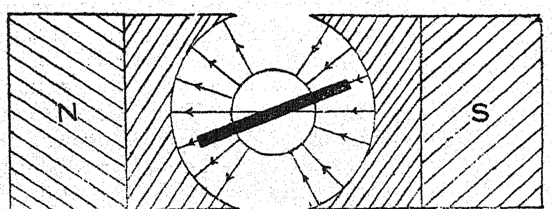


Fig. 32·5 (*c*)

$$HAni = C\theta$$

whence $$i = \frac{C}{HAn}\,\theta$$

or $$i = K\theta$$

where $K = \dfrac{C}{HAn}$

Thus for radial field the deflection is directly proportional to the strength of the current. The deflections are observed by means of a pointer or a beam of light reflected from a small mirror attached to the rigid support of the coil.

If the field is not radial, then as soon as the coil rotates from its rest position in which it lies with its face parallel to the field, the couple is reduced to $HAni \cos\theta$, where θ is the inclination of the plane of the coil to the field, and

$$C\theta = HAni \cos\theta$$

whence $$i=\frac{C}{AHn} \cdot \frac{\theta}{\cos \theta}$$

A scale which varied as $\theta/\cos \theta$ would be very inconvenient and hence the desirability of making the field radial which gives a linear scale.

The instrument can detect currents of the order of 10^{-8} amperes and possesses the following advantages:—

(1) It is practically independent of the earth's magnetic field since no sensitive magnetic needle is used.

(2) The field in which the coil moves is so strong that other external magnetic fields do not affect the readings to any appreciable extent. This effect can be still further reduced by magnetic screening.

(3) It is remarkably dead-beat (see § 32·5) in its action when the coil is wound on a metallic frame, the damping being produced by eddy currents. Further, when the current is stopped any oscillation of the coil may be prevented by short-circuiting it for a moment.

(4) It may be set up in any convenient position.

(5) The deflection is directly proportional to the current, and the evenly divided scale is a great advantage in practice.

(6) The instrument can be made very sensitive by increasing the number of turns in the coil and decreasing the value of C, the controlling couple per unit twist.

32·4. Sensitiveness or Figure of Merit of a Galvanometer. In the case of pointer or **non-reflecting galvanometers,** *the* **sensitiveness** *is defined as the deflection in degrees produced by a current of one micro-ampere.* Alternatively, *the* **figure of merit** *is defined as the current in amperes necessary to produce a deflection of one degree.* In the case of **reflecting galvanometers,** *the sensitiveness is usually defined as the number of mms. deflection on a scale one metre away by one micro-ampere*; and *the* **figure of merit** *as the number of amperes required to produce one mm. deflection on a scale one metre from the galvanometer mirror.*

Exercise. *To determine the figure of merit of a reflecting galvanometer.*

1. Method. Shunt the reflecting galvanometer with a low resistance wire and then connect it through a commutator to an accumulator of known E. M. F., E connected in series with a variable high resistance R (Fig. 32·6), usually of the order of megohm. Adjust the resistance R so that the spot of light remains on the scale when the current is started.

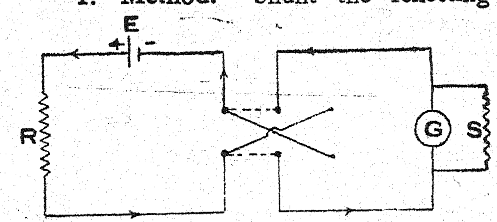


Fig. 32·6

Note down the deflection in mm. on the scale with the current direct and reversed and find its mean value. If S be the resistance of the shunt and G that of the galvanometer, the main current is given by

$$i = \frac{E}{R + \frac{SG}{S+G}}$$

the resistance of the accumulator, being very small when compared with R, has been neglected. Since the current divides through the shunt and the galvanometer in the inverse ratio of their resistances, the galvanometer current is given by

$$i_g = \frac{E}{R + \frac{SG}{S+G}} \times \frac{S}{S+G} \text{ amperes}$$

$$= \frac{ES}{R\,(S+G)+SG} \text{ amperes}$$

If the deflection on the scale be δ mm., the figure of merit of the galvanometer is given by

$$\eta = i/\delta \text{ amp. per mm.}$$

II. Method. Take a P. O. box and connect an accumulator of known E. M. F., E in the usual 'battery arm' AC through a tapping key K_1. Connect the galvanometer in the usual 'galvanometer arm' BD through a commutator K_2. Adjust P=10 ohms, Q=1000 ohms and R=5000 to 10000 ohms, leaving the usual 'unknown resistance arm' disconnected. Press the key K_1 and note down the deflection on the scale in mm. with current direct and reserved and thus determine the mean value of the deflection for a current in the galvanometer.

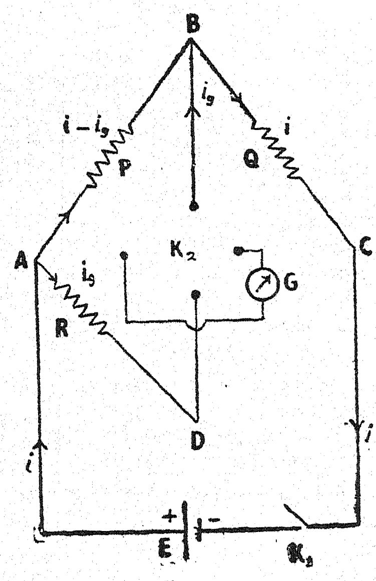


Fig. 32·6 *a*

Assuming the cell resistance to be negligible and applying Kirchhoff's II law, we have for the circuit ABDA (Fig. 32·6*a*)

$$(i-i_g)\,P - i_g\,G - i_g\,R = 0$$

or $i\,P - i_g\,(P+R+G) = 0$ (32·2)

and for the circuit ABCEA

$$(i-i_g)\,P + i\,Q = E$$

or $\quad i\,(P+Q) - i_g\,P = E \quad \ldots \quad (32·3)$

Multiplying equ. (32·2) by (P+Q) and equ. (32·3) by P and then subtracting equ. (32·2) from equ. (32·3), we have

$$i_g\,[\,(P+Q)\,(P+R+G)-P^2] \quad = EP$$

or
$$i_g\quad[\,(P+Q)\,(R+G)\,+\,P\,Q\,] \quad = EP$$

whence
$$i_g = \frac{EP}{(P+Q)\;(R+G)+P\;Q}$$

Since $P \lll R$, we can neglect PQ in the denominator and hence

$$i_g = \frac{E\,P}{(P+Q)\;\;(R+G)} \qquad \ldots \qquad (2{\cdot}4)$$

If δ mm. be the mean deflection, the figure of merit of the galvanometer is given by

$$\eta = i_g/\delta \text{ amp. per mm.}$$

The value of i_g can be known from equ. (32·4).

32·5. Meaning of term 'Dead-beat' and Methods of Damping. When the moving part of a galvanometer, if disturbed, continues to vibrate about its equilibrium position for a long time, the amplitude of vibration gradually diminishing until it comes to rest, the galvanometer is said to be *periodic*. But, if the moving part of a galvanometer, if disturbed, rapidly comes to rest, its motion is said to be 'damped' and if the damping is critical, *i.e.*, such that the vibrations are almost instantaneously completely stopped, the galvanometer is known as *'dead-beat'* or 'aperiodic.'

When current is passed through a periodic galvanometer, then on account of oscillation, the moving part does not at once attain the corresponding deflection, and hence some time elapses before a reading can be taken. Similarly, when the current is stopped, the moving part takes some time to come to rest. On the other hand, when current is passed through a dead-beat or aperiodic galvanometer the moving part almost *instantaneously* reaches its steady position without overshooting it, and when the current is stopped it returns to zero with the same rapidity. Consequently dead-beat galvanometers are very suitable for the measurement of steady deflections and are, therefore, especially used in all null methods. It should be clearly understood that in dead-beat galvanometers, the damping does not alter the final position of the moving part, but simply enables it to attain that position in a shorter time.

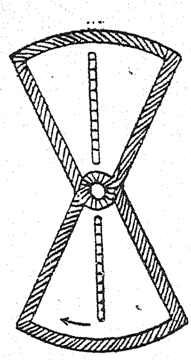


Fig. 32·7

There are three systems of damping in general use :—

(a) *Air Friction Damping.* There are two methods of applying air friction damping. In the first method two vanes (Fig. 32·7) made of thin aluminium sheet are mounted on the spindle of the moving system and move in a closed, sector-shaped box.

In the second method a light aluminium piston P (Fig. 32·8) is attached to the spindle S of the moving system

and moves in a rectangular or circular air chamber C closed at one end, the clearance between the piston and the walls of the chamber being about a few thousandths of an inch. When the piston moves rapidly into the chamber, the air in the closed space is compressed as a result of which it exerts pressure and opposes the motion of the piston and hence that of the whole moving system. When the piston moves out of the chamber rapidly, the pressure in the closed space falls, and as the pressure on the open side of the piston is then greater than that on the other side, it opposes the motion of the piston. In this damping system care should be taken to ensure that the arm of the piston is not bent, otherwise the piston will touch the walls of the chamber and a serious error will result in the deflection. This method is more efficient than the first one.

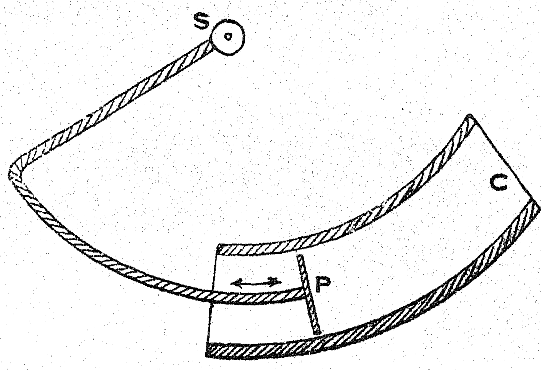


Fig. 32·8

(*b*) *Fluid Friction Damping.* In this method a disc (Fig. 32·9) attached to the spindle of the moving system ippe dipped completely into a pot of damping oil. As the disc rotates, its motion is opposed by the viscous drag which increases with the speed of the rotation of the disc. In order to avoid surface tension effects, the suspending stem of the disc should be cylindrical and of small diameter where it enters the oil surface.

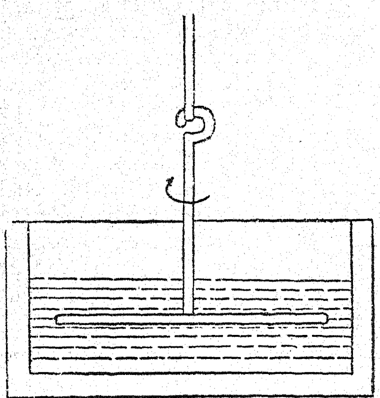


Fig. 32·9

Increased fluid friction damping can be obtained by the use of vanes in vertical planes, carried on a spindle and immersed in oil as shown in fig. 32·10.

The method of fluid friction damping suffers from two disadvantages, firstly it can be used only in instruments which are used in a vertical position and secondly, owing to creeping of the damping oil, it is difficult to keep the instrument clean.

(*c*) *Electromagnetic or Eddy Current Damping.* This method of

damping depends upon the fact that when a sheet of conducting material is rotated in a magnetic field so as to cut through lines of force, eddy currents are set up in it, their magnitude being proportional to the velocity of the movement of the conductor. On account of interaction between these currents and the permanent magnetic field, a force acts on the conductor in a direction opposing its motion. The magnitude of the force is proportional to the magnetic field and to the currents, and hence to the velocity of movement of the conductor. The damping force is, therefore, zero when there is no movement of the conductor.

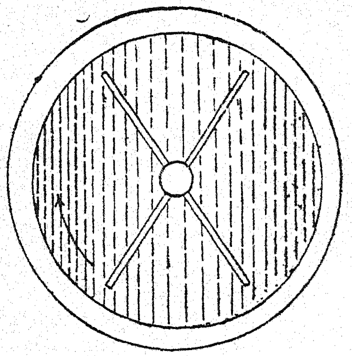


Fig. 32·10

There are two methods of applying electro-magnetic damping tn instruments using permanent magnets. In the first method a ihin disc D (Fig. 32·11) of conducting but non-magnetic material, usually copper or aluminium, is mounted on the spindle S which carries the pointer P of the instrument, the edge of the disc being in the gap of the permanent magnet M. When the spindle rotates, the edge of the disc cuts through the lines of force in the gap of the magnet. This produces eddy currents in the disc which damp the motion of the disc and hence that of the spindle.

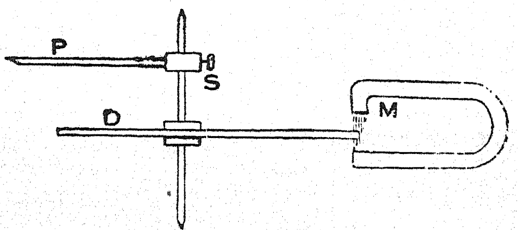


Fig. 32·11

In the second method the coil is wound on a light metal former P (Fig. 32·12). As the coil rotates in the field of the permanent magnet NS, eddy currents are produced in the metal former and the coil is rapidly brought to rest. In this case the damping depends upon the magnitude of the external resistance and is often very high for a low external resistance.

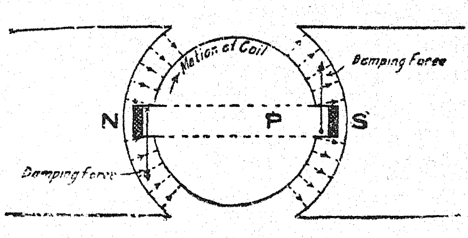


Fig. 32·12

In moving-magnet galvanometers, electro-magnetic damping is produced by placing a disc of copper or aluminium under the magnet. As the magnet moves, eddy currents are produced in the disc which oppose the motion of the magnet. The damping can be increased by enclosing the magnet in a metal case.

32·6. Ballistic galvanometer. When a transient current, *i.e.*, a current of very short duration, *e.g.*, the discharge of a condenser, passes through a galvanometer, its movable part, the magnet or the coil, experiences only a *sudden* impulse. This impulse produces a swing or throw of the movable part of the galvanometer which then performs a number of oscillations about its position of equilibrium with gradually decreasing amplitude and finally settles down to rest again. If the duration of impulse on the system is very small compared with a quarter of its period so that *the whole of electricity passes through the galvanometer before its movable part has moved sensibly from its zero position*, the magnitude of the swing of the galvanometer is independent of the duration of impulse, and further if *the motion of the movable part is entirely undamped* so that no energy is lost during the swing, the magnitude of the *first* swing or throw, for a given galvanometer, is dependent only upon the quantity of electricity passing through it. A galvanometer which satisfies the above two conditions is known as a *ballistic galvanometer* and is used to measure a quantity of electricity passed through it.

It is evident from above that a ballistic galvanometer is not a galvanometer of any special type or construction, but any pattern whose period of oscillation is large, usually 10 to 15 seconds and in which the damping is negligibly small can be used as a ballistic galvanometer. The period of oscillation of a galvanometer is given by

$$T = 2\pi\sqrt{\frac{I}{c}}$$

where I is the moment of inertia of the movable part about the axis of rotation and c the restoring reactional couple per unit twist. Hence the period of the galvanometer can be made large by increasing the moment of inertia of the movable apart and by reducing the controlling forces. The moment of inertia is increased by loading the movable part while the controlling forces are reduced by the use of quartz fibres or phospho-bronze strips as suspensions which have small torsional rigidity. In moving coil ballistic galvanometers which use non-conducting materials for suspension, the current is led into the coil by delicate spirals of very thin copper strip.

The damping of the motion of the movable part of the galvanometer consists of air damping and eddy current or electro-magnetic damping. The air damping is usually very small and the electro-magnetic damping is minimised by winding the coil on a non-conducting frame, *e.g.*, of wood or ebonite and by using a non-metallic case. In the moving coil galvanometer electro-magnetic damping is dependent upon the resistance of the external circuit and is sufficiently high when the external resistance is low.

As indicated above both the moving magnet and the moving-coil galvanometers are used for ballistic purposes. In the moving-magnet type the magnet is set initially at right angles to the axis of the coil so that when the current passes, the deflecting and the controlling fields may be mutually at right angles. If then θ be the magnitude of the first swing or throw of the magnet when a quantity Q is passed through the coil, it can be shown that

$$Q = \frac{H}{G} \cdot \frac{T}{\pi} \sin \frac{\theta}{2}$$

where H is the intensity of the restoring field, G the galvanometer constant, *i.e.*, field due to unit current in the coil and T the period of oscillation of the magnet. Thus the quantity of electricity passing through a moving magnet ballistic galvanometer is proportional to the sine of half the angle of the first throw.

For a moving coil galvanometer, the relation between the quantity of electricity passed through the galvanometer and the consequent magnitude of the first throw is given by

$$Q = \frac{c}{nAH} \cdot \frac{T}{2\pi} \theta$$

where c is the torsional rigidity of the suspension, H the field of the permanent magnet, n the number of turns in the coil, A the face area of the coil and T the period of the galvanometer. Thus the quantity of electricity passing through a moving-coil ballistic galvanometer is proportional to the magnitude of the first throw.

The above relations between Q and θ are derived on the assumption that the motion of the movable part of the galvanometer is entirely undamped and that no energy is lost during the swing. But this is not the case in practice for there is always some air damping present. The result is that the observed throw is less than what would be if there were absolutely no damping. Hence to get the value of undamped throw of the galvanometer, the observed value of θ should be multiplied by 'damping factor.' It can be shown that the value of this correcting factor is $(1+\lambda/2)$, where λ is the logarithmic decrement. Hence the complete expressions for the quantity of electricity passing through the two types of galvanometer are

$$Q = \frac{H}{G} \cdot \frac{T}{\pi} \sin. \left[\frac{\theta}{2}\left(1 + \frac{\lambda}{2}\right)\right] \qquad (32 \cdot 5)$$

for a moving-magnet ballistic galvanometer ; and

$$Q = \frac{c}{nAH} \cdot \frac{T}{2\pi} . \theta \left(1 + \frac{\lambda}{2}\right) \qquad (32 \cdot 6)$$

for a moving-coil ballistic galvanometer. The value of logarithmic decrement λ may be calculated from the expression

$$\lambda = 2 \cdot 3026 \ \log_{10} \frac{\theta_1 + \theta_2 + \theta_3 + \ldots + \theta_{n-1}}{\theta_2 + \theta_3 + \theta_4 + \ldots + \theta_n}$$

where θ_n is the nth throw of the galvanometer. While determining the value of λ, the galvanometer should be in the circuit of the experiment.

In actual practice, ballistic galvanometers are reflecting instruments and the value of θ under ordinary circumstances does not exceed 6 or 7 degrees. Hence, if d be the movement of the spot of reflected light on the scale corresponding to θ, we have, since the angular motion of the spot of light is twice that of the mirror

$$\sin \theta/2 = \theta/2 \quad \text{and} \quad 2\theta = d/L$$

where L is the distance of the scale from the mirror. Thus for small angles

$$\theta = d/2L \quad \text{and} \quad \sin \theta/2 = \theta/2 = d/4L$$

Putting these values of θ and $\sin \theta/2$ in equations (32·5) and (32·6) we have, for a moving-magnet galvanometer

$$Q = \frac{H}{G} \cdot \frac{T}{\pi} \cdot \frac{d}{4L}(1+\lambda/2) = K_1 d\,(1+\lambda/2)$$

where $K_1 = \dfrac{H}{G} \cdot \dfrac{T}{\pi} \cdot \dfrac{1}{4L}$ is the ballistic constant of the moving-magnet galvanometer ; and for a moving-coil galvanometer

$$Q = \frac{c}{nAH} \cdot \frac{T}{2\pi} \cdot \frac{d}{2L}(1+\lambda/2) = K_2 d\,(1+\lambda/2)$$

where $K_2 = \dfrac{c}{nAH} \cdot \dfrac{T}{2\pi} \cdot \dfrac{1}{2L}$ is the ballistic constant of the moving-coil galvanometer.

In fact the equation which holds generally for all types of galvanometers, provided that the deflections are sufficiently small, can be written as

$$Q = Kd\,(1+\lambda/2)$$

where K is the ballistic constant of the type of instrument used.

While noting down the deflection of the spot of light on the scale, the scale is usually kept at a distance of one metre from the galvanometer mirror. Hence using the above equation we may define *the* **ballistic constant** *of a galvanometer as the charge required to produce a deflection of one millimetre of the spot of light on a scale placed at a distance of one metre from the galvanometer mirror, in the absence of damping.*

Experiment 32.1

Object. To determine the *ballistic constant* of a ballistic galvanometer by the *steady deflection method.*

Apparatus. A ballistic galvanometer whose ballistic constant is to be determined, a lamp and scale arrangement, a Post Office box, an accumulator, a voltmeter, a commutator and a stop-watch.

Theory. We know that the ballistic constant K of a ballistic galvanometer is connected to its figure of merit η by the relation

$$K=\eta.\ \frac{T}{2\pi} \qquad (32\cdot7)$$

where T is the period of the galvanometer. Thus, if the figure of merit η of the ballistic galvanometer is determined and its period T observed, the ballistic constant K of the galvanometer can be calculated from the above equation.

To determine the figure of merit η of the galvanometer, it is connected in the usual 'galvanometer arm' BD of a post office box as shown in fig. 32·13. Then keeping the usual 'unknown resistance arm' CD

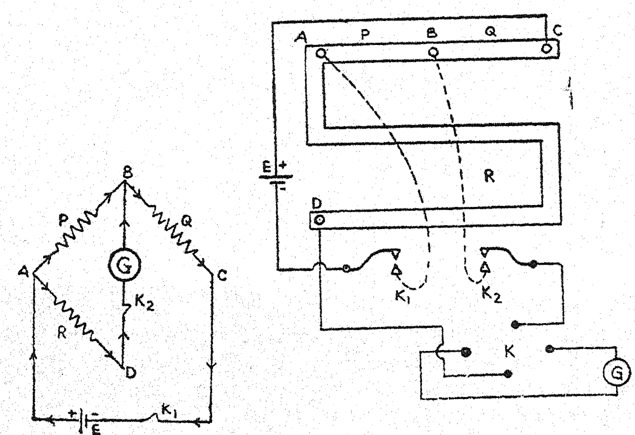


Fig. 32·13

of the post office box *disconnected*, if an accumulator of known E. M. F. E connected in the usual 'battery arm' AC, produces a steady deflection δ on the galvanometer-scale, the figure of merit of the galvanometer is from equa. (32·4) given by

$$\eta=\frac{EP}{(P+Q)\ (R+G)}\cdot\ \frac{1}{\delta} \qquad (32\cdot8)$$

where P and Q are the resistances in the ratio arms, R the resistance in the rheostat arm of the post office box and G the resistance of the galvanometer.

The above equation can also be written as

$$R+G=\frac{EP}{(P+Q)\,\eta}\cdot\frac{1}{\delta} \qquad (32\cdot9)$$

so that, if δ_1 and δ_2 are the steady deflections on the galvanometer-scale for two values R_1 and R_2 of the resistance in the rheostat arm, we have

$$R_1+G=\frac{EP}{(P+Q)\,\eta}\cdot\frac{1}{\delta_1}$$

and
$$R_2+G=\frac{EP}{(P+Q)\,\eta}\cdot\frac{1}{\delta_2}$$

which on subtraction gives

$$R_1-R_2=\frac{EP}{(P+Q)\,\eta}\cdot\left(\frac{1}{\delta_1}-\frac{1}{\delta_2}\right)$$

whence
$$\eta=\frac{EP}{(P+Q)(R_1-R_2)}\cdot\left(\frac{1}{\delta_1}-\frac{1}{\delta_2}\right) \qquad (32\cdot10)$$

This equation can be used to calculate the figure of merit η of the galvanometer, without requiring any knowledge of its resistance G.

Alternatively, steady deflection δ for various values of R in the rheostat arm are observed and a graph plotted taking R along the X-axis and $1/\delta$ along the Y-axis. This will come out to be a straight line as shown in fig. 32·14, and as is evident from equ. (32·9), its —ve intercept on the X-axis will give G, for when $1/\delta=0$, $R=-G$. Using this value of G the figure of merit η of the galvanometer can then be calculated from equ. (32·8).

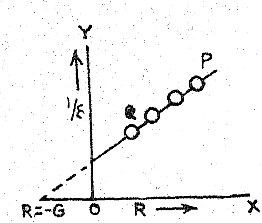


Fig. 32·14

Method. Level the base of the ballistic galvanometer by means of the levelling screws and release its coil. Throw light on the mirror of the galvanometer and get the bright spot of reflected light on the scale set up at a distance of *one metre* from it. Adjust the spot of light on the scale at zero. Connect the galvanometer in the usual 'galvanometer arm' BD of the post office box through a commutator K and an accumulator in the usual 'battery arm' AC, leaving the arm of 'unknown resistance' CD disconnected as shown in fig. 32·13.

Adjust the ratio arm P to 10 ohms, the ratio arm Q to 1000 ohms and the rheostat arm R to a high resistance, say 5000 ohms. Press the battery key and note down the *steady* deflection of the spot of light produced on the scale. Then reverse the current through the galvanometer with the help of the commutator K and again note down the steady deflection of the spot of light on the scale. Thence find the *mean* value of the galvanometer deflection δ for the resistance R=5000 ohms in the rheostat arm.

Now keeping the resistances P and Q in the ratio arms the *same*, increase the resistance R in the rheostat arm in steps of 1000 ohms and determine the mean value of δ as above for at least six values of R.

Next measure the E. M. F. of the accumulator with a voltmeter; and noting down the time of oscillation of the galvanometer coil for a known number of oscillations, also determine the period of the galvanometer.

Plot a graph taking the various values of resistance in the rheostat arm of the post office box along the X-axis and the corresponding values of *reciprocal* of deflection δ along the Y-axis. This will come out to be a straight line as shown in fig. 32·14. Measure its *negative* intercept OR on the X-axis which gives G, the galvanometer resistance. Finally calculate the figure of merit of the galvanometer from equations (32·8) and (32·10), and the ballistic constant from equ. (32·7).

Sources of error and precautions. (1) The base of the galvanometer should be carefully levelled by means of the levelling screws and the coil should be unclamped.

(2) A tapping key should be connected across the galvanometer terminals in order to bring the galvanometer coil to rest in a *short* time by tapping the key, when so desired.

(3) In the case of plug type of post office box all sockets must be *clean* and the plugs made *tight*.

(4) The accumulator connected in the usual battery arm of the post office box should be fully charged so that it may have a *constant* E. M. F., thus producing *steady* deflection in the galvanometer.

(5) The scale should be set up at a distance of *one metre* from the galvanometer mirror.

(6) The resistance R in the rheostat arm should be *very large* when compared to the resistance P in the ratio arm so that the latter may be neglected in comparison to the former.

(7) The graph between $1/\delta$ and R should be *smoothly* drawn with (0, 0) at the origin.

(8) While determining the period T of the galvanometer, the galvanometer circuit must be kept *open*.

Observations. [*A*] *Determination of figure of merit of galvanometer.*

(*i*) Measurement of δ for different values of R

Rest position of spot of light=zero.

S. No.	Ratio arms P ohms	Q ohms	Rheostat arm R ohms	Deflection δ in galvanometer in mm.	$1/\delta$ in (mm.) $^{-1}$	$\frac{1}{\delta_1}-\frac{1}{\delta_1}$ for $R_1-R_2=3000$ ohms
1.	10	1000	5000		*a*	
2.	,,	,,	6000		*b*	
3.	,,	,,	7000		*d*	
4.	,,	,,	8000		*e*	
5.	,,	,,	9000		*f*	
6.	,,	,,	10000		*g*	
					Mean	*x*

(*ii*) E. M. F. of the accumulator = volts

[*B*] *Determination of period of the galvanometer.*

S. No.	No. of oscillations	Time		Period
		Min.	Sec.	
			Mean	

Calculations. From equ. (32·10)

$$\eta = \frac{EP}{(P+Q)(R_1-R_2)} \cdot \left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right)$$

$$= \frac{EP}{(P+Q)(R_1-R_1)} \cdot x$$

$$= \quad \text{amp./mm.}$$

From graph (32·14)

Galvanometer resistance G = intercept OR= ohms

[Using this value of G prepare the following table from table No.

S. No	R ohms	(R+G) ohms	δ mm.	(R+G)δ ohms × mm,
1.	5000			
2.	6000			
3.	7000			
4.	8000			
5.	9000			
6.	10000			
			Mean	*y*

$$\eta = \frac{EP}{(P+Q)\ (R+G)\delta}$$

$$= \frac{EP}{(P+Q).\ y}$$

$$= \qquad \text{amp./mm.}$$

$\therefore$ Mean value of $\eta =$ amp./mm.

Hence $K = \eta . \dfrac{T}{2\pi}$

$$=$$

$$= \qquad \text{coulombs/mm.}$$

Result. The ballistic constant of the ballistic galvanometer = coulombs/mm.

32·7. Ammeters. Amperemeters, or ammeters as they are usually called, are instruments used for the measurement of current directly in amperes. To measure a current, an ammeter is inserted in the circuit so that the whole current to be measured passes through it. This introduction of the instrument into the circuit must have least possible influence on the circuit, *i.e.*, the instrument must be able to record the value of the current without changing it. In addition the 'voltage drop' IR across the instrument and the power loss I^2R and consequent heat produced in the instrument must, in spite of large currents, be negligible. This requires that an ammeter should be a *low* resistance instrument. Consequently the coil or the wire used in an ammeter is always of low resistance. When the resistance of the coil is not small, it is shunted with a wire of very low resistance. The shunt makes the total resistance low as well as allows most of the current to pass through it so that only a safe part goes through the coil.

There are three types of ammeters in common use: (*a*) Moving iron, (*b*) Hot wire and (*c*) Moving coil.

(a) **Moving iron ammeter** (*i*) *Repulsion type.* It consists of two rods A and B (Fig. 32·15) of soft iron lying inside and parallel to the axis of a long coil C. One of the rods A is fixed and the other B is pivoted and attached to a spindle carrying a pointer. When the current to be measured is passed through the coil, the iron rods are similarly magnetized and the repulsion of the movable rod from the fixed one ensues. With the movement of the movable rod, the pointer is deflected on the scale until equilibrium is established by controlling forces provided either by a balance weight W or a spring. The scale

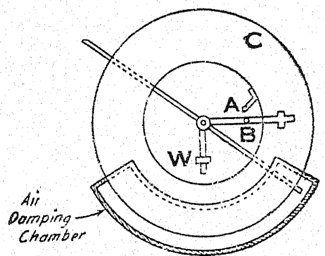


Fig. 32·15

is graduated in amperes so that by noting the position of the point on it, the current can be read off directly.

(*ii*) *Attraction type*. It consists of a coil A and a soft iron core C mounted eccentrically (Fig. 32·16). The iron core is made of several thin discs of soft iron. When the current to be measured is passed through the coil, a magnetic field is set up and the iron core moves from weaker magnetic field outside the coil into the stronger field inside it. With the iron core moves the pointer attached to it until equilibrium is established by controlling forces which may be gravitational or due to a spring, and the current is read off directly on the scale. The shape of the discs in the iron core is such that a suitably divided scale is obtained.

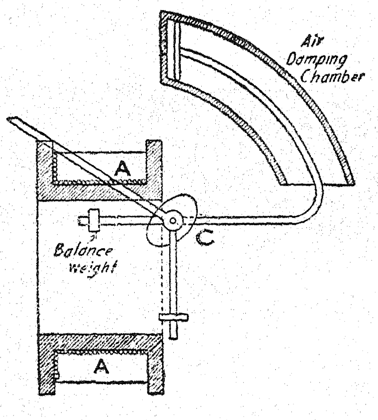


Fig. 32·16

The moving iron ammeters are very cheap and can stand much rough usage without injury and are, therefore, most generally used. The force of repulsion or attraction is proportional to the square of the current and is independent of the direction of the current ; hence these instruments can be used for either direct or alternating current measurements. The main disadvantages of these instruments are:—(i) errors due to hysteresis in the iron and to stray magnetic fields and (ii) the non-uniformity of the scale. The error due to hysteresis can be reduced by the use of short iron pieces so that they demagnetize themselves and the effect of strong fields can be minimised by magnetic screening.

The moving iron ammeters are dead-beat instruments, the damping being obtained by air friction device.

(b) Hot wire ammeter. The commonest form of double-sag hot wire ammeter consists of a wire AB Fig. (32·17) of platinum-iridium, at the middle of which is attached another wire W of phosphor bronze, the latter being attached to a fine silk thread which passing round a small pulley P is fastened to a spring S which keeps the whole system taut. A light pointer is carried by the spindle upon which

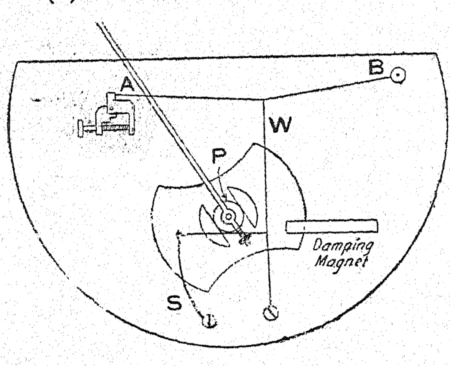


Fig. 32·17

the pulley is mounted. When the current to be measured or a definite fraction of it, is passed through the hot-wire AB, it gets heated and expands. The sag produced in it by expansion is taken up by the spring. This causes the pulley to rotate and the pointer to deflect through a corresponding angle over the scale calibrated to give the current directly in amperes.

The hot-wire must be made of a material whose resistance varies only slightly with temperature and whose coefficient of expansion is proportional to the rise of temperature. Further, as the hot-wire is made as thin as possible in order that it may attain a steady temperature quickly, its material must have sufficient mechanical strength to withstand the stresses placed upon it by the tension of the spring system. The box of the instrument must be made of materials whose coefficient of expansion is equal to that of the hot-wire so that changes in temperature other than those caused by the current through the hot-wire may not affect the deflection.

The instrument is dead-beat. The damping is electro-magnetic and is caused by eddy currents produced in a thin aluminium disc carried by the spindle of the pulley, the edge of the disc being situated in the air-gap of a permanent magnet. This damping is very essential to stop movements due to vibration or to sudden changes in the current, otherwise the hot-wire will be subjected to excessive stresses.

The hot-wire ammeter is very simple and cheap. Its sensitivity can be greatly increased by placing the hot-wire in vacuum. Since the heat produced is independent of the direction of the current the instrument can be used for either direct or alternating-current measurements. Moreover, since the heating effect by an alternating current is independent of its wave-form of frequency, the calibration of the ammeter is the same for both direct and alternating current. The working of the instrument is not affected by stray magnetic fields.

Since the hot-wire is always very fine, its current-carrying capacity is limited. Consequently the double-sag hot-wire ammeter can be used without any shunt for current of the order of one ampere only. For higher currents the instrument must be shunted so that only a safe part of the current may flow through the hot-wire. The instrument has a high power consumption. It is sluggish owing to the time taken by the wire to heat up. Since the heating effect is proportional to the square of the current, the scale is not uniform, it being cramped at the bottom end.

(c) **Moving-coil ammeter.** It is simply a pivoted type moving-coil galvanometer (Fig. 32·18) whose coil has been shunted with a wire R of low resistance. When a definite fraction of the current to be measured is allowed to pass through the coil, it is deflected on account of interaction between the permanent magnet field and the magnetic field of the coil. The motion of the coil is controlled by one

or two phosphor-bronze springs according as the instrument is a uni-pivot or bi-pivot one. The field of the permanent magnet is kept radial by the use of concave pole-pieces and hence the current is proportional to the deflection of the coil which it produces. The instrument is dead-beat, damping being produced by eddy currents induced in the aluminium former upon which the coil is wound.

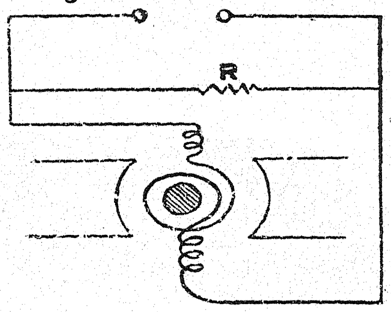


Fig. 32·18

In uni-pivot instrument, the friction torque at the pivot is reduced by the elimination of one point. This makes the instrument highly accurate. But the instrument requires careful handling and must be used only in a horizontal position.

The moving coil ammeter is sufficiently accurate. Its power consumption is low. The scale is remarkably uniform. The same instrument can be used to measure a large range of a current by the use of shunts. The instrument is free from errors due to hysteresis and from errors due to stray magnetic fields.

The instrument suffers from friction and heating errors which are, of course, present in other types of instruments also. Unless the permanent magnet is carefully aged during manufacture, its weakening with time may introduce a considerable error. As the direction of deflection depends upon the direction of current the instrument can be used only for direct-current measurements.

The range of an ammeter can be increased by shunting the instrument with a suitable low resistance wire. If r is the resistance of an ammeter which gives a full-scale deflection with a current of i amperes, then the shunt resistance required to increase the range of the instrument from i amperes to I amperes is given by $S = \frac{i}{I-i}\,r$. These shunts used to increase the range of ammeters are known as range multipliers.

Experiment 32.2

Object. To convert a weston galvanometer into an ammeter of given range.

Apparatus. A weston galvanometer, an accumulator, a high resistance box, a voltmeter, an ammeter of nearly the same range as given for conversion, a plug key and apparatus for determining galvanometer resistance by Kelvin's method.

Theory. Let G be the galvanometer resistance and I_g the current which when passed through it produces a full-scale deflection. To convert the galvanometer into an ammeter reading up to I amperes, it is to be shunted with a low resistance whose value from § 26·5 is given by

$$S = \frac{I_g}{I - I_g} . G \qquad (32\cdot11)$$

The length of the shunt wire having a resistance S is given by

$$l = \frac{\pi r^2 S}{\rho} \qquad (32\cdot12)$$

where r is its radius and ρ the resistivity of the material.

Method. Determine the resistance of the weston galvanometer by Kelvin's method.

Next connect (Fig. 32·19) the weston galvanometer to an accumulator in series with a high resistance box including a plug key K in the circuit. Adjust the resistance box to a high value, say 5000 ohms. Note down the zero reading of the galvanometer. Close the key K and again adjust the resistance box so that almost full-scale deflection is obtained in the galvanometer. Note down the resistance R introduced into the circuit by the resistance box and also the deflection n produced in the galvanometer taking into account the zero error of the instrument. Measure the E. M. F., e of the accumulator with an accurate voltmeter and calculate the figure of merit of the galvanometer from the formula

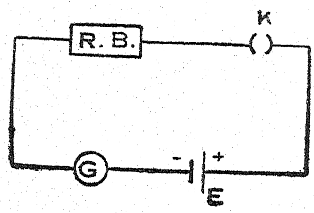


Fig. 32·19

$$k = \frac{e}{n\,(R+G)}\,.$$

Repeat the experiment with different values of R and thus find the mean value of k. Multiply this by the total number of divisions N on one side of zero of the galvanometer scale which gives I_g the current required to produce full-scale deflection in the galvanometer.

Now calculate the shunt resistance S from equation (32·11) and then selecting a proper gauge, say S. W. G. 22, calculate from equation (32·12) the length l of a manganin or constantan wire which will have a resistance equal to S, the value of r and ρ being known from tables. Then cut off a length of the wire about 4 cm more than the calculated value and mark two points on it at the calculated distance apart, leaving some spare parts on each side. Connect the wire across the galvanometer such that the marked points are just outside the screws. Graduate the scale divisions in amperes taking N divisions equal to the range I amperes. This converts the weston galvanometer into an ammeter of the given range.

Next connect (Fig. 32·20) the shunted galvanometer in series

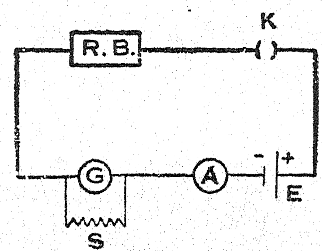


Fig. 32·20

with an ammeter of nearly the same range, a high resistance box, and an accumulator including a key K in the circuit. Adjust the resistance box to a high value. Close the key K and note down the reading of the galvanometer and the ammeter. Convert the galvanometer reading into amperes and find the difference between the readings of the two instruments, if any, which gives the error in the galvanometer reading. Alter the resistance of the circuit and in this way take at least six sets of observations, finding the error in each reading of the galvanometer. Plot a graph taking galvanometer readings as abscissae and corresponding ammeter readings as ordinates. The graph will be nearly a straight line and will represent the calibration curve of the shunted galvanometer.

Sources of error and precautions. (1) The resistance box should be of high resistance and should be preferably of dial pattern. At no stage of the experiment the resistance in the resistance box should be reduced to zero for then a heavy current will flow through the circuit damaging the galvanometer or ammeter or both.

(2) The accumulator used should be freshly charged so that its E. M. F. may remain constant throughout the determination of I_g.

(3) The ammeter used for calibration of the shunted galvanometer should be of nearly the same range and its readings should be corrected for zero error of the instrument.

(4) The +ve marked terminal of the ammeter should be connected to the positive pole of the cell or the high potential point of the circuit.

(5) While connecting the shunt across the galvanometer, care should be taken to see that only the exact required length of the wire lies between its terminals.

(6) The calibration curve of the shunted galvanometer should be a straight line and should be smoothly drawn.

Observations and Calculations. [*A*] *Determination of galvanometer resistance G.*

[Note down observations for Kelvin's method for determination of galvanometer resistance G].

Galvanometer resistance G= ohms

[*B*] *Determination of* I_g

E. M. F. of the accumulator = volts

S. No.	Resistance introduced by the resistance box R ohms	Deflection in galvanometer n scaled div.	Figure of merit of galvanometer k amp./div.
		Mean	

Total No. of divisions N on one side of zero of the galvanometer scale ... ... =

∴ The current I_g reqd. to produce full-scale deflection = amp.

[*C*] *Calculation of length of the shunt wire*

$$S=\frac{I_g}{I-I_g}.G$$
$$= \quad \text{ohms}$$

Gauge selected S. W. G. No......

From tables $\begin{cases} \text{resistivity } \rho \text{ of the material } (......) & = \text{ohm} \times \text{cm.} \\ \text{radius } r \text{ of the wire} & = \text{cm.} \end{cases}$

∴ $l=\pi r^2 S/\rho$ = = cm.

[*D*] *Calibration of the shunted galvanometer.*

S. No.	Galvanometer reading in		Ammeter reading I′ amp.	Error I—I′ amp.
	Scale divisions	amperes I		

Results. (*i*) The length of the shunt wire of.........S. W. G. ......required to convert the weston galvanometer into an ammeter of the given range = cm

(*ii*) The graph shown in fig.........is the calibration curve of the shunted galvanometer and is a straight line.

Criticism of the method. If performed carefully, the experiment yields quite satisfactory results. The accuracy of the result depends upon the constancy of E. M. F. of the accumulator and the accuracy of the measuring instruments, namely, the voltmeter and the ammeter. For higher accuracy the shunted galvanometer should be calibrated by means of a potentiometer.

32·8. Voltmeters. Voltmeters are instruments used for the measurement of P. D. directly in volts. To measure the P. D. between two points of a circuit, a voltmeter is connected across the two points whose P. D. is required. In order that the instrument should record the actual P. D. between two points, it is necessary that in connecting the instrument across the points, the P. D. between them should not be altered. To fulfil this condition the instrument must take only an infinitesimal current. In addition the power consumption V^2/R and consequent heat produced in the instrument should be negligible. Hence it is clear that a voltmeter must have a high resistance whose value must always be very great in comparison with the resistance between the points whose P. D. is to be measured. In fact the resistance of an ideal voltmeter should be infinite.

Voltmeters can be divided into four classes : (*a*) Electro-static (*b*) Moving iron, (*c*) Hot wire and (*d*) Moving coil.

The electro-static voltmeters are in general of two types ; (*i*) The quadrant type which is used for voltages up to 10 or 20 kilovolts and (*ii*) The attracted disc type which is used for voltages above 10 or 20 kilovolts. The construction and working of these voltmeters is similar to those of Lord Kelvin's quadrant and attracted disc electrometers.

The electro-static voltmeters are ideally perfect in having infinite resistance. The power consumption is extremely small. They work equally well on both direct and alternating currents. Their working is free from errors due to hysteresis and are not affected by stray magnetic fields and eddy currents. They are, however, not sufficiently sensitive to read low voltages, their useful range being from about 500 volts upwards to several hundred kilovolts.

The moving iron voltmeters and the hot-wire voltmeters are constructed from moving iron ammeters and hot-wire ammeters respectively by connecting a high non-inductive resistance in series with the solenoid and the hot-wire respectively, and then calibrating the scale to read the P. D. directly in volts. These instruments are suitable for measuring either direct or alternating voltages.

The moving-coil voltmeters are constructed from moving-coil galvanometers or ammeters by connecting a high non-inductive resistance in series with the moving coil (Fig. 32·21). The higher the resistance connected in series with the coil, the more accurately will it indicate the P. D. to be measured. Cheap voltmeters usually have fairly low resistance and consequently they do not give reliable results. Moving-coil voltmeters can be used for direct voltages only.

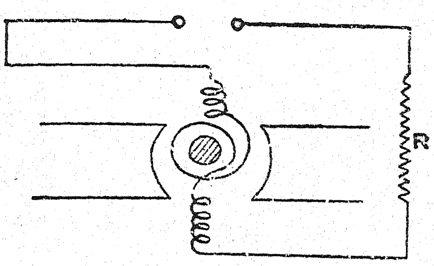


Fig. 32·21

The range of moving-iron, hot-wire or moving-coil voltmeters may be increased by using a suitable high resistance in series with it. Thus, if a voltmeter gives a full-scale deflection when a current i passes through it and if its resistance is r, then the series resistance R required to increase the range of the instrument from v volts to V volts is given by

$$\frac{V}{R+r}=i=\frac{v}{r}$$

whence
$$R=\frac{V-v}{v}.r$$

The resistance connected in series with a voltmeter to increase its range is called 'multiplying resistance' or 'multiplier.' The value of these multipliers must be constant and hence they must have a low temperature coefficient and since they absorb appreciable power, ample provision for cooling must be made.

An ammeter can be converted into a voltmeter by placing a high resistance in series with it, and a voltmeter can be used as an ammeter if the former is shunted with a low resistance.

Experiment 32.3

Object. To convert a weston galvanometer into a voltmeter of given range.

Apparatus. A weston galvanometer, an accumulator, a voltmeter of nearly the same range as specified for conversion, a plug key, a rheostat and apparatus for determining galvanometer resistance and its sensitivity.

Theory. To convert a weston galvanometer into a voltmeter reading upto V volts, a *high* resistance is to be connected *in series* with it. If R be the magnitude of this series resistance, I_g the current required to produce full-scale deflection in the galvanometer and G the resistance of the galvanometer,

$$I_g = \frac{V}{R+G}$$

whence
$$R = \frac{V}{I_g} - G \qquad (32{\cdot}13)$$

Method. Determine the resistance G of the weston galvanometer and its figure of merit η by adopting any one of the following two procedures :—

(*i*) The galvanometer resistance by Kelvin's method (expt. 63) and its figure of merit by a separate method as described in § 32·4

(*ii*) Both the galvanometer resistance and its figure of merit by a *single* method using P. O. box as described in expt. 86, where galvanometer resistance may be obtained graphically and its figure of merit by way of calculations from equ. (32·10).

Next note down the total number of divisions N on one side of zero of the galvanometer scale and, multiplying it by the figure of merit of the galvanometer, calculate the current I_g required to produce full-scale deflection in the galvanometer. Then calculate the resistance R required to convert the galvanometer into a voltmeter of given range V from equ. (32·13). Procure a resistance of this value and connect it in series with the galvanometer. Graduate the galvanometer scale in volts taking full-scale deflection equivalent to V volts. This converts the galvanometer into the voltmeter reading upto V volts.

To calibrate the instrument, connect (Fig. 32·22) the whole of a rheostat S, using its fixed terminals at the base, to an accumulator including a plug key K in the circuit, connect the galvanometer together with its series resistance R, in parallel with a part of the rheostat using one of the fixed terminals at the base and the screw joined to the sliding contact maker. Connect between the same points a voltmeter of nearly the same range.

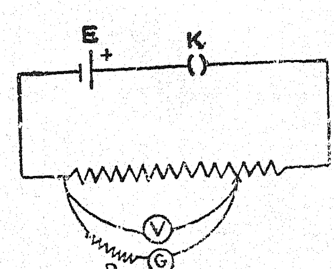


Fig. 32·22

Close the key K. Adjust the rheostat to get full-scale deflection in the galvanometer. Note down the reading of the instruments and find the difference between them which gives the error in the galvanometer reading. Decrease the deflection in the galvanometer with the help of the sliding contact maker of the rheostat and in this way take at least six sets of readings of the two instruments, finding error corresponding to each reading of the galvanometer. Plot the calibration curve of the galvanometer taking the galvanometer readings as abscissae and the corresponding readings of the voltmeter as ordinates. The graph will be a straight line.

Sources of error and precautions. Besides the precautions

observed in the determination of the galvanometer resistance G and its figure of merit η, the following precautions should also be taken :

(1) The voltmeter used for calibration of the galvanometer after conversion into a voltmeter, should be of nearly the *same* range and its readings should be corrected for zero error of the instrument.

(2) The +ve marked terminal of the voltmeter should be connected to the higher potential terminal of the rheostat.

(3) The calibration curve of the galvanometer after its conversion into a voltmeter should be a straight line and should be smoothly drawn.

Observations and Calculations. [*A*] *Determination of G and* η.

(Record observations for determining galvanometer resistance G and its figure of merit η)

Galvanometer resistance G = ohms.

Its figure of merit η = amp./div.

Total no. of divisions N on one side of the zero of the galv. scale=

$\therefore$ The current I_g required to produce full-scale deflection = amp.

[*B*] *Calculation of resistance* R *required for conversion of galvanometer into voltmeter.*

$$R = \frac{V}{I_g} - G$$

=

= ohms

[*C*] *Calibration of galv.-scale to read upto V volts.*

S. No.	Galvanometer reading in		Voltmeter reading v′ in volts	Error v—v′
	Scale divisions	volts v		

Results. (*i*) The series resistance required to convert the weston galvanometer into a voltmeter of the given range = ohms

(*ii*) The graph shown in fig....... is the calibration curve of the galvanometer converted into a voltmeter and is a straight line.

Oral questions

GALVANOMETERS

What is a galvanometer ? How many classes of galvanometers do you know of and what are the main principles on which they are constructed ? Give examples of each. Describe the construction of a moving magnet galvanometer. How is the needle deflected in it ? Describe a tangent galvanometer. What is it used for ? What advantages does Helmholtz double coil tangent galvanometer possess over an ordinary tangent galvanometer ?

How can you increase the sensitiveness of a moving-magnet galvanometer ? Why can you not use a large number of turns in the coil ? What is the harm if the radius of the coil is very small ? What is control magnet ? Explain how is it used to decrease the controlling field ? In case the controlling field is reduced to zero, how can the necessary restoring couple be provided ? What is an astatic pair ? How does it increase the sensitiveness of the galvanometer ? Which is the most sensitive moving-magnet galvanometer ?

What is the principle of construction of moving-coil galvanometer ? How many types are they ? How is the coil deflected ? What provides the controlling couple ? Why is a phosphor-bronze strip used for suspension ? Why are springs in a pivoted type moving-coil galvanometer made thin ? Why have they large number of turns and why are they coiled in opposite directions ? What is the core of the coil made of and what purpose does it serve ? Why are concave pole-pieces used ? What is the harm if the field is not radial ? What factors determine the sensitiveness of a moving coil galvanometer ? What are the advantages of a moving coil galvanometer over a moving-magnet galvanometer ?

What is the advantage of a mirror galvanometer ? What advantages does the lamp and scale method of measuring deflections possess over the use of mechanical pointer ? What is the relative sensitiveness of the optical and mechanical methods of measuring deflections ? What is meant by the sensitivity of a galvanometer ? Which is the most sensitive galvanometer ? Is it possible to have a galvanometer which may be regarded very sensitive from one point of view and very insensitive from the other ? How can you determine the sensitivity of a sensitive moving col galvanometer ?

Distinguish between periodic an aperiodic galvanometer. What is a dead-beat galvanometer ? How can you make a galvanometer dead-beat ? How many methods of obtaining damping do you know ? Describe them and mention the instruments in which they are used. Which is the most efficient method of damping ? What are eddy currents and how are they produced ? What is a ballistic galvanometer ? What are its special features ? What is it used for ? What is logarithmic decrement ? How and why do you determine it ?

How can you use a galvanometer (*i*) as an ammeter (*ii*) as a voltmeter ? Can you use all types of galvanometers as ammeters and voltmeters ? If so, how ?

AMMETERS AND VOLTMETERS

What are ammeters and voltmeters ? How do they differ from galvanometers ? How many types of ammeters do you know and which is the best ? Why should an ammeter have low resistance and a voltmeter a high resistance ? What is the harm if the resistance of the voltmeter is not so ? Do you know of any voltmeter whose resistance is infinity ? Why is an ammeter connected in series and a voltmeter in parallel ? Why can you not connect a voltmeter in series with a circuit ? Describe a hot-wire ammeter. Of what material is the hot wire made and why ? How is the damping produced in the instrument ? Explain the construction and working of the moving-coii ammcter and voltmeter. What advantages does an ammeter possess over a tangent gaivanometer as a current measuring instrument ?

Why are voltmeters and ammeters marked × on or near tke terminals ? Are all voltmeters and ammeters so marked ? If not, why ? Why does the needle of these instruments move to some extent to give a deflection and then stop there ? How can you test the accuracy of an ammeter or a voltmeter ? How can yon increase the range of an ammeter or a voltmeter ? Can a voltmete, be us an ammeter and vice versa, if so, how ?

CHAPTER XXXIII
A. C. MEASUREMENTS

33·1. Circuit containing capacitance, inductance and resistance. When a harmonically varying E. M. F., $E_0 \sin \omega t$ is applied to a circuit containing a resistance R, an inductance

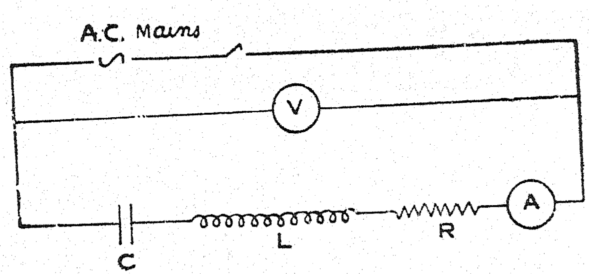


Fig. 33·1

L and a capacitance C in series (Fig. 33·1), we have

$$Ri+L\frac{di}{di}+V=E_0 \sin \omega t$$

where i is the current and V the potential to which the condenser is charged. Let q be the charge on the condenser at the instant when its potential is V, then $V=q\,C$ and we get from the above equation

$$Ri+L\frac{di}{dt}+\frac{q}{C}=E_0 \sin \omega t \qquad (33·1)$$

Let the solution be

$i = A \sin(\omega t - a)$,

where A and a are to be determined

$$\therefore \quad \frac{di}{dt}=A\,\omega \cos(\omega t-a)$$

$$\therefore \quad dq=idt=A \sin(\omega t-a)\,dt$$

$$\therefore \quad q=\int dq=\frac{A}{\omega}\cos(\omega t-a)$$

Substituting the values in equation (33·1)

$$R\,A \sin(\omega t-a)+[\,L\omega-1/(\omega C)\,]\,A \cos(\omega t-a)=E_0 \sin \omega t$$

Comparing the coefficients of $\sin \omega t$ and $\cos \omega t$ on the two sides of the above equation, we get

$$R\,A \cos a+[L\omega-1/(\omega C)]\,A \sin a \quad =E_0 \qquad (33·2)$$

$$-R\,A \sin a+[L\omega-1/(\omega C)]\,A \cos a \quad =0 \qquad (33·3)$$

Squaring and adding equations (33·2) and (33·3), we have

$$A^2 [R^2 + \{L\omega - 1/(\omega C)\}^2] = E_o^2$$

$$\therefore \qquad A = \frac{E_o}{\sqrt{R^2 + [L\omega - 1/(\omega C)]^2}}$$

Also from equation (33·3)

$$\tan \alpha = \frac{L\omega - 1/(\omega C)}{R}$$

Hence $$i = \frac{E_o}{\sqrt{R^2 + [L\omega - 1/(\omega C)]^2}} \sin\left[\omega t - \tan^{-1}\left(\frac{L\omega - 1/\omega C}{R}\right)\right] \qquad (33\cdot4)$$

The quantity $L\omega$ is the *inductive reactance* of the circuit and $1/(C\omega)$, the *capacitive reactance*. The total reactance of the circuit is the resultant of the inductive and capacitive reactances. The quantity $\sqrt{(L\omega - 1/\omega C)^2 + R^2}$ is called the *impedance* of the circuit and is denoted by Z. The total reactance of the circuit is denoted by X, the inductive reactance by X_l and the capacitive reactance by X_c. It is clear from the expression for the impedance of the circuit that

$$Z^2 = X^2 + R^2$$

From equation (33·4) it is evident that the maximum value of the current is given by

$$i_o = \frac{E_o}{\sqrt{[(L\omega - 1/(\omega C)]^2 + R^2}} \qquad \ldots \qquad (33\cdot5)$$

Thus the less the impedance of the circuit the greater is the value of the maximum current in it. The unit of impedance is ohm. After all impedance is the effective resistance of an A. C. circuit. For a circuit in which $L\omega = 1/(\omega C)$ the current attains a value $i_o = E_o/R$. Under such conditions the true ohmic resistance alone determines the current in the circuit.

The value of the angle α depends on the relative magnitudes of the inductive and the capacitive reactances in the circuit, ohmic resistance being constant. When $L\omega > 1/(\omega C)$ the current lags behind the applied E. M. F., when $L\omega < 1/(\omega C)$ the current leads the applied E. M. F. However when $L\omega = 1/(\omega C)$ (the condition of resonance), the current is in phase with the applied E. M. F.

The vector diagram representing the p. ds. across the various components may be conveniently drawn. There will be three components to be laid off in the vector diagram.

(i) Ri the active component of the applied e. m. f. along OA,

(ii) $L\omega i$ drawn 90 ahead of Ri, and

(iii) $\frac{i}{C\omega}$ drawn 90° behind Ri.

This is shown in figure 33.2. These three vectors can be combined by obtaining the resultant of any two and then combining this resultant with the third one. In the figure $L\omega i$ and $i/C\omega$ have been first combined giving the resultant

$$OX = \left(L\omega - \frac{1}{C\omega}\right) i$$

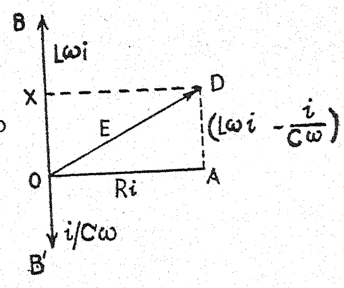


Fig. 32.2

which when combined with OA gives the resultant OD. Evidently

$$OD = E = Zi = i.\sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}$$

In the case illustrated the resultant e. m. f. is ahead of the current by the angle

$$AOD = \alpha = \tan^{-1} \frac{\left(L\omega - \frac{1}{C\omega}\right)}{R}$$

and the circuit behaves like a slightly inductive circuit. The current may be ahead or lag behind the impressed e. m. f. E according as 1/Cw or Lw has the larger value.

33·2. Power in an A. C. Circuit. In an *inductive* circuit

$$E = E_o \sin \omega t$$

$$i = i_o \sin (\omega t - \alpha)$$

where $\alpha = \tan^{-1}(L\omega/R.)$

Therefore the rate of working at any instant is

$$E = E_o\, i_o \sin {}_o(\omega t - \alpha)$$

$$= E_o\, i^o \cos \alpha \sin^2 \omega t - \tfrac{1}{2} E_o i_o \sin \sin \alpha\, 2\omega t$$

The mean value of $\sin^2 \omega t$ for a cycle is $\frac{1}{2}$ and of $\sin 2\, \omega t$ is zero.

$\therefore$ Mean rate of working $= \frac{1}{2}\, E_0\, i_o \cos \alpha$

$$= \frac{E_o}{\sqrt{2}} \cdot \frac{i_o}{\sqrt{2}} \cos \alpha$$

$$= \text{(virtual volts)} \times \text{(virtual am ps.)} \times \cos \alpha$$

Thus on measuring separately the virtual volts and virtual amperes for a circuit by means of a voltmeter and an ammeter

and taking their product, we get the apparent watts. This does not measure the actual power absorbed in the circuit, for the apparent watts should be multiplied by $\cos \alpha$ to get true watts. The ratio of true watts to apparent watts, *i. e.*, $\cos \alpha$ is called the *power factor* of the circuit.

The graphical method given below is very instructive as it throws additional light on the problem discussed above. Let E_0 and i_0 represent the maximum values of the E. M. F. and the current in the circuit (Fig. 33·2). Resolving i_0 parallel and perpendicular to E_0, we have the components $i_0 \cos\alpha$ and $i_0 \sin\alpha$ respectively. The component parallel to E_0 gives the mean rate of working $\frac{1}{2} E_0 i_0 \cos\alpha$, whereas the latter component $i_0 \sin \alpha$, as its lags $90°$ in phase behind the E. M. F., gives the mean rate of working

$$\frac{E_0 i_0 \int_0^{2\pi} \sin \omega t \sin (\omega t - 90)\, d(\omega t)}{\int_0^{2\pi} d(\omega t)}$$

$$= -\frac{E_0 i_0}{2\pi} \int_0^{2\pi} [\tfrac{1}{2} \sin 2\, \omega t\, d(\omega t)]$$

$$= \frac{E_0 i_0}{8\pi} \Big[\cos 2\, \omega t \Big]_0^{2\pi} = 0$$

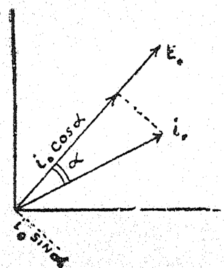


Fig. 33·2

Thus for the component $i_0 \sin\alpha$ perpendicular to E_0, the mean rate of working is zero. This component is termed the Idle or Wattless current. The name is self-explanatory ; it does not contribute to the rate of working in the circuit. The current in entirely wattless when $\alpha = 90°$ which is the limiting case when the inductance in the circuit is very large and R is negligible.

33·3. A.C. Measuring Instruments. The mean values of alternating E. M. F. and current for a complete cycle are each zero Hence an ordinary electromagnetic voltmeter or ammeter whose moving system is comparatively massive will indicate this mean value. Thus such instruments cannot be employed for the measurement of alternating current and voltage. In the first half-cycle the average current is $i_0 . 2/\pi$ (i_0 being the maximum value of A. C.) and in the later half $-i_0 . 2/\pi$, hence effects being directly proportional to the current, the suspended system receives equal and opposite impulses during a complete cycle and it is in such a quick succession that the needle fails to respond to it. Hence A. C. measuring instruments should be independent of

the direction of flow of the current. This is possible when the deflection is proportional to the square of the current *e.g.*, hot wire instruments. Hence only such instruments are employed for the measurement of A. C.

To interpret the readings of these instruments let us calculate the mean value of $i_0^2\sin^2\omega t$ for it is this value that is proportional to the deflection of the instrument. The mean value of this quantity for a complete cycle

$$=\int_0^{2\pi} i_0^2 \sin^2 (\omega t)d(\omega t) \Bigg/ \int_0^{2\pi} d(\omega t)=i_0^2/2.$$

Therefore the continuous current whose square would have the same mean value as that of the A. C. is $i_0/\sqrt{2}$; this continuous current would give the same reading on the hot wire instrument. This is termed the virtual current and is equivalent to A. C. of the maximum value i_0. That is the A. C. is measured by the strength of the steady current which would produce the same heating effect. Hence it is also termed as "effective current."

If the steady value of the current be I, then

$$I=i_0/\sqrt{2}$$

$$\therefore \quad I^2=i_0^2/2= \text{Mean value of the square of the current.}$$

Thus $I=\sqrt{i_0^2/2}=i_0/\sqrt{2}=$Sq. root of the mean of the square of the current. This value is, therefore, termed as the Root Mean Square value (R. M. S. value) of the current. In the same manner $E=E_0/\sqrt{2}$.

From the above treatment it is evident that a *virtual ampere* is one which will produce the same heating effect in a resistance as a steady current of 1 ampere will produce in the same time. A *virtual volt* is one which when applied to the ends of a resistance will produce the same heating effect as a steady P. D. of 1 volt applied across it for the same time.

Experiment 33.1

Object. To determine the impedance of a given A. C. circuit.

Apparatus. A condenser, an inductance, resistance, an A. C. ammeter, an A. C. voltmeter, a step-down transformer having a number of tapping in the secondary and flexible cord for connections.

Theory. Let a condenser of capacitance C, an inductance L and a resistance R be connected *in series*; and let an alternating E. M. F. of E virtual volts be applied across the combination as shown in fig.

33·4. Then, if i virtual amperes is the current in the circuit, equ. (33·5) gives

$$\text{Independence} = \sqrt{[L\omega - 1/C\omega]^2 + R^2} = \frac{E_0}{i_0} = \frac{E_0/\sqrt{2}}{i_0/\sqrt{2}} = \frac{E}{1}$$

or $$\text{Impedance } Z = \frac{\text{E. M. F. as measured by an A. C. voltmeter}}{\text{Current as measured by an A. C. ammeter}}$$

Alternatively, the current i in the circuit may be measured by applying various known e.m. fs., E in the circuit and a graph plotted taking E along the X-axis and the corresponding values of i along the Y-axis. This will come out to be a straight line as shown in fig. 33·3 i and its slope QR/PR with the i-axis will give impedance of the circuit.

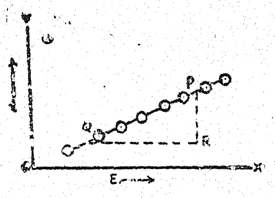


Fig. 33·3

Method. Connect the primary of a step-down transformer to the A. C. mains and connect to one of the tappings of the secondary, a suitable condenser, an inductance and a resistance, all *in series*, including an A. C. ammeter in the circuit, as shown in fig. 33·4. Connect an A. C. voltmeter across the tapping of the secondary of the transformer to measure the virtual E. M. F. applied in the circuit. Having properly made the connections, switch on the current and note down the readings of the voltmeter and the ammeter which give E and i respectively.

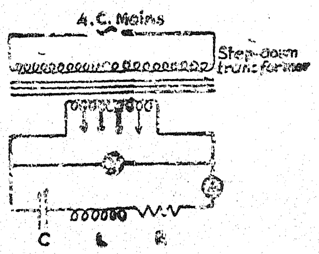


Fig. 33·4

Next switching off the current, change the tapping of the secondary of the transformer and in this way take at least six sets of observations for i and E. Calculate the impedance of the circuit from each set of observations separately and then take the mean.

Next plot a graph taking applied e. m. f. E along the X-axis and the corresponding values of i along the Y-axis. This will come out to be a straight line. Measure its slope QR/PR with the i-axis which will give impedance of the circuit.

Also calculate impedance Z from the formula

$$Z = (\sqrt{\{L\omega - (1/\omega C)\}^2 + R^2}$$

if the values of L, C and R are known.

Sources of error and precautions. (1) Both the ammeter and the voltmeter must be A. C. measuring instruments.

(2) Connect the ammeter in series and the voltmeter in parallel in the circuit.

(3) Zero error, if any, in the above measuring instruments should be taken into account.

(4) The condenser used should have a working voltage greater than the A. C. mains voltage.

(5) The inductance and the resistance used in the circuit should have a proper current rating so that they do not get overheated and thereby get short-circuited and burnt.

(6) Before changing the tapping of the transformer see that the current has been switched off.

(7) The graph between i and E should be a straight line and should be smoothly drawn.

Observations

L. C. of voltmeter = volts

L. C. of ammeter = amp.

S. No.	Voltmeter reading E volts	Ammeter reading i amp.	Impedance ohms
1.			
2.			
3.			
...			
...			
...			
		Mean	

Calculations. I Set

$$Z = \frac{E}{i} = \qquad = \text{ohms}$$

[Make similar calculations for other sets.]

$\therefore$ Mean value of Z= ohms

From graph (33·3)

QR = PR =

$\therefore$ Impedance $Z = \frac{QR}{PR} = \qquad = \text{ohms}$

Now L= henrys, C = farads

R= ohms and $\omega = 2\pi n =$

22104

$$\therefore \qquad Z=\sqrt{\{L\omega-(1/\omega C)\}^2+R^2}$$

$$=$$

$$= \qquad \text{ohms}$$

Result. The impedance of the circuit = ohms

Experiment 33·2

Object. To use an electrical vibrator to determine (*i*) the frequency of A. C. mains and (*ii*) the capacitance of a condenser.

Apparatus. Electrical vibrator, table clamp pulley, pan wt box, fishing cord, 6—10 volt battery, a microammeter, a voltmeter (0—15V) and a condenser.

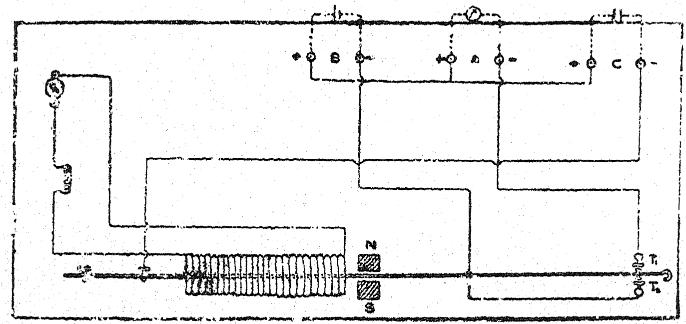


Fig. 33·5

Description of Apparatus. *The Electrical Vibrator.* The electrical vibrator (Fig. 33·5) consists of a solenoid through which passes a thin rod of steel which can be clamped at one end. At its free end the steel rod carries a hook to which a string under tension can be connected. The solenoid is connected in series with a 25-watt lamp and is used directly with A. C. mains. A permanent horse-shoe magnet is also mounted on the baseboard, the steel wire passing through the pole-pieces. When A. C. is fed through the solenoid, the rod is magnetised longitudinally with the polarity reversing with the change of sign of the current. Owing to the interaction of this with the field due to the permanent magnet the rod vibrates with the frequency of A. C. The length of the steel rod can be adjusted so as to get resonance indicated by a large amplitude of vibration of its free end. A continuous vibration is then maintained.

While using the vibrator for the determination of the capacitance of a condenser, a battery, a microammeter and a condenser can be connected to the terminals marked B, A and C on it. The steel rod carries a small iron piece with flat ends which makes contact alternately with the two flat steel discs T_1, T_2; this during

one half cycle charges the condenser and during the latter half discharges it through the microammeter.

Theory. (i) Frequency of A. C. Mains. When a cord of mass per unit length m, is connected to the vibrating rod of the vibrator and stretched with a tension T, the cord vibrates in segments as in Melde's experiment. If the length of the cord is then adjusted until the nodes are clearly marked, the frequency of the stretched string is the same as of the vibrating rod which is vibrating with the frequency of A. C. mains. Then, if l be the length of one loop of the vibrating string, its frequency of vibration is given by

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \ldots \quad (33.7)$$

and this is also the frequency f, of A. C. mains.

(ii) Capacitance of a Condenser. As depicted in the figure, if the connections are made as at B, A and C, the vibrating rod makes contact between the battery and the condenser during one half of the cycle, thus charging the latter to a voltage V of the battery. During the next half cycle the condenser makes contact with a microammeter and discharges through it. If f be the frequency of A. C. mains, the process is repeated f times per sec. At each discharge, the quantity of electricity passing through the microammeter is given by CV where C is the capacitance of the condenser.

$\therefore$ quantity of electricity flowing per sec. $= C.\ V.\ f$

But $q = i\ t$ and for f times charging per sec., $t = 1$ sec.

$\therefore$ $q = i$

or $q = i = CVf$

$$\therefore \quad C = \frac{i}{Vf} = \frac{\text{Current}}{\text{Voltage} \times \text{frequency}} \quad \ldots \quad (33\cdot8)$$

Thus recording i and V and taking the value of f from the first experiment, and substituting these in equation (33·8), the value of C can be calculated.

Method. (i) A. C. Mains Frequency. Switch on the current and see that the rod of the electrical vibrator begins to vibrate. Adjust the length of rod till it is found that its free end attains the maximum amplitude. Now switch off the current. Tie a fishing cord to the hook at the free end of the rod. Pass the cord over a table-clamp pulley and tie it to a light pan. Place some suitable weights on the pan, say a gm. or two. Switch on the current when the string will be found to vibrate in segments as in Melde's experiment. Alter the length of the vibrating cord by shifting the vibrator backward till the nodes are sharply defined. Mark the positions of the extreme nodes leaving out the first and the last loop. Measure the length between them and divide it by the number of intervening loops to get the value of l. For the same tension take two more sets by altering the length of the cord vibrating in resonance with the steel rod. Calcu-

late the mean value of l for this fixed tension. Weigh the pan in the balance and then compute the total tension applied to the cord. Also weigh in a chemical balance, say 200 cm. of the fishing cord used in the experiment and calculate m, its mass per unit length.

Repeat the experiment with different tensions. Finally calculate the frequency from each set of observations and get the mean value of f.

(ii) Capacitance of a Condenser. Connect a 6 volt battery across the terminals marked B, a microammeter across the terminals marked A and the condenser across C. Untie the cord for this part of the experiment. Adjust the distance of the flat steel discs T, T so that the small iron piece with flat ends (attached to the steel rod) makes contact with them when the vibrator is working. When this has been properly adjusted switch on the current. The condenser is first charged to a voltage V (=E. M. F. of the battery) and then discharges through the microammeter and this process takes place f times per sec. This means a steady current in the microammeter. Take the reading of the microammeter. Also record the E. M. F. of the battery with a suitable voltmeter.

Repeat the experiment by altering the P. D. used to charge the condenser to, say 8 and 10 volts. For each set calculate the value i/V and then with its mean value, calculate the value of the capacitance of the condenser using equation (33.8).

Sources of error and precautions. (1) The length of the steel rod must be initially adjusted so that it vibrates in resonance with A. C. frequency. This is attained when the free end of the rod vibrates with maximum amplitude.

(2) There should be no friction in the pulley for then we cannot be sure of the tension applied to the cord.

(3) The nodes and antinodes on the cord should be sharply defined. The extreme loops should be left out while making measurement for l, as in these cases we cannot be certain of the extreme position of the nodes.

(4) The fishing cord should preferably be employed in this experiment as it possesses a fairly constant mass per unit length.

(5) Zero error of the microammeter, if any, should be taken into account or else the pointer should be initially adjusted on the zero mark.

(6) The contact of the vibrating rod with the flat steel discs T, T should be properly adjusted so that the microammeter shows a steady deflection when the vibrator is working.

Observations. [A] Determination of A. C. Mains frequency.

S. No.	No. of loops	Length of cord	l	Mass of pan	Mass placed on pan	T
1.						
2.						
3.						

Mass of......cm. of the cord = gm.

[B] Determination of Capacitance of condenser.

S. No.	Reading of the microammeter i amp.	Reading of the voltmeter V volt	i/V
1.			
2.			
3.			
		Mean	

Calculations. [A] A. C. mains frequency.

Set I. $f=n=\dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$

$=$

$=$ cycles/sec.

[B] Capacitance of a condenser

$$C=\frac{i}{Vf}$$

$=$

$=$ microfarads

Result. (*i*) The frequency of A. C. Mains = cycles/sec.

(*ii*) The capacitance of the given condenser = microfarads

Experiment 33·3

Object. To determine the power in a circuit without a watt-meter by using three voltmeters in conjunction with non-inductive resistance R and also to determine the power factor of the circuit.

Apparatus. The unknown circuit X (usually an inductive circuit), a known non-inductive resistance, an ammeter, a variac, three voltmeters, an ammeter and connexion wires.

Theory. Refer to fig. 33.7. The circuit X usually an inductance in which power is required is connected in series with a known non-inductive resistance R of appropriate current carrying capacity and the three voltages, V, V_R and V_L are measured. Let cos ϕ be the po-

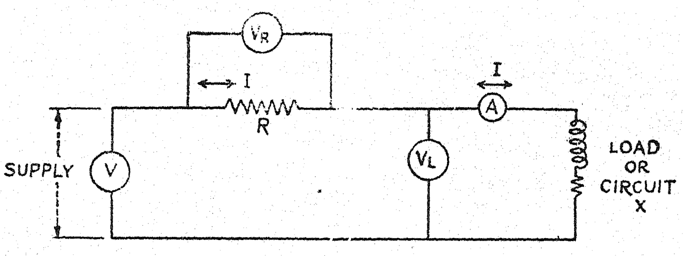


Fig. 33·7

wer factor of the load X. The drop V_R across R is in phase with the current and the vector diagram for the circuit is given below.

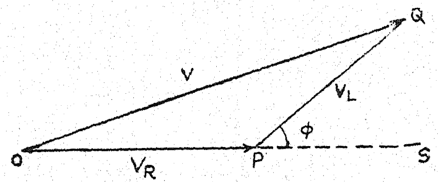


Fig. 33·8

From the vector diagram we have

$$V^2 = V_R^2 + V_L^2 + 2V_R V_L. \cos\phi \qquad (33.9)$$

Neglecting the currents taken by the voltmeters across R and the load, the current in R is the same as the load current I. Hence

$$V_R = IR$$

Substituting for V_R in the equation above

$$V^2 = V_R^2 + V_L^2 + 2\,(IV_L.\ \text{Cos}\ \phi)\,R$$
$$= V_R^2 + V_L^2 + 2\ WR$$

where W ($= IV_L$ Cos ϕ) is the power consumption of the load

X. Hence

$$W = I.\ V_L \cos\phi = \frac{V^2 - (V_R^2 + V_L^2)}{2\ R} \qquad (33.10)$$

From equation(33.9), the power factor is given by

$$\cos\phi = \frac{V^2-(V_R^2+V_L^2)}{2\ V_R\ V_L} \qquad (33.11)$$

Thus from the measurement of the three voltages and the resistance R, W and Cos ϕ can be calculated. Further, if the load current I is measured with an ammeter, the apparent rate of power consumption IV_L can be calculated. If will be found that it differs from the actual rate of power consumption W; Cos ϕ may also be calculated from the relation

$$\cos\psi = \frac{W}{I\ V_L} \qquad (33.12)$$

Method. If the non-inductive resistance R is not given as known, determine its value with the help of a post office box.

Make the connexions as shown in the figure 33·7. Adjust the variac dial on 100 volts and switch on the current. Take the readings of the voltmeters V, V_R and V_L and the ammeter A. Care must be exercised in reading the voltmeters; where the pointer does not stand on a particular graduation of the voltmeter scale, approximation must be made to read the fraction. This has to be done because the voltages appear in the second power in the expressions for W and cos ϕ.

Repeat the experiment by altering the value of V with the variac in steps of 20 volts.

For each set of observation calculate separately the value of W and cos ϕ. Find the mean value of cos ϕ; this gives the power factor of the circuit.

Power consumption will be different for each set, being dependent on the current I. For each set calculate IV_L and plot a graph between W and the corresponding value of IV_L. Calculate cos ϕ from the graph using the relation

$$\cos\phi = \frac{W}{I.V_L}$$

Draw vector diagram for each set of observation as given in figure 33·8 and measure the angle ϕ. Calculate cos ϕ.

Sources of Error and precautions. (1). The method is a very suitable one for the determination of power in coils taking a small current and working at a low power factor. Hence the coil which constitutes the load X must be suitably chosen.

(2) Since the squares of voltages have to be used, it is necessary to use accurate instruments. Due approximation must be made where the pointer does not fall on a graduation of the scale.

The possibility of error will certainly be reduced if only one voltmeter is used, a transfer switch connecting it to the points required as given in the figure below (33.9). Two equivalent resistances U and W,

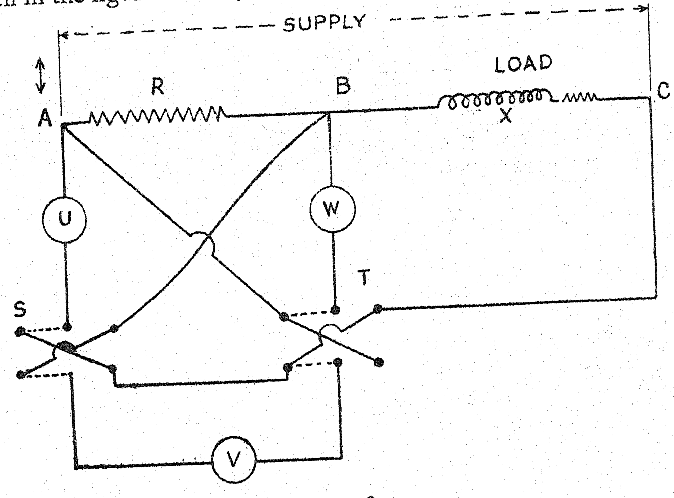


Fig. 33.9

each equal to the resistance of the voltmeter, occupy the places of the missing voltmeters. With both switches S and T thrown to the right, the voltmeter *V* is put across AC and measures the supply voltage V. When both the switches are thrown to the left, the voltmeter is across BC and measures the p. d. V_L across the load X. When the switch S is thrown to the left and the switch T to the right, the voltmeter is across AB and measures the voltage V_R across the non-inductive resistance R only. The resistances U and W are simultaneously transferred to the positions not occupied by the voltmeter.

(3) It is desirable that V_R and V_L should be as nearly equal as possible, hence R must be reasonably chosen to satisfy this condition.

Observations. Least Count of the voltmeter V = volt.
Least Count of the voltmeter V_R = volt.
Least Count of the voltmeter V_L = volt.
Least Count of the ammeter A = amp.
Non-inductive resistance R = Ohm.

S. N.	Ammeter, Reading, I amps.	P. D. across Both L and R V volt	R V_R Volt	L V_L volt	W True watts	Cos ϕ	IV_L Apparent watts	$\frac{W}{IV_L}$
1								
2								
3								
4								
5								
					Mean		Mean	

Calculations. For the first set

$$W = \frac{V^2 - (V_R^2 + V_L^2)}{2\ R}$$

$$= \qquad\qquad = \text{ watts}$$

and

$$\cos\phi = \frac{V^2 - (V_R^2 + V_L^2)}{2\ V_r V_L} =$$

Similar calculations for other sets.

Also, apparent power consumed (for the first set)

$$= I\ V_L$$

$$= \qquad \text{apparent walts}$$

Actual power consumed (for the first set)

$$= W = \text{true walts}$$

Hence from the first set,

$$\cos\phi = \frac{W}{I\ V_L} = \qquad =$$

Similar calculations for other sets.

Also, from the graph $\cos\phi$ =

Also, from the vector diagrams for each set find $\cos\phi$.

Result. (i) The power consumption of the load X at the current .... amp. = watts.

(ii) Power factor, $\cos\phi$ =

Exercise. To determine the power factor by three ammeter method.

Make the connexions as shown in fig. 33·10(a).

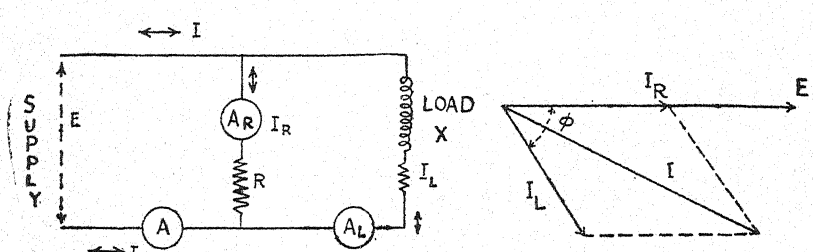


Fig. 33·10 (a) and (b)

Fig. 33·10 (b) represents the current vectors I, I_R and I_L and the supply voltage E vector. The current measured by the ammeter A is the vector sum of the load current I_L and I_R that taken by the non-inductive resistance R, this being in phase with the applied voltage E. From the vector diagram.

$$I^2 = I_R^2 + I_L^2 + 2I_R . I_L . \cos\phi \qquad (33.13)$$

But $$I_R = \frac{E}{R}$$

$$\therefore \quad I^2 = I_R^2 + I_L^2 + 2.\frac{E}{R} I_L . \cos\phi$$

$$= I_R^2 + I_L^2 + 2.\frac{W}{R}$$

where $$W = E.I_L^2 \cos\phi = \left[\frac{I^2 - (I_R^2 + I_L^2)}{2}\right] R \qquad (33.14)$$

Also from equation (33.13)

$$\cos\phi = \frac{I^2 - (I_R^2 + I_L^2)}{2\, I_R \;\; I_L} \qquad (33.15)$$

whence from equations (33.14) and (33.15) W and cos ϕ can be calculated from the observations of I, I_R, I_L and the known value of R.

Experiment 33·4

Object. To study a series resonant a. c. circuit.

Apparatus. A variac to give 110 volts a. c. or less, an inductance coil of an inductance a few henry, capacity box with a working voltage of 500 volts, one kilo-ohm resistor, a vacuum tube voltmeter and connexion wires.

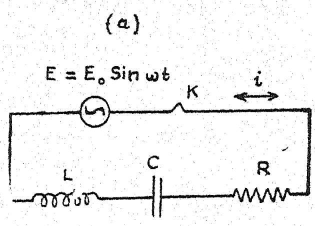


Fig. 33.11

Theory. Consider the circuit depicted in fig. 33.11. It consists of an inductance L, capacitance C and resistance R and the supply voltage is $E = E_0 \sin \omega t = E_0 \sin 2\pi nt$ where n is the frequency of the a. c. supply, usually 50 c. p. s. The current in the circuit at any instant is given by

$$i = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \sin\left\{\omega t - \tan^{-1}\frac{\left(L\omega - \frac{1}{C\omega}\right)}{R}\right\}$$

The peak value of the current is given by

$$i_0 = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}}$$

and hence the r. m. s. value E and I are related by the equation

$$I = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

and the angle of lag is given by

$$\tan \alpha = \frac{\left(L\omega - \frac{1}{C\omega}\right)}{R} = \frac{\text{Reactance}}{\text{Resistance}}$$

For the circuit to be resonant

$$L\omega - \frac{1}{C\omega} = 0$$

or

$$\omega = 2\pi n = \frac{1}{\sqrt{LC}}$$

For the circuit to be resonant for an inductance value of, say, ten henry, at 50 c. p. s., the capacitance C should be of the order of 1 μ F (microfarad).

If I_1, be the current in the circuit at resonance,

P. D. across inductance (V_L) $= L\omega I_1$

P. D. across capacitor (V_C) $= \frac{I_1}{C\omega}$

and that $L\omega I_1 = \frac{I_1}{C\omega}$

i.e., $V_L = V_C$

Hence to study the condition of resonance, we can take an inductance of a fixed value L and a variable capacitance C. Observations of V_L and V_C are made by varying C, keeping R and E constant. For values of C less than that for resonance condition

$$\frac{I}{\omega C} > L\omega I$$

or

$$V_C > V_L$$

As the capacitance is increased in the circuit V_C goes on diminishing while V_L goes on increasing. At resonance

$$V_C = V_L$$

After the condition of resonance has been reached, if the capacitance is further increased in the circuit,

$$V_C < V_L$$

Hence by plotting graphs between capacitance values along X-axis and the corresponding values of V_C and V_L along Y-axis, we can see how V_C and V_L vary with the value of C. The point of intersection of the two curves is the point where $V_C = V_L$ and hence the

condition of resonance. The value of C corresponding to this point of intersection of the two graphs (Fig. 33·11) gives the capacitance at which the circuit becomes resonant.

Further, at resonance, the p. d. across the resistance

$$V_R = RI_1$$

should be a maximum. Hence the measure of p. d. across R may be simultaneously made. It will be found to increase with the increasing value of C in the initial stages, then reach a maximum, after which further increase of C decreases the value of V_R in the circuit. Hence a plot of a graph between p. d. across resistance V_R against the capacitance values C can also be used to study for what value of C the given circuit of L and R is resonant. From the graph the value of C corresponding to the maximum value of V_R can be determined.

Further, at resonance, the p. d. across both L and C combined, V_{CL} should be theoretically zero. This should be so if the choke coil is of negligible resistance and there are no other losses. Practically V_{CL} is never zero at resonance and its value is a measure of the effective resistance of the choke. The term effective is used to indicate that the resistance is not just the d. c. resistance of the winding but also includes power losses in the iron due to hysterisis.

Hence if V_{CL} is also measured simultaneously for varying values of C, V_{CL} will be found to vary in a manner opposite to that of V_R. At resonance V_R is a maximum while V_{CL} is a minimum. From the minimum value of V_{CL} on the (V_{CL}, C) graph, the value of C can be computed that makes the given circuit, with constant values of L and R, resonant at the supply frequency.

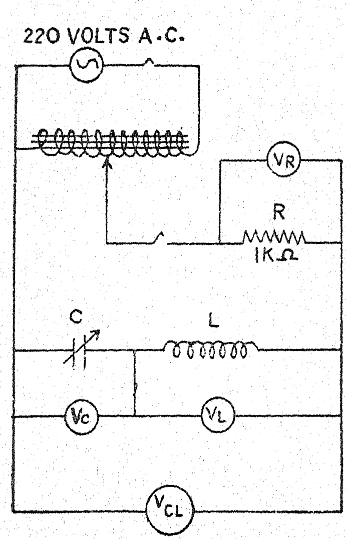


Fig. 33·12

Method. Make the connexions as shown in fig. 33.12. If four voltmeters are available of suitable ranges, they may be connected across R, across C, across L and across both C and L to record V_R, V_C, V_L and V_{CL} respectively. If a V. T. V. M. is available, measure the above voltages by connecting it across the requisite points in succession. C is a dial condenser box having capacitances in hundredths, tenths and units of a microfarad. R is a resistance of 1 kilo-ohm and L an inductance of about 10 henry.

Introduce a capacitance of say 0·4 microfarad in the condenser box and switch on the supply. Measure V_C, V_L, V_{CL} and V_R with the help of the vacuum tube voltmeter.

Repeat the observations by altering the capacity in regular steps of 0·1 microfarad.

Plotting of Graphs. 1. On the same sheet of graph and with the same scale, draw the graphs (V_C, C) and (V_L, C). Find the co-ordinates of the point of intersection of the two graphs. The value of C for this point is the one for which the circuit with the fixed values of L and R will be in resonance at the frequency of the supply. This is reproduced below from the data given in the observation table.

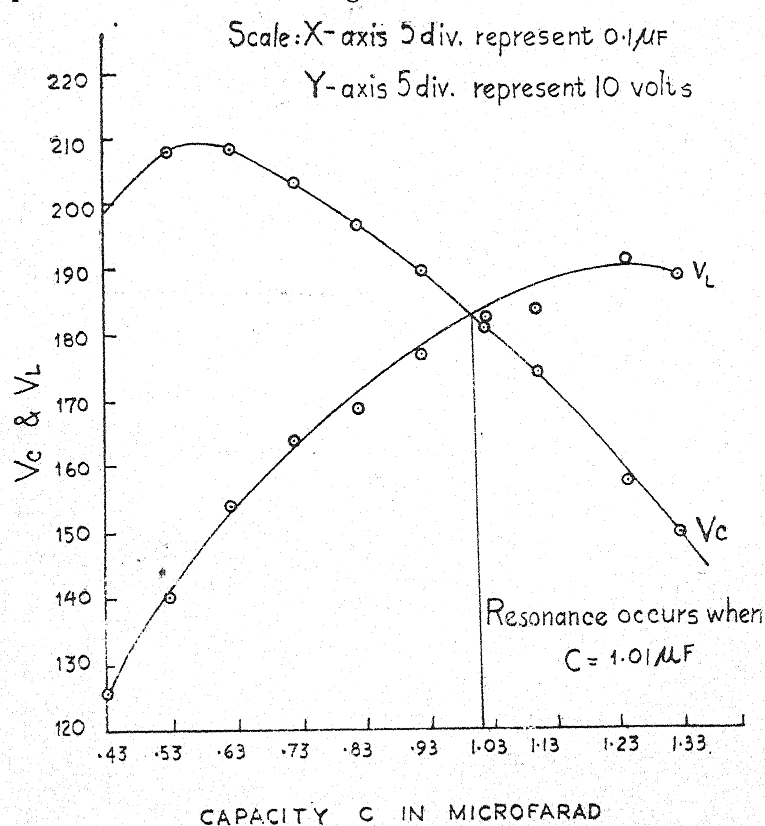


Fig. 33·13

2. Plot graphs with V_R and V_{CL} along Y-axis and C along the axis of X. Find the value of C corresponding to the maximum value of V_R from the (V_R, C) graph.

Also find the value of C corresponding to the minimum value of V_{CL} from the (V_{CL}, C) graph. The values of C so obtained from these two graphs, (V_R, C) and (V_{CL}, C) will be found to be very nearly the same. These graphs therefore also give the value of C which will make the circuit resonant at the frequency of the supply.

Compare this value of C with the one obtained from the coordinates of the point of intersection of (V_C, C) and (V_L, C) graphs. These values will be found to be approximately the same; the difference may be due to experimental errors.

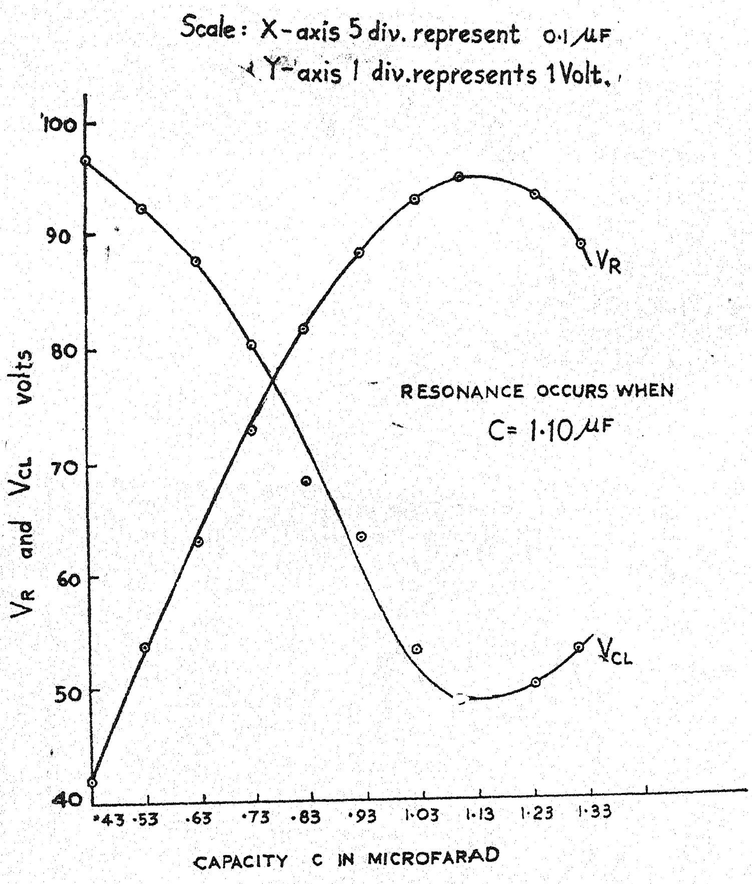


Fig. 33·14

Sources of error and precautions. 1. The vacuum tube voltmeter should be very carefully handled. The zero adjustment should be carefully made before starting the observations.

2. With the help of the variac adjust the p. d. across the L, C, R circuit to be about 110 volts or less.

3. V_R, the p. d. across R and V_{CL}, the p. d. across both L and C will not exceed the applied p. d. of 110 volts. These can be measured with the V. T. V. M. by adjusting the selector switch at × 3 V so that the full scale reading is 150 volts. The p.d. across C and that across L can exceed the supply voltage of 110 volts from the variac and

as such due care must be taken in their measurement by having the selector switch at × 10 V so that the full scale voltage is 500 volts. Only when the p. d. comes out to be less than 150 volts, the selector switch should be turned on at × 3 V and then the reading taken.

4. Observations may be taken by varying C in steps of 0·1 μF or 0·05 μF.

5. The graphs must be smoothly drawn, choosing a convenient scale for the values of p. d. and capacitance.

Observations. (Here we reproduce one set of observations)

R = 1 kilo-ohm

L = 10 Henry approx.

Supply voltage = 105 volts 50 c. p. s.

Table—For C, V_R, V_L, V_C, V_{CL}

S. No.	C in μ F	V_C volts	V_L volts	V_{CL} volts	V_R volts
1	0.43	200	126	97.5	42
2	0.53	210	141	93.0	54
3	0.63	210	155	88.5	63.5
4	0.73	205	165	81	73.5
5	0.83	198	170	69	82.5
6	0.93	191	178	64	89
7	1.03	182	183	54	94
8	1.10	175	185	49.5	96
9	1.25	158	192	51	94.5
10	1.33	150	190	54	90

Calculations. (i) From the graphs (V_C, C) and (V_L, C), the point where $V_C = V_L$

gives C = microfarad.

(ii) From the graph (V_R, C) the point where V_R is a maximum gives C = microfarad

(iii) From the graph (V_{CL}, C) the point where V_{CL} is a minimum gives C = microfarad.

Hence mean value of C = microfarad.

Result. The circuit with the given values of L and R becomes resonant when a capacitor of value.........microfarad is put in series.

At resonance, (i) $V_C = V_L$

(ii) V_R is a maximum

and (iii) V_{CL} is a minimum.

33·4 **Inductive Circuits in Series.** When two inductive circuits are joined in series, the same current must flow through them both. But, in general, the e. m. f. over one will not be in phase with

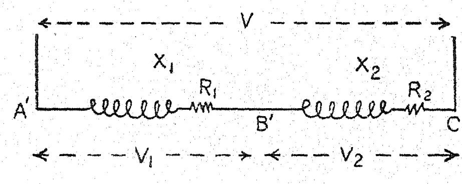


Fig. 33·15

that over the other and as such the total e. m. f. required to maintain the current in the circuit will be less than the sum of the two e. m. fs.

The circuit is shown in fig. 33·15. The supply p. d. is the vector sum of the p. ds. across the two coils, and each of these consists of two components, one overcoming the resistance, and the other equal and opposite to the reactance e. m. f. The component IR_1 overcoming the resistance in the first coil is in phase with the current vector while the component IX_1, equal and opposite to the reactance e. m. f. is 90° ahead of the current vector. Usually it is more convenient to use the vector triangle rather than the parallelogram and hence IX_1 is set up,

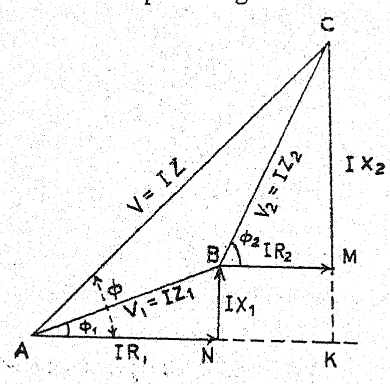


Fig 33·16

not at A, but at the end of IR_1 (fig. 33·16). The resultant of these $V_1 = IZ_1$ is the p. d. across the first coil and leads the current by an angle ϕ_1. V_1 is evidently the reading recorded by a voltmeter connected across A'B'.

Similarly, the p. d. triangle for the second coil B'C' is shown by BMC, which is drawn in the position shown because the point B in each triangle represents the one point B' between the two parts of the circuit and as such it should occupy only one position in the diagram. At B, IR_2 has been set off horizontally and at its extremity IX_2 is drawn

vertically, the resultant being $BC = V_2 = IZ_2$, the p. d. across the second coil as measured by a voltmeter. This p. d. leads the current by a larger angle ϕ_2. The p. d. across the two coils in series, *i.e.*, the supply voltage is

$$V = IZ = \overrightarrow{AC},$$

the vector sum of V_1 and V_2 leading the current by an angle ϕ intermediate between the angles ϕ_1 and ϕ_2 in each part of the circuit considered separately.

It is evident from the diagram that

$$\begin{aligned} V^2 &= (IZ)^2 = AC^2 = CK^2 + AK^2 \\ &= (CM + MK)^2 + (AN + NK)^2 \\ &= (IX_2 + IX_1)^2 + (IR_1 + IR_2)^2 \\ \text{or} \quad Z^2 &= (X_1 + X_2)^2 + (R_1 + R_2)^2 \\ \text{and } \tan\phi &= \frac{CK}{AK} = \frac{X_1 + X_2}{R_1 + R_2} \end{aligned}$$

The total resistance in the circuit is the sum of the resistances of each part, and the total inductance is the sum of the separate inductances. Of course, the two parts are supposed to be far enough apart to avoid mutual induction between them.

33·5. Inductive Circuits in parallel. The problem of a divided circuit, having two inductances in parallel, can be more conveniently solved graphically. Fig. 33·17 shows the circuit.

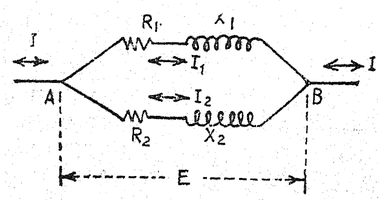


Fig. 33·17

As each branch has the same impressed e. m. f. E, the hypotenuse of each triangle will be identical. Let this be laid off to scale as AB. Since each triangle is right angled it can be inscribed within a semicircle with AB as diameter. From the constants of the circuit, the angle

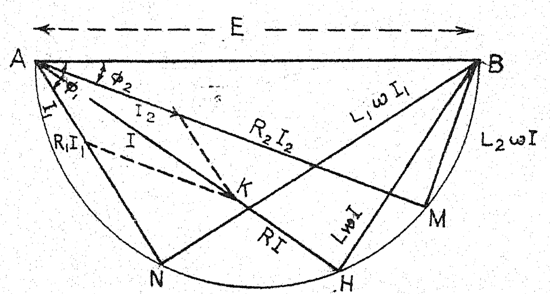


Fig. 33·18

of lag ϕ_1 and ϕ_2 in each branch can be calculated. The base AN of the first triangle can be laid off by making an angle ϕ_1 with BA. Join NB. The value of the current $I_1 = AN/R_1$ and is laid in the direction AN.

In the same manner the triangle AMB is laid off for the second circuit R_2L_2. The value of the current is determined by the relation $I_2 = AM/R_2$ and laid along AM. The current I in the circuit is the vector sum of I_1 and I_2 and is given by the vector AK. Produce AK to cut the semi-circle at H. Then if R and L be the equivalent resistance and inductance of the divided circuit, these can be computed from the relation

$$RI = AH$$

and

$$L\omega I = BH,$$

triangle AHB representing the equivalent circuit.

33·6. Condenser Circuits in Series. Fig. (33·19) represents two

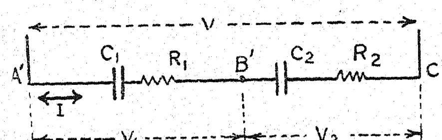


Fig. 33·19

capacitance circuits in series. The supply p. d. is the vector sum of the p. ds. across the two parts of the circuit, and each of these consists of two components, one overcoming the resistance, and the other equal and opposite to the reactance e. m. f. The component IR_1, overcoming the resistance in the first part of the circuit, is in phase with the current vector while the component IX_1 ($= I/C_1\omega$) equal and opposite to the reactance e. m. f. is 90° behind the current vector. This gives the vector triangle ABN.

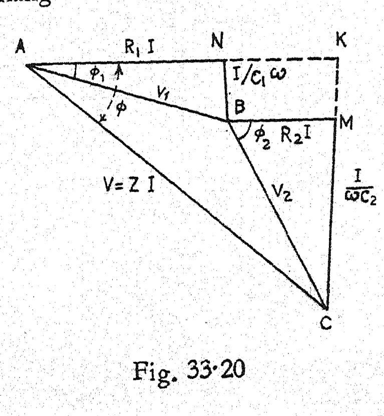


Fig. 33·20

Similarly, for the second part of the circuit we have the vector triangle BMC.

Evidently

$$V = ZI = AK^2 + CK^2$$
$$= (R_1 I + R_2 I)^2 + \left(\frac{I}{\omega C_1} + \frac{I}{\omega C_2}\right)^2$$

or

$$Z^2 = (R_1 + R_2)^2 + \left(\frac{1}{\omega C_1} + \frac{1}{\omega C_2}\right)^2$$

and

$$\tan\phi = \frac{\frac{1}{\omega C_1} + \frac{1}{\omega C}}{R_1 + R_2}$$

Hence it is evident that the total resistance of the circuit is the sum of the resistances of each part and that the total capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

33·7. **Condenser Circuits in parallel.** The figure (33·21 a) represents two capacitance circuits in parallel. The graphical solution for such a circuit is easily constructed. As the two circuits are in parallel, each one is subjected to the same applied e. m. f. V. Let V be laid off to scale along a straight line AB. At A construct the triangle AKT by laying off AK $= R_1$ along AB and KT $= 1/\omega C_1$ perpendicular to AKB at the point K. Both are drawn to scale. Join AT and produce it so as to cut at C the semi-circle drawn on AB as diameter.

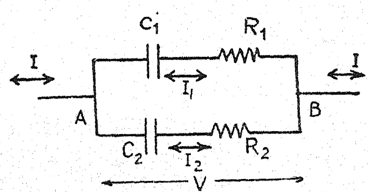


Fig. 33·21 a

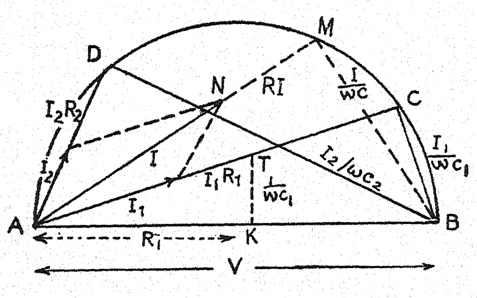


Fig. 33·21 b

ACB is then the right angled triangle representing the p. ds. in the first circuit of R_1 C_1 (fig. 33·21 b).

The current I_1 in this branch will be in phase with AC, the active component, V_{R_1} = AC = $R_1 I_1$, of the applied voltage V. This current can be shown on this diagram as I_1 laid off to an appropriate scale along AC.

In the same manner the p. d. triangle for the other circuit $C_2 R_2$ can be drawn within the same circle with the same scale. This is shown by ADB. Here the current in this branch is shown by I_2.

The equivalent circuit for this combination can be easily determined. The current through such an equivalent circuit is I given by the vector sum of I_1 and I_2 and this is represented by the diagonal of the parallelogram by AN. Produce AN to meet the semi-circle at M. AM gives the active component, V_R = AM, of the applied voltage V. This value of V_R is obtained by measuring AM using the same scale as used in laying off AB. In the same manner V_X = MB gives the value of the reactive component of the e. m. f. for the equivalent circuit.

The impedance triangle can be constructed from the p. d. triangle AMB of the equivalent circuit. Dividing the sides of the p. d. triangle AMB by the value of the resultant current I_1 gives the corresponding impedance triangle, the sides of which give the resistance and the reactance of a single circuit that is equivalent to the two given circuits in parallel.

33·8. **Comparison of Capacities—A. C. Bridge Method.** The arrangement is depicted in fig. 33·22 where R_1 and R_2 are non-inductive resistances and C_1 and C_2 are two capacitances, one of which is known and the other is to be determined. The output of the audio-frequency oscillator set at a frequency of 1000 c. p. s. is fed across AE. Between B and D is a headphone. Silence in the head-phone indicates that the current through it is zero. A sound in the headphone does not indicate whether R_2 is too large or too small for a balance, but R_2 can be varied in the direction that gives less sound until a minimum sound is heard. By varying R_2 back and forth from a value that is slightly too small to a value on the other side of the balance point that is just large enough to give the same loudness of sound as the smaller value, and decreasing this range in R_2 until the sounds are just inaudible, the balance point can be located.

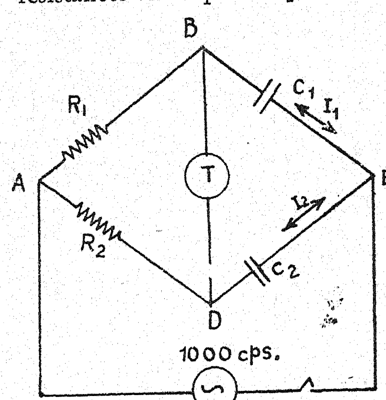


Fig. 33·22

The condition for balance is evidently

$$\frac{R_1}{R_2} = \frac{-\dfrac{j}{\omega C_1}}{-\dfrac{j}{\omega C_2}}$$

or $$\frac{R_1}{R_2} = \frac{C_2}{C_1}$$

whence $$C_1 = C_2 \frac{R_2}{R_1} \qquad (33.16)$$

This relationship can be conveniently established by a careful study of the corresponding vector diagram. Using the method for drawing the vector diagram for such a circuit as discussed in Art. 33·7, we find that the vector △ ABE for the circuit R_1 C_1, and the triangle ADE for the circuit R_2B_2. Between B and D is connected the head-phone T. (Fig. 33·23). Since B and D are some distance apart as shown in this diagram, it means a corresponding voltage is applied to

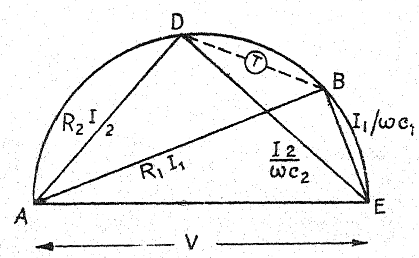


Fig. 33·23

the headphone circuit. By increasing R_2 the point D will be moved over the semi-circle towards B or by decreasing R_1, B will be moved towards D. For proper values of R_1 and R_2, D and B can be made to coincide when there would be no current in the head-phone. The bridge is then balanced. In such a case △s ABE and ADE have been made similar and hence

$$\frac{R_1 I_1}{R_2 I_2} = \frac{I_1/\omega C_1}{I_2/\omega C_2}$$

or $$C_1 = C_2 \, . \, \frac{R_2}{R_1}$$

From the figure it is apparent that a slight change in R_2 will change the position of D by a greater amount when AD and DE are equal than when they are widely different. Hence for greatest sensitiveness R_2 should be nearly equal to $1/\omega C_2$.

In this discussion it has been assumed that the condenser arms of the bridge contain capacitance only. If either condenser shows

residual charge, it will not be possible to obtain complete silence in the headphone. The nearest balance will be a minimum of sound. For a better balance it is necessary to use a modification of this—*The Wien Capacitance Bridge*.

33·9. **Power factor of a Condenser.** In practice every condenser dissipates some power when an alternating p. d. is applied across it. To account for this power loss, a condenser may be regarded as made up of a capacitance associated with a resistance—either a small resistance in series or a large resistance in parallel. Fig. 33·24 shows

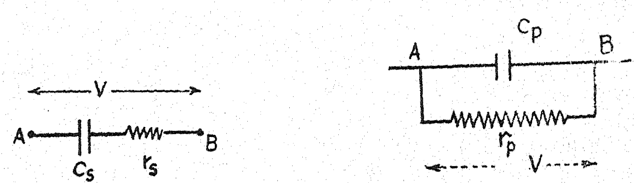


Fig. 33·24

these two modes of representation of a condenser, the essential condition being that for the same applied voltage the current through the combination must be identical both in magnitude and phase in both the cases. This would be so when the vector impedances of the two alternative modes of representation are identical. We thus have

$$r_s+\frac{1}{j\omega C_s}=\frac{r_p/j\,\omega C_p}{r_p+j\omega C_p}$$

which gives

$$\frac{r_s\,j\,\omega C_s+1}{j\,\omega C_s}=\frac{r_p}{r_p\,j\,\omega C_p+1}$$

$$\therefore \qquad (-r_s r_p\omega^2 C_s C_p+1)+j\,(r_p\omega C_p+r_s\omega C_s-r_p\omega C_s)=0$$

Whence we have

$$r_s r_p\omega^2 C_s C_p=1 \qquad (33.17)$$

and $$r_p\omega C_p+r_s\omega C_s-r_p\omega C_s=0$$

or $$r_p(C_s-C_p)=r_s C_s \qquad (33.18)$$

Dividing (33.18) by (33.17)

$$\frac{C_s-C_p}{r_s\omega^2 C_s C_p}=r_s C_s$$

$$\therefore \qquad C_s-C_p=(r_s\omega C_s)^2\,.\,C_p$$

$$\therefore \qquad C_p=\frac{C_s}{1+(r_s\omega C_s)^2} \qquad (33.19)$$

This expresses C_p in terms of C_s, r_s and ω.

Again from (33.17) we have

$$r_p = \frac{\{1+(r_s\omega C_s)^2\}}{r_s\omega^2 C_s \times C_s}$$

$$= \frac{r_s\{1+(r_s\omega C_s)^2\}}{(r_s\omega C_s)^2}$$

$$= r_s\left\{1+\frac{1}{(r_s\omega C}\right\} \qquad (33{\cdot}20)$$

This equation expresses r_p in terms of r_s, C_s and ω.

The factor $r_s C_s \omega$ is called the *power factor of the condenser.*

In the case of mica condensess, the power factor is extremely, small, say 0.0001, hence C_s and C_p differ by a negligible amount whereas r_p is approximately $\frac{1}{r_s\omega^2 C_s^2}$. These values of C_s, C, r_s and r_p depend upon the frequency. In the case of ordinary condensers using paraffin wax as dielectric, the power factor is of the order of 0.03 but even then the difference between C_s and C_p though sensible is not large.

Experiment 33·5

Object. To determine the capacitance of a condenser with Wien's series resistance bridge for capacity measurement.

Apparatus. Two four-dial resistance boxes with the decimal ohm dial, one decimal ohm dial box having units and tenths of an ohm, audio-oscillator, headphone, a condenser of known capacity, the experimental condenser, a key and connexion wires.

Theory. *Wien's Series resistance Bridge.* The bridge connexions are shown in fig. 33·25. R_1 and R_2 are pure ohmic resistances. C_2 is a standard mica condenser in series with a variable standard resistance r_2. In the arm BE is a paraffin paper condenser which will be regarded as equivalent to a perfect condenser of capacitance C_1 with a resistance r_1 in series with it. Headphone is placed across BD. The equivalent series resistance of C_2 will be considered negligible. The output of an audio-frequency oscillator of frequency 1000 c. p. s. is connected across AE through a key or a switch K.

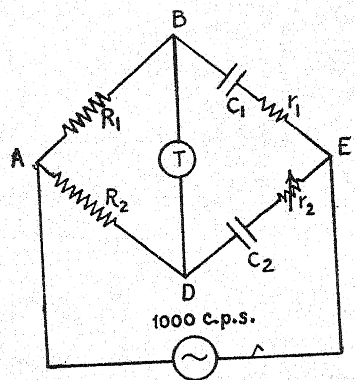


Fig. 33·25

When the bridge is balanced, *i.e.*, there is silence in the head-phones, the condition for the balance is

$$\frac{R_1}{R_2}=\frac{r_1+\dfrac{1}{j\omega C_1}}{r_2+\dfrac{1}{j\omega C_2}}$$

or

$$\frac{R_1}{R_2}=\frac{(r_1 j\omega C_1+1)\,j\omega C_2}{(r_2 j\omega C_2+1)\,j\omega C_1}$$

$$=\frac{r_1 j\,\omega C_1C_2+C_2}{r_2 j\,\omega C_1C_2+C_1}$$

whence

$$\frac{R_1}{R_2}=\frac{r_1}{r_2} \tag{33.21}$$

and

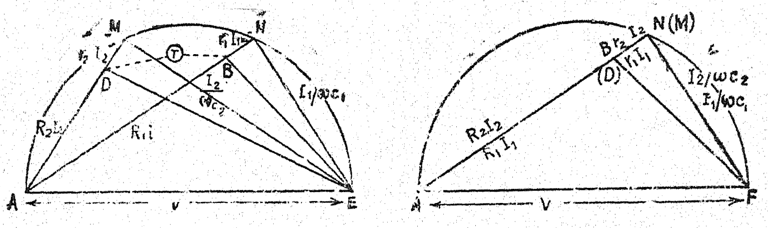
$$\frac{R_1}{R_2}=\frac{C_2}{C_1} \tag{33.22}$$

Equations (33.21) and (33.22) represent the two conditions of balance These may be satisfied by varying R_2 and r_2 alternately. Thus when the bridge is balanced, we have from equation (33.22)

$$C_1=\frac{R_2}{R_1}\times C_2 \tag{33.23}$$

whence C_1 can be calculated.

The vector diagram for the bridge may now be studied to throw additional light on the adjustments to be made for balancing the bridge. The vector diagram has been drawn in accordance with the method dealt with in the text earlier.



(a) Unbalanced Bridge (b) Balanced Bridge

Fig. 33·26 (a) and (b)

The vector diagram for the condenser circuits C_1r_1 and C_2r_2 are the triangles NBE and MDE and the telephone is connected across BD. For balancing the bridge it is not suffi. ient that M should coincide with N which can be adjusted by altering R_2 but it is also necessary that B should coincide with D which can be achieved by r_2. Hence

both R_2 and r_2 are to be adjusted alternately by trial for balancing the bridge. When the balance is achieved the vector diagrams for the bridge appear as in fig. 33·26 (b). There are two sets of similar triangles in this figure, from which we have

$$R_1I_1 = R_2I_2 = r_1I_1 : r_2I_2 = \frac{I_1}{\omega C_1} : \frac{I_2}{\omega C_2}$$

giving as before, $\dfrac{R_1}{R_2} = \dfrac{r_1}{r_2}$

and $\dfrac{C_1}{C_2} = \dfrac{R_2}{R_1}$

whence $C_1 = \dfrac{R_2}{R_1} \cdot C_2$

Evidently r_2 must be set equal to $r_1 \dfrac{C_1}{C_2}$ in order to find the value of R_2 that gives silence in the headphone.

Method. Make the connexions as shown in fig. 33·25. For R_1 and R_2 use two four-dial resistance boxes giving any resistance from 0·1 ohm to 1000 ohm. For r_2 use a decimal ohm box having two dials—units and tenths of an ohm. C_2 is a standard mica condenser whose equivalent series resistance can be considered negligible. The experimental condenser is an ordinary condenser using paraffin wax as dielectric and is connected in the arm BE. The out-put of an audio-frequency oscillator is fed across AE and the frequency is set on its dial at 1000 c. p. s.

Adjust the resistance R_1 in the arm AB to 100 ohms. It is desirable that R_1 should be nearly equal to the impedance in the branch BE.

Press the key K and put the head-phone on the ears and listen to the sound. Adjust R_2 by changing the resistance in the dial box in the arm AD so that the sound in the head-phone decreases. Now adjust r_2 to minimise the sound in the head-phone. After a few trials the bridge can be balanced. Note the values of R_2 and r_2.

Take more sets by altering R_1 and finding the corresponding values of R_2 and r_2.

Tabulate your observations as shown in the observation table. Calculate C_1 from each set and find its mean value. Also from each set calculate r_1 which equals $\dfrac{r_2C_2}{C_1}$, and find its mean value. From a knowledge of ω, C_1 and r_1, calculate the power factor $\omega C_1 r_1$.

Sources of Error and precautions. (1) For the determination of C_1 it is not necessary that R_2 and r_2 should be specifically non-

inductive; but for the determination of the power factor it is essential to use non-inductive resistances.

(2) It is important to record the frequency of the oscillator, usually 1000 c. p. s., at which measurements are made since both the capacity C_1 and the resistance r_1 of the condenser are dependent on requency.

(3) C_2, the standard condenser, must be chosen of a capacity nearly equal to that of C_1 and R_1 should be set nearly equal to the impedance of C_1 branch of the bridge. The reason for this is that the bridge is most sensitive when the impedances of all the arms are equal.

(4) The headphone employed should preferably have an impedance of the same order as that uf the other branches of the bridge.

(5) In obtaining the balance position of the bridge for a minimum of sound in the head-phone, a number of separate attempts should be made. These will vary from one another by slight amounts, and then the arithmetic mean of the results so obtained should be taken.

Observations. Capacity C_2 of the standard condenser = μ F

Frequency at which measurements are made = c.p.s.

Set No.	R_1	R_2	Mean R_2	r_2	Mean r_2	Calculated Capacity C_1	Calculated resistance r_1
1		(1) (2) (3)		(1) (2) (3)			
2		(1) (2) (3)		(1) (2) (3)			
3		(1) (2) (3)		(1) (2) (3)			
					Mean		

Calculations. From set 1

$$C_1 = \frac{R_2}{R_1} \times C_2 = \qquad = \mu F$$

Similar calculations from other sets.

$\therefore$ Mean value of $C_1 = \mu F$

From set 1, $r_1 = r_2 \dfrac{C_2}{C_1}$ or $= r_2 \dfrac{R_1}{F_2} = \qquad =$ ohm

....

$\therefore$ Mean value of $r_1 =$ ohm

Also $f = 1000$ c. p. s.

$\omega = 2\pi f =$

$\therefore$ Power factor of the condenser $C_1 = r_1 \omega C_1$

$=$

$=$

Result. The capacity of the unknown condenser $=$ μF
Power factor of the condenser $=$

Experiment 33·6

Object. To determine the self-inductance of a given coil by Maxwell's Inductance Bridge.

Apparatus. Two four-dial resistance boxes (units to thousands), one three dial resistance box (units, tenths and hundredths), unknown inductance, a known standard self-inductance preferably having a value as close to the unknown as possible, audio-frequency oscillator, head-phone, a key or switch and connection wires.

Theory. Refer to fig. 33·27. R_3 and R_4 are the resistances in the arms AB and BE which are provided with two four dial resistance boxes with non-inductive windings. In the arm AD lies the unknown inductance L_1 of resistance R_1 while in the arm DE is the known standard inductance L_2 having resistance R_2 along with a non-inductive variable resistance R. Across BD is the head-phone while across AE is fed the output of the audio frequency oscillator set usually at a frequency of 1000 c. p. s.

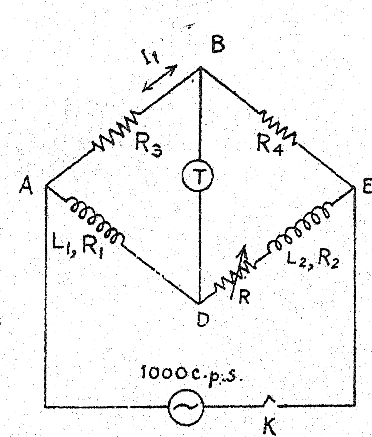


Fig. 33·27

The condition for the balance of the bridge is

$$\frac{R_2}{R_1 + jL_1\omega} = \frac{R_4}{R + R_2) + jL_2\omega}$$

or $R_3 (R + R_2) + jR_3L_2\omega = R_1R_4 + jL_1R_4\omega$

whence $R_3 (R + R_2) = R_1R_4$

or
$$\frac{R_3}{R_4} = \frac{R_1}{(R + R_2)} \tag{33.24}$$

Also $R_3L_2\omega = R_4L_1\omega$

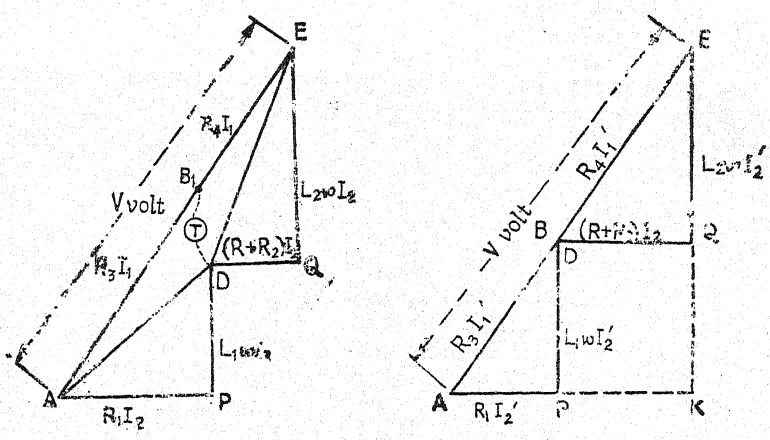
or
$$\frac{L_1}{L_2} = \frac{R_3}{_2} \tag{33.25}$$

or
$$L_1 = L_2 . \frac{R_3}{R_4} \tag{33.26}$$

from which L_1 can be calculated from a knowledge of L_2, R_3 and R .

When the conditions embodied in equations (33.24) and (33.25) are satisfied, the sound in the headphone will be a minimum. Balance may be approached by alternate adjustment of R_4 and R. The two conditions of balance are then semi-independent for while a change of R_4 affects condition embodied in equation (33.24), a change in R does not affect the condition embodied in equation (33.25). If for L_2 we have an inductometer so that L_2 is a known variable inductance, with constant value of R_2, the two adjustments are quite independent and the exact balance is conveniently obtained by alternate adjustment of R_4 and L_2.

The vector diagram for the unbalanced bridge is of great help in understanding the adjustments to be made for obtaining the balance. The graphical representation of the inductive branch ADE is shown by two e.m.f. triangles APD and DQE (fig. 33·28). The corresponding



(a) Bridge in balanced (b) Bridge balanced

Fig. 33·28 (a) and (b)

diagram for the non-inductive circuit ABE is a straight line commencing at A and ending at E, since this circuit is also subjected to the same impressed voltage V. The head-phone is connected across BD. For balancing the bridge there should be no current in the headphone. This would happen if B and D coincide (fig. 33·28 b). This could be achieved by (i) altering R_4 so that B lies opposite D and (ii) then altering R so that D approaches B. With fixed inductances L_1 and L_2, the only possible adjustments for a known value of R_3 are the values of R_4 and R. With an inductometer, adjustments

of R_4 and L_2 would have attained the balance conveniently. It is also apparent that if B and D are to coincide at the middle point of AE, the four arms of the bridge must have equal impedances.

From the figure for the balanced bridge, we have from similar triangles

$$\frac{L_1}{L_2} = \frac{R_1}{R + R_2} = \frac{R_3}{R_4}$$

the conditions already deduced earlier.

Method. Make the connexions as given in figure 33·27. In the dial resistance box in the arm AB, introduce a suitable resistance R_3, of the same order as the impedance in the arm DE. In the dial resistance box in the arm BE introduce the resistance R_4 of the same order as R_3. Switch on the audio frequency oscillator set at 1000 c.p.s. and listen to the sound in the head-phone. Adjust R_4 and R alternately and obtain the balance when the sound in the head-phone is a minimum. Repeat this observation twice.

Alter the value of R_3 and repeat the experiment. From each set calculate the value of L_1 and then find its mean value.

Sources of Error and Precautions. (1) The resistances R_3 and R_4 should be of the same order of magnitude and equal to the order of magnitude of the impedance in either inductive arm of the bridge. This is to ensure that the bridge is in the sensitive condition and as such can be accurately balanced.

(2) In obtaning the balance position of the bridge for a mindmum of sound in the headphone, a number of separate attempts shouls be made. These will vary from one another by slight amounet and then the arithmetic mean of the results so obtained should be taken.

(3) Referring to equation (33.24) we have

$$R_1 = \frac{R_3 (R + R_2)}{R_4}$$

$$= \frac{R_3}{R_4} R2 + \frac{R_3 R}{R_4}$$

If R_1 is less than $\frac{R_3}{R_4} R_2$, R will have to be negative. When such is the case R will have to be placed in the same arm as L_1 and not in the arm in which we have L_2.

To avoid this difficulty it is desirable to have a decimal ohm box in each of the arms AD and DE and use both or one of them whichever is necessary.

(4) For different sets of observation, variation in R_3 should not be large.

Observations and Calculations.

Known standard inductance L_2 = mH

Set. No.	R_3 ohms	R_4 ohms	Mean value of R_4 ohms	R ohm	Mean R ohms	$L_1 = L_2 \frac{R_3}{R_4}$
1		(i) (ii) (iii)		(i) (ii) (iii)		
2						
					Mean	

Result. The inductance of the given coil = mH.

33·10. **Maxwell's Self-inductance Bridge—Coils in parallel.** When the two inductances, the unknown L_1 and the known L_2, are placed in adjacent and parallel arms of the bridge, as shown in figure

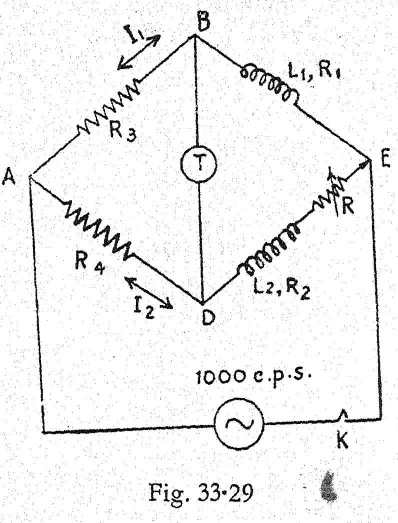


Fig. 33·29

33·29, there is a non-inductive resistance in series with each coil. The condition for the balanced bridge is evidently

$$\frac{R_3}{R_4} = \frac{R_1 + jL_1\omega}{(R_2 + R) + jL_2\omega}$$

$$\text{or } R_3\ (R_2+R) + jR_3L_2\omega = R_1R_4 + jL_1L_4\omega$$

whence, equating reals on each side and likewise unreal,

$$R_3\ (R + R_2) = R_1R_4$$

or
$$\frac{R_3}{R_4} = \frac{R_1}{R + R_2} \qquad (33.27)$$

and
$$L_2R_3 = L_1R_4$$

or
$$\frac{L_1}{L_2} = \frac{R_2}{R_4} \qquad (33.28)$$

giving

$$L_1 = L_2 \times \frac{R_3}{R_4} \qquad (33.29)$$

Hence for no sound in the headphone, the two conditions embodied in equations (33.27) and (33.28) have to be satisfied. Balance may be approached by alternate adjustment of R_3 and R for a fixed value of R_4. Then L_1 can be calculated in terms of L_2, R_3 and R_4.

The vector diagram of the bridge is given in figure 33·30. The p. d. triangle for the branch consisting of R, R_1 and L_1 is ABE while for the other parallel circuit is the triangle ADE. The headphone is

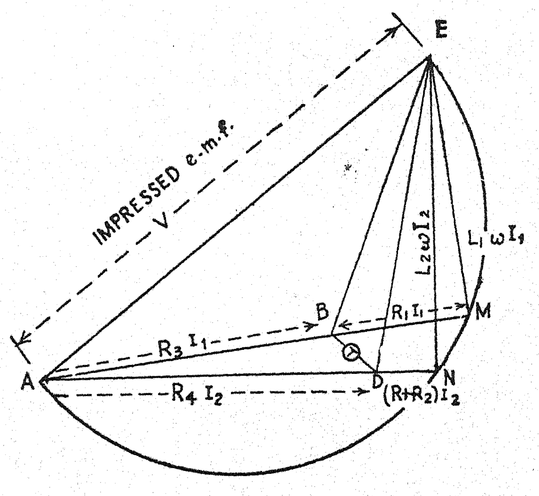


Fig. 33·30 Unbalanced Bridge

connected across BD. For the balanced bridge B should coincide with D. This can be achieved by altering R_3 and R to the desired values for a fixed value of R_4. Fig. 33·31 depicts the vector diagram for the balanced bridge, and we then have

$$\frac{R_3}{R_4} = \frac{R_1}{R + R_2} = \frac{L_1}{L_2},$$

the same conditions as deduced earlier.

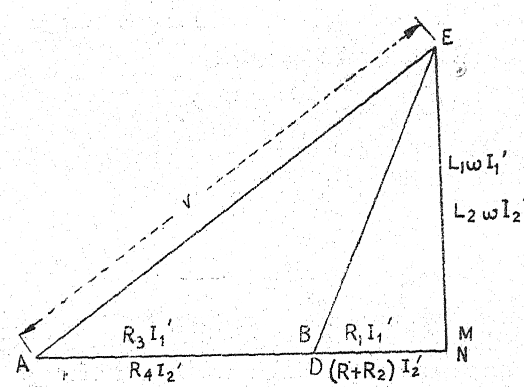


Fig. 33·31 Balanced Bridge

Oral Questions

IMPEDANCE OF AN A C. CIRCUIT

What do you underst nd by the impedance of an A. C. circuit ? What is reactance ? What is the relation between impedance, reactance and ohmic resistance ? What is inductive reactance of a circuit ? What part does it play in a circuit ? What is wattless current ? What is choke ? What is the difference between a H. F. and a L. F. choke ? What is capacitive reactance ? Which would you prefer to use and why when you want to reduce the current in an A. C. circuit—a choke, condenser, or a resistance ? What is the relation between the total applied E. M. F. in a circuit and P. Ds. across the resistance, inductance and capacitance in that circuit ? Compare an A. C. circuit with a D. C. circuit. What is the difference in the behaviour of a condenser when used on D. C. and when used on A. C. ?

What is the difference in the measuring instruments used for A. C., and D. C. ? Can a D. C. instrument be used on A. C. ? Can an A. C. instrument be used on D. C. ? What is virtual, effective or R. M. S. current in A. C. circuit ? Explain the significance of each term ? What is the maximum voltage when the voltmeter reading is 220 volts A. C. ? What is power factor ? What is the difference between apparent watts and true watts.

A. C. FREQUENCY AND CAPACITANCE OF A CONDENSER

What do you mean by the frequency of A. C. mains ? Explain how you find it out experimentally with an electrical vibrator. What is the function of the solenoid and the permanent magnet in the vibrator ? What for do you use a 25-watt lamp in series with the solenoid ? Is it necessary to adjust the length of the steel rod before starting the experiment ? If so, why ? What are the sources of error in the experiment ? What precautions do you observe and why ?

Explain how the condenser is first charged and then discharged through the micrometer. Why do you get a continuous current as indicated by the micro-ammeter ? What is the function of the steel discs T, T. ? Can you use it to compare the capacitances of two condensers ? If so, how ? Is this method of comparing capacitances superior to the ballistic galvanometer method ? Point out the relative merits and demerits of the two methods.

POWER FACTOR OF AN A. C. CIRCUIT

What is the expression for the power consumed in an ohmic resistance ? What is the expression for the power consumed in an inductive circuit ? What do you mean by power factor ? What is the relation between apparent watts and true watts consumed in a circuit ? What is the power factor of a pure inductive circuit ? What is the order of the ohmic resistance or d. c. resistance of the inductance that you are using ? Draw the p. d. Vector diagram for the circuit. Can you calculate from this Vector diagram (i) the resistance of the inductive or load circuit and (ii) the inductance of the coil ? Will the calculated value of the resistance of the load circuit be the same as the d.c. resistance ? If there is a difference, how do you account for this difference ? Which is greater—the calculated resistance from Vector diagram or the measured d. c. resistance ? What precautions do you take in the experiment ? Why should the values of p. d. across various points be measured very accurately ? Should you make an aproximation while taking the voltmeter reading when the pointer lies between two scale divisions. Is there any other simple method of measuring the power factor ?

STUDY OF SERIES RESONANCE

What do you mean by resonance ? When does resonance occur in a series a. c. circuit ? When resonance occurs what is the relationship between p. d. across inductance and the p. d. across capacitance ? Can these values of p. d. be greater than the supply e. m. f. at resonance ? What is theoretically the value of p. d. across both L and C, and why ? Do you achieve this condition practically ? How do you account for this difference ? At resonance, what is the p. d. across the resistance theoretically ? Do you get this result practically ? How do you account for this difference ? Why do you use an inductance coil of about 10 henry in the circuit. How is such a coil made ? How does the inductive reactance vary with frequency ? How does the capacitive reactance vary with frequency ? Draw graphs to illustrate your above answers. From these graphs, draw the graph of the net reactance with frequency. What conclusion can you draw from the net reactance graph ? Explain the nature of graphs (V_C, C), (V_L, C) that you have drawn in this experiment. At what value of C does resonance occur ? Explain the nature of graphs (V_R, C), (V_{CL}, C) that you have drawn. At what value of C does resonance occur ? What precautions do you take in the experiment.

WIEN'S SERIES RESISTANCE BRIDGE FOR CAPACITY

For what purpose do you use this bridge ? Draw the Vector diagram for an unbalanced bridge. From the diagram point out what adjustments would be necessary to balance the bridge. What are the conditions for the bridge to be balanced ? How do you test this balance ? What is the impedance of the headphone ? Does it vary with frequency ? Why do you use a mica condenser of known capacity, why not a paper condenser ? Why do you consider a condenser as equivalent to a capacitance with a resistance in series with it ? Which of the resistances in the bridge should be specifically non-inductive for the determination of the power factor of the condenser ? Why is it important to record the frequency of the oscillator, usually 1000 c. p. s. ? When is the bridge most sensitive ? What precautions do you observe in the experiment.

MAXWELL'S INDUCTANCE BRIDGE

(Coils in Series)

For what purpose do you use this bridge ? Draw the Vector diagram for an unbalanced bridge. From the diagram point out what adjustments would be necessary to balance the bridge (a) if the standard inductance is fixed, (b) if this is variable in a known manner. What are the conditions for the bridge to be balanced ? What should be the order of the resistance R_3 and R_4 ? What is the purpose of choosing R_3 and R_4 of this order ? Why do you use a decimal ohm box in series with both L_1 and L_2 ? Would one decimal ohm box in series with L_2 not serve the purpose ? What difficulty may you experience by using only one decimal ohm box in series with L_2 ? What precautions do you take in the experiment ? What is an inductometer ? Why is it more useful for this experiment that a fixed inductance ?

Chapter XXXIV

DIODE, TRIODE AND TETRODE VALVES

34·1. Thermionic Valves. A thermionic valve (Fig. 34·1) consists essentially of a heated conductor emitting electrons, known as the cathode and a surrounding plate or cylinder called the anode. In between the cathode and the anode, all valves except a diode have one or more additional electrodes called the grids. The electrodes are mounted on a suitable supporting structure inside a vessel made of glass or metal. A glass pinch is formed at the base of the tube which carry the connecting wires between the electrodes and the outside terminals.

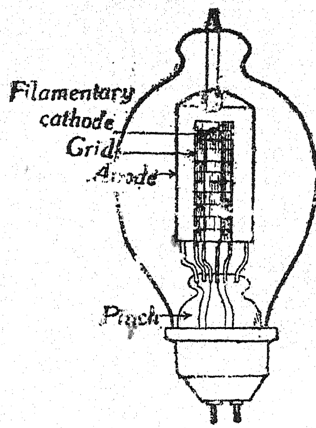
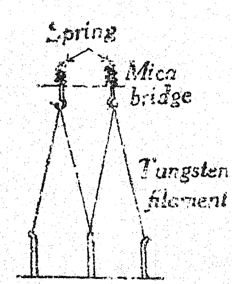


Fig. 34·1

The cathode is in general of two types : (*a*) The directly-heated or filamentary type and (*b*) the separate-heater or indirectly-heated type. The filamentary type cathode as illustrated in fig. 34·2 (*a*) consists of wires or ribbons made of (*i*) pure tungsten or (*ii*) thoriated tungsten or (*iii*) a metal or an alloy coated with specially active material, such as alkaline earth metals and their oxides. Pure tungsten filaments are now not used to a great extent on account of lower energy consumption and absence of true saturation effects in thoriated tungsten and all coated filaments. However, in tubes which are operated at high anode potentials, tungsten filaments are still preferred because of their greater ruggedness and stability.



(*a*) Fig. 34·2 (*b*)

The separate-heater or indirectly-heated type cathode consists of metal tubes (Fig. 34·2*b*) with insulated heater wires of pure tungsten at the centre. The metal tubes are externally coated with electron-emitting oxides and the wires are often spiralled.

The grids of high vacuum tubes (Fig. 34·3) are usually made of a spiral or mesh of molybdenum wire wound in grooves on the supporting wire. In cheaper type iron-nichrome, iron-nickel or manganese-nickel alloys are used for grid materials. The anode is usually a circular or flattened cylinder of nickel or iron surrounding the grid and the cathode but may consist of two parallel plates situated on opposite sides of the cathode.

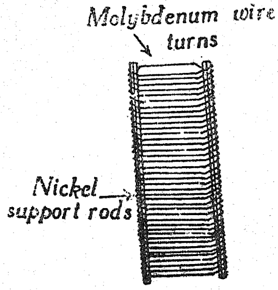


Fig. 34·3

Most of the valves are of high vacuum type so that the whole vessel is completely exhausted, but in some valves a trace of some inert gas like helium or mercury vapour is inserted Completely exhausted valves are known as high vacuum tubes or hard valves and those in which a trace of gas is inserted are called gas-filled tubes or soft valves. Soft valves require careful adjustment owing to then anode current depending largely on the pressure of the gas which is affected by temperature and occlusion.

When the filament or the separate-heater cathode of a high vacuum tube is heated by an electric current, electrons are emitted into the region surrounding the cathode. If an external field is present which removes the electrons as fast as they are liberated, there is a steady loss of electrons from the metal which is equivalent to a current flowing into the metal. The magnitude of this current depends upon the number of electrons liberated per second and hence upon the temperature of the metal. If the external field is sufficient to remove the electrons as soon as they are emitted, the current per sq. cm. of the heated surface is given by $i=aT^2e^{-b/T}$, where T is the absolute temperature and a and b are constants for the given substance. In case the external field is not sufficiently great to remove the electrons as fast as they are liberated, cloud of electrons is formed near the cathode surface which exists a repulsive force on the electrons just leaving the metal. This electron cloud is known as *space charge*. In the absence of the space charge the current i is related to the external field E by the equation $i \propto E^{3/2}$. When there is no external field, the space charge may accumulate until it reaches such a magnitude that it repels all the electrons as soon as they are emitted with the result that there is no flow of current.

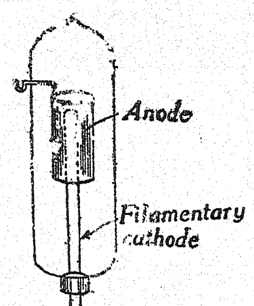


Fig. 34·4

34·2. Diode valve. Diode valve was invented by Fleming in 1904 and as its name suggests consists of two electrodes (Fig. 34·4), namely, the cathode and the anode. When the cathode is heated by an electric current, electrons are emitted from it. When the anode

is at a positive potential with respect to the cathode, electrons are attracted to it to form an anode current.

In the absence of space charge, the current is proportional to $E^{3/2}$ and hence to $V^{3/2}$, where V is the anode potential and E is the field due to it around the cahode. As the positive potential on the anode is raised, the node current increases until a saturation effect is produced, when further anode-potential increase does not cause any further appreciable rise of anode current. This behaviour of the diode is illustrated in fig. 34·5. Over the portion AB of the curve, the valve is said to be saturated and the corresponding current is called the saturation current. The saturation current can be increased by increasing the temperature of the cathode. When the anode is kept at a negative potential with respect to the cathode, no electron can reach it and consequently no current flows across the valve.

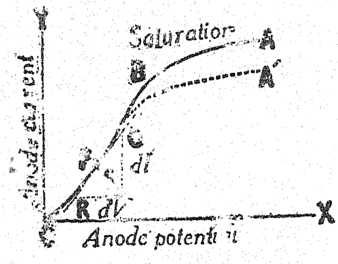


Fig. 34·5

In an ideal valve the portion AB and CB would intersect at a sharp angle, but owing to end effects of the electrodes, etc., this angle is rounded off in actual practice. It may be noted that the slight rise in the portion BA of the curve is due to the slight ncrease in the saturation current of the diode on account of Schottky effect. According to this effect the total electron emission from a thermionic cathode at a specific temperature is increased to a small extent by the application of a positive electric field at the cathode surface. This effect is somewhat more marked if an indirectly heated cathode is used.

A diode valve is chiefly used as (*i*) a rectifier for a power supply, (*ii*) a detector of wireless signals and (*iii*) a high frequency voltmeter.

Experiment 34.1

Object. To draw the anode-current anode-voltage characteristic curve of a diode and then to find its internal resistance.

Apparatus. A diode valve mounted on a panel, a low-tension battery, a high-tension battery, two rheostats, a voltmeter, a milliammeter, an ammeter and two one-way plug keys.

Theory. The internal resistance of a diode valve is defined as the ratio of a small change of anode voltage to the corresponding small change of anode current. Expressed symbolically

$$R = \frac{\partial V}{\partial I}$$

The resistance of a diode is often spoken of as its anode impedance or slope resistance. Since it is not constant, it must be measured at a specified mean anode potential.

Referring to fig. 34·5 which represents the characteristic curve of a diode, it is evident that the anode resistance of the valve at the potential corresponding to a point P on the curve is equal to the reciprocal of the slope of the curve at P, *i.e.*,

$$R = \frac{\delta V}{\delta I} = \frac{RS}{QS} \qquad (34 \cdot 1)$$

Method. Connect the cathode terminals of the valve to a low-tension battery (Fig. 34·6) in series with a suitable resistance R_1 and a sensitive ammeter A, including a plug key K_1 in the circuit. Join a high-tension battery across a rheostat R_2, including a key K_2 in the circuit. Connect the anode terminal of the valve to the −ve marked terminal of the milliammeter, the +ve marked terminal of which is connected to the sliding contact maker of the rheostat R_2. Join the −ve terminal of the H. T. battery to the −ve terminal of the L. T. battery and finally place a 150 volts voltmeter across the rheostat R_2 as *shown*.

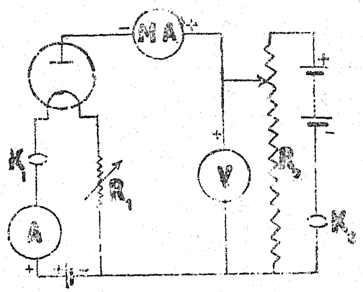


Fig. 34·6

Insert the valve in its socket, close the key K_1 and adjust the current in the cathode-circuit by means of the rheostat R_1 to a value specified by the manufacturer. Close the key K_2 and apply a low potential to the anode by adjusting the rheostat R_2. Note down the readings of the milliammeter and the voltmeter. Increase the anode potential in small steps, say of 5 volts, noting its value and the corresponding value of the anode current at each step till the anode current reaches a saturation value.

Plot a graph taking the various values of anode potential as abscissae and the corresponding values of anode current as ordinates. The graph will be as illustrated in fig. 34·5. Take a convenient point P on the portion CB of the curve. Draw a tangent at the point P to the curve. Take two points Q and R on the tangent one on each side of P and draw lines QS and RS parallel to the Yo and X axes respectively. Measure QS and RS, and according to the scale adopted, find the change in anode potential corresponding to RS and the change in anode current corresponding to QS. Finally calculate the internal resistance of the valve R from equ. (34·1).

Repeat the experiment with a lower value of filament current (dotted curve).

Sources of error and precautions. (1) The −ve marked terminal of the milliammeter should be connected to the anode terminal of the valve.

(2) The cathode current must not exceed the value specified by the manufacturer and should remain constant throughout.

(3) The maximum anode voltage should not exceed the limit permissible.

(4) Only the portion CB of the curve should be used for calculation of resistance of the valve.

Observations. Cathode heating current = amp.

S. No.	Anode potential volts	Anode current amp.

Calcations. From the graph (34·5), we have

QS= amp. RS= volts

$$R=\frac{RS}{QS}= \qquad = \qquad \text{ohms}$$

Result. The anode-current anode-voltage characteristic curve of the given diode valve is shown in fig. ...... and the value of its internal resistance= ohms.

34·3. Triode Valve. The three-electrode or triode valve was invented by Lee De Forest in 1907. It consists of three elements, namely, the anode or plate, the grid and the filament as illustrated in fig. 34·7. The electrodes are mounted on a suitable supporting structure and contained in a vessel of glass or metal.

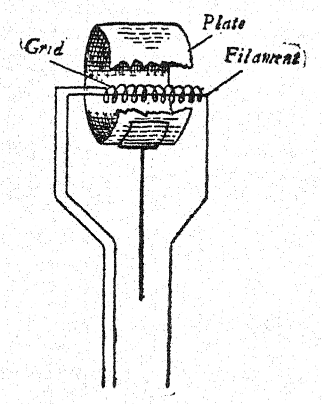


Fig. 34.7

When the cathode is heated to emit electrons, they are attracted by the anode which is always maintained a at high positive potential with respect to the cathode. The grid may be raised to a positive or negative potential with respect to the cathode and consequently it will attract or repel the electrons coming from the cathode. Thus the field around the filament is produced jointly by the grid and the anode. Let the potential of the grid be V_g and that of the anode V_a both with respect to the cathode. In the absence of space charge, the electrodes may be treated as ordinary electrostatic condensers and in that case

$$E \propto (C_{af} V_a + C_{gf} V_g)$$

where C_{af} and C_{gf} are the capacitance of the anode-filament and the grid-filament respectively. Now

$$i \propto E^{3/2}$$

Hence $$i \propto (C_{af} V_a + C_{gf} V_g)^{3/2}$$

or $$i = K (V_a + \mu V_g)^{3/2}$$

where K is a constant containing C_{af} and $\mu = C_{gf}/C_{af}$. It is evident from the above equation that the current leaving the filament is determined *jointly* by the potentials of the grid and the anode and that a potential V on the grid is μ times as effective in producing current as a potential V on the anode. The quantity μ is called the *amplification factor* of the valve. It is a structural constant of the valve and may be varied over a wide range.

The electron current leaving the filament divides itself into two branches one part goes to the grid and the other to the anode, but the way in which the division takes place does *not* depend upon the relative potentials of the grid and the anode. The potentials applied to the valves in wireless are very high so that when the electrons reach the plane of the grid they are moving rapidly. Consequently although the grid may exert a great attractive force on them, it produces only small deviations in their paths. Most of the electrons, therefore, shoot through the grid spaces and finally reach the anode. It follows from this that, provided the potentials on the grid and the anode are sufficiently high, the way in which the current divides between the grid and anode is determined by the *shadow ratio*, *i.e.*, the ratio of the projections of the grid and anode as seen from the filament. Since in most valves the spaces in the grid subtend a much larger area than the wires of the grid, most of the electron-current reaches the anode. But as the grid is closer to the supply of electrons than is the anode and as the electrons going to the anode must all pass through the grid, it is a good controller of the anode current. When the grid is made negative with respect to the filament, it repels electrons and in a sense assists the space charge in forcing the electrons back towards the filament. This results in reduction of the anode current that would otherwise flow to the anode. Thus the grid acts as a control electrode in a triode.

34·4. Constants of a Triode. There are three constants or coefficients of a triode valve, namely, (*a*) amplification factor, (*b*) grid-plate transconductance and (*c*) plate resistance.

(a) Amplification factor. It is a measure of the effectiveness of the grid with respect to the anode or plate and *may be defined as the ratio of the change in anode voltage required to produce a certain change in the anode current to the change in the grid voltage which would cause the same mall change of the anode current.* In other words, the amplification factor is the ratio of the change in anode voltage to the change in grid voltage, as this change is made infinitely small

95 — 100

with the anode current being held constant. Thus, if V_a and V_g be the anode voltage and grid voltage respectively and i_a the anode current, the amplification factor is given by

$$\mu = \frac{\delta V_a}{\delta V_g} \qquad (i_a \text{ constant})$$

The amplification factor has no unit. Its value is always greater than unity and in the case of triodes it may be as high as 150.

(b) **Grid plate transconductance** (*mutual conductance*). *It is defined as the rate of change of anode current with grid voltage, keeping the plate voltage constant*; or expressed mathematically

$$g_m = \frac{\delta i_a}{\delta V_a} \qquad (V_a \text{ constant})$$

where g_m stands for the grid plate transconductance. It is given by the slope of the grid voltage-anode current curve and is measured in ohms.

(c) **Plate resistance.** It is the internal opposition between the filament and plate to the flow of alternating current component and *may be defined as the reciprocal of the rate of variation of anode current with anode voltage, grid voltage being kept constant.* Expressed mathematically

$$r_p = \frac{\delta V_a}{\delta i_a} \qquad (V_g \text{ constant})$$

where r_p is the plate resistance. The plate resistance is given by the reciprocal of the slope of the plate-voltage anode-current curve and is measured in ohms. It is sometimes referred to as *plate impedance* but for all except the high radio frequencies it is essentially a resistance.

The above quantities are not exactly constant, but vary with the direct electrode voltages. They are approximately constant over the *straight* portion of the characteristic curves and vary considerably as *the bends in the curves* are approached. The tube constants are related by the expression

$$\mu = g_m \ r_p$$

34·5. Characteristic Curves of a Triode. In a high vacuum triode there are three independent variables, namely, the cathode temperature, the node voltage and the grid voltage, and two dependent variables, namely, the anode current and the grid current. Curves may be drawn by permitting any one of the first three to vary, keeping the other two constant. Such curves are known as characteristic curves of the tube. The most important of these curves for a triode is that between grid voltage and plate current. It should be noted that these curves portray accurately the instantaneous action of the tube under any circumstances whatsoever.

Experiment 34 2

Object. To draw the grid voltage-plate current characteristic curves of a triode and to determine from them the value of the tube constants.

Apparatus. A triode valve with its socket fitted to a board, a low-tension battery of two accumulators, a grid-bias battery of cells of, say 30 volts, a high tension battery of dry cells or any other convenient source of potential from 0 to 100 volts, a milliammeter, a sensitive ammeter, a voltmeter reading up to 150 volts, a high resistance zero-centred voltmeter reading up to 20 volts on either side, suitable rheostats and plug keys.

Theory. Let the fig. 34·8 represent the grid voltage-plate current characteristic curves for three different plate voltages, say V_a=40, 60 and 80 volts. Referring to the curves PACM and QBN, it is evident that the plate current can be changed by an amount AB either (*i*) by lowering the plate voltage from V_1=60 to V_2=40 volts, keeping the grid voltage constant or (*ii*) by decreasing the grid voltage by an amount BC, keeping the plate voltage constant at V_1=60 volts. Hence from § 34·4 (*a*), the amplification factor is given by

$$\mu = \frac{V_1 - V_2}{BC} \tag{34.2}$$

Referring to the curve PACM, a change of grid potential by an amount BC produces a change of plate current by an amount AB.

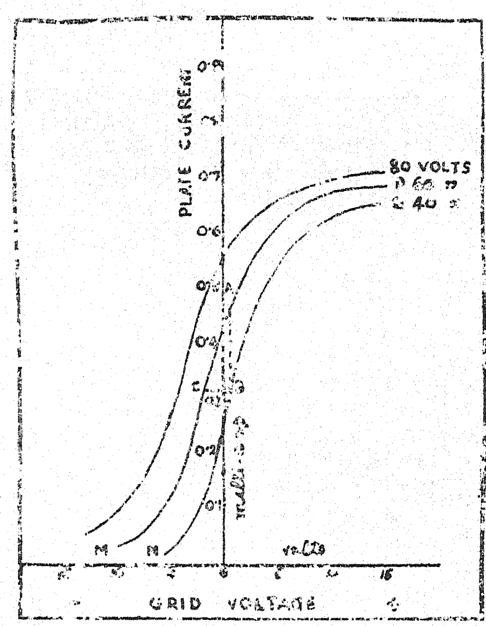


Fig. 34·8

Hence from § 34·4 (*b*), grid-plate transconductance is given by

$$g_m = \frac{AB}{BC} \tag{34.3}$$

These two equations can be used to calculate μ and g_m and having evaluated them, the plate resistance can be calculated from the expression

$$r_p = \frac{\mu}{g_m} \tag{34.4}$$

Method. Connect (Fig. 34·9) the filament terminals of the

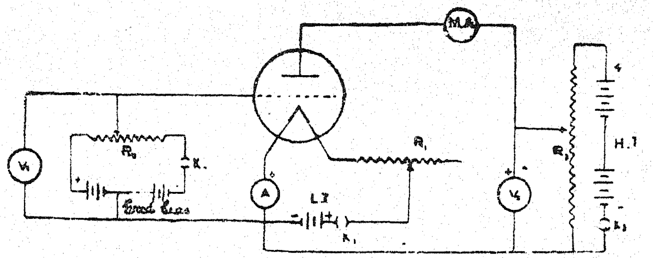


Fig. 34·9

triode valve to the low-tension battery in series with a fine rheostat R_1 and a sensitive ammeter A including a plug key K_1 in the circuit. Connect the grid-bias battery across a rheostat R_2 including a plug key K_2 in the circuit. Join the mid-point of the grid-bias battery to the $-$ve of the low-tension battery and connect the sliding contact maker of the rheostat R_2 to the grid terminal of the valve, placing a high resistance zero-centred 20 volt voltmeter across the grid and the filament as shown in the figure. Join the high-tension battery to a rheostat R_3 including a plug key K_3 in the circuit. Connect the plate terminal of the valve to the — marked terminal of milliammeter, the + marked terminal of which is connected to the sliding contact maker of the rheostat R_3 placing a 150-volt voltmeter across the rheostat R_3 as shown and joining the —ve of the high tension battery to the —ve of the low-tension battery.

Insert the valve in the socket, close the key K_1 and adjust the rheostat R_1 till the heating current as recorded by the ammeter A equals that specified by the manufacturer. Close the key K_3 and adjust the rheostat R_3 till the voltmeter V_2 registers, say 40 volts. This applies a +ve potential of 40 volts to the plate with respect to the filament. Then close the key K_2 and adjust the rheostat R_2 till the voltmeter V_1 reads say —15 volts, *i.e.*, a —ve potential of 15 volts is applied to the grid with respect to the filament.

Note down the plate current as registered by the milliammeter in the plate circuit. Keeping the plate potential and the filament current constant with the help of the rheostats meant for the purpose, increase the grid potential in steps of 2 volts or less till it is about +15 volts, noting the plate current for each value of the grid voltage. Plot a curve between the grid voltage and the plate current taking the various values of grid voltage as abscissae and the corresponding values of plate current as ordinates.

Next keeping the filament current constant, alter the plate potential to, say 60, 80 volts, etc., and take at least two more sets of observations for variation of plate current with grid potential; and plot similar grid voltage-plate current curves on the same graph. The curves will be as shown in fig. 34·8.

Take a convenient point B on straight portion, near the middle, of the curve QBN and draw from it two straight lines BC, and BA parallel to X and Y axes cutting the upper curve PACM at C and A respectively. According to the scale adopted, find the change in plate current corresponding to AB and the change in grid voltage corresponding to BC. Note down the difference between the plate potentials V_1 and V_2 corresponding to the curves PACM and QBN. Calculate the value of μ and g_m from equations (34·2) and (34·3) and then that of r_p from equation (34·4).

Sources of error and precautions. (1) The filament heating current must be maintained constant throughout the experiment by adjusting the rheostat R_1 in series with the low-tension battery, if necessary.

(2) The −ve marked terminal of the milliammeter should be connected to the anode.

(3) The potentials applied to the grid should be both positive and negative with respect to the filament. This can be done conveniently by dividing the grid-bias battery into two parts as shown in the figure, and using a zero centred voltmeter to register the grid potentials.

(4) The maximum voltage applied to the grid should not exceed 20 volts otherwise the filament may be broken due to excessive mechanical strain.

(5) While taking observations for the plate current with different grid potentials, the plate potential must be kept constant for each set by adjusting the rheostat R_3, if necessary.

(6) The straight portion of the characteristic curves must be used to evaluate the tube constants.

Observations. Filament heating current = amp.

S. No.	Grid potential volts	Plate current with plate potential equal to		
		40 volts. milliamp.	60 volts. milliamp.	80 volts. milliamp.

Calculations. Form the graph (34·8)

Difference between plate potentials of the curves PACM and QBN $=V_1-V_2=$ volts

Change in grid potential corresponding to BC= volts

Change in plate current corresponding to AB= amp.

$$\therefore \qquad \mu=\frac{V_1-V_2}{BC}= \qquad =$$

$$g_m=\frac{AB}{BC} \qquad = \qquad = \text{ ohms}$$

and

$$r_p=\frac{\mu}{g_m} \qquad = \qquad = \text{ ohms}$$

Result. The grid voltage-plate current characteristic curves of the given triode valve are shown in fig......and the values of the tube constant are :

$\mu=$, $g_m=$ mhos and r_p = ohms

34·6. Uses of a Triode. The main uses of a triode valve are as follows :

(a) As an amplifier. We have seen in § 34·3 that a potential V applied in the grid circuit of a triode is μ times more effective in producing change of plate current than an equal potential V applied in the plate circuit. Consequently, if a signal is applied to the grid of the valve, it will produce corresponding large variations of plate current resulting in a large voltage drop across a load including in the plate circuit, *i.e.*, the valve can be used as an amplifier.

(b) As an oscillator. If the load in the plate circuit of a triode is inductive, then for its certain values, the input resistance of the valve reflected into the grid circuit is *negative*. Consequently, if the grid of the valve is connected to an oscillatory circuit consisting of inductance and capacitance and the load in the plate circuit is properly adjusted, the effective ohmic resistance of the oscillatory circuit may be made zero or even negative. In such a case if oscillations are once started in the oscillatory circuit, they may continue to build up, the energy for their maintenance being fed back from the plate circuit through the grid-plate capacitance of the valve.

(c) As a rectifier. When a signal is applied to the grid of a triode, there is an increase of plate current during the positive half-cycle and a decrease in the negative half-cycle. If we work on the curved portion of the characteristic curve and, if initially there is zero average potential on the grid, the alternate increases of plate current will be greater than the alternate decreases, there being a mean increase of plate current. Consequently the valve can be used as a rectifier.

The above uses of a triode have made possible radio broadcast and reception, the long distance telephone, facsimile-picture transmission, public address systems, automatic and remote control of power and machinery, television, direct-current power transmission and numerous other achievements.

34.7. **Inter-electrode Capacitances.** Various electrostatic fields exist between the charged electrodes of a triode, such as the field between the plate and the grid, between plate and cathode, and between grid and cathode. The presence of an electrostatic field is equivalent to an electric capacitor. Capacitances thus exist between any two pieces of metal separated by a dielectric, the value of the capacitance depending on the size of the metal plates, the distance between them and the type of dielectric (usually a vacuum in triodes). The inter-electrode capacitances of a triode are shown in fig. 34·10. C_{gp} represents the grid to plate capacitance, C_{gk}, the grid to cathode capacitance and C_{pk}, the plate to cathode capacitance. These capacitances are usually small, of the order of 2 to 10 micro-micro-farads.

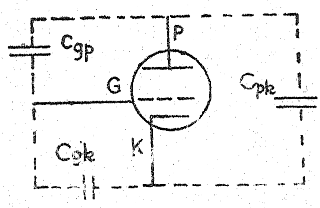


Fig. 34.10

"Their reactance at the higher radio-frequencies becomes low and thus leads to undersirable coupling effects. C_{gp}, the grid to plate capacitance has the property of feeding back energy from the plate (out-put) circuit to the grid (input) circuit, which may lead to instability and oscillations. This property is generally undesirable."

A reduction in inter-electrode capacitances can be achieved by additional shielding electrodes which leads to the design of multi-electrode tubes.

34·8. **Tetrodes.** The undesirable effect of grid-to-plate capacitance in triodes has led to the development of the tetrode. This capacitance, as mentioned earlier, leads to coupling effects and instability in radio-frequency amplifiers that cannot be usually eliminated. This problem of feed back can be conveniently solved by inserting an additional shielding electrode in the triode between the control grid and the plate. We call this the screen grid (G_2 in fig. 34·11). This acts as an effective electrostatic shield between plate P and control grid G_1, and thereby reduces the grid-to-plate capacitance to a value as low as 0·01 $\mu\mu$F, which effectively cancels the feed-back action and brings stability in r.f. amplifiers. The arrangement of the four electrodes in the tetrode is depicted in fig. 34·11. The screen grid is similar to the control grid but of a somewhat coarser mesh. The

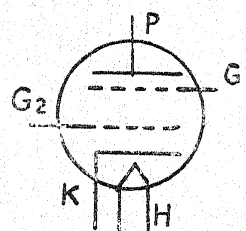


Fig. 34.11

tetrode is operated in the same manner as the triode, with the cathode near ground potential, the control grid at a small negative bias voltage, and the plate at a fairly high positive potential. The screen grid is also placed at a high positive potential with respect to the cathode, but some what lower than the plate potential. This positive voltage on the screen grid accelerates the electrons on their way to the plate and thus aids the electrostatic field of the plate. However, some of the electrons strike the screen grid and produce a *Screen Current*, which is not useful, but most of the electrons pass to the plate through the coarse open mesh of the screen to produce the useful plate current.

As a result of the shielding effect of the screen grid, the electrostatic field of the plate has little effect on the electron space charge near the cathode. Hence variations in plate voltage have little effect on the plate current, and thus a much greater change in the plate voltage is required to produce the same change in the plate current than would would be necessary in a triode. This means that the plate resistance of a tetrode is far greater than that of a triode. It is usually of the order of 0·5 to 1 megohm.

The action of the control grid has about the same effect on the plate current as in a triode as it is not shielded from the space charge by the screen grid. As the tetrode requires a very large change in plate vol age to produce a small change in plate current, the amplification factor of a tetrode is far higher than that of a comparable triode.

Experiment 34·3

Object. To plot the static characteristics of a tetrode valve.

Apparatus. A tetrode valve, high tension and low tension power supplies, two milliammeters 0 – 10 mA, two voltmeters 0 – 160 volts, one voltmeter 0 – 5 volts, four rheostats, key and connection wires.

Method. (i) Make the connexions as shown in the circuit diagram so that a requisite d. c. potential may be applied to the grids and the plate. Before switching on these voltages keep the variable point of the rheostat in a position so that the applied voltages are a minimum.

(ii) Record the recommended values of the various potentials from the tube manual.

(iii) Keep the control grid potential V_c at zero volt and the screen grid potential V_s at a value less than the recommended one. For the tetrode of type 24 A, let it be say, 88 volts.

(iv) Start with the anode potential V_p at zero value and note the anode (or plate) and screen grid currents I_p and I_s respectively.

(v) Increase the plate potential in steps of 5 volts with the help of the rheostat R_1 and record the plate and the screen grid currents in each case. Continue your observation to values of plate potential,

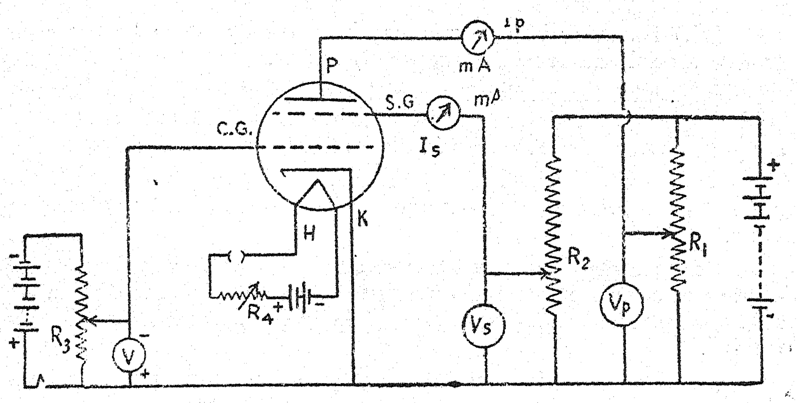


Fig. 34.12

greater than screen grid potential, such that the variation of plate current is meagre with the variation of plate potential. This completes one set of observations.

(vi) Keeping the screen grid potential constant, change the control grid voltage to a slightly negative value, say—3 volts and repeat the set of observations.

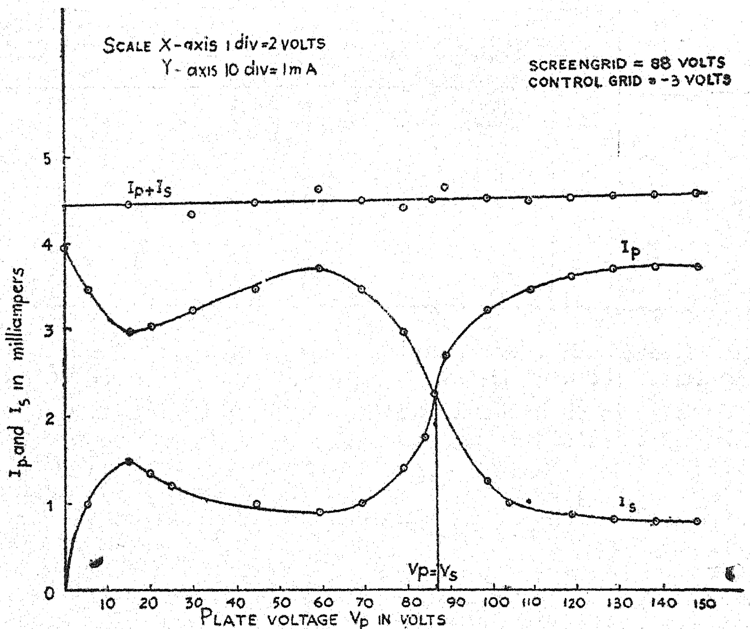


Fig. 34·13

(vii) Further observations may be taken by altering the screen grid potential and repeating steps (iii) to (vi).

(viii) Plot a graph showing the variation of plate and screen grid currents, I_p and I_s respectively, with anode potential, stating on the graph the fixed values of grid bias and screen grid potential.

The graph above is a sample graph of observations when the control grid potential was -3 volts and the screen grid potential 88 volts.

Sources of Error and Precautions. (1) The filament heating current must be maintained constant throughout the experiment by adjusting the rheostat R_4 in series with the low tension supply, if necessary.

(ii) The negative marked terminals of the milliammeters should be connected to the anode and the screen grid.

(iii) The heater voltage must not exceed the recommended value as this may damage the cathode because of excessive emission.

(iv) Anode and screen grid voltages should also be kept below the recommended values.

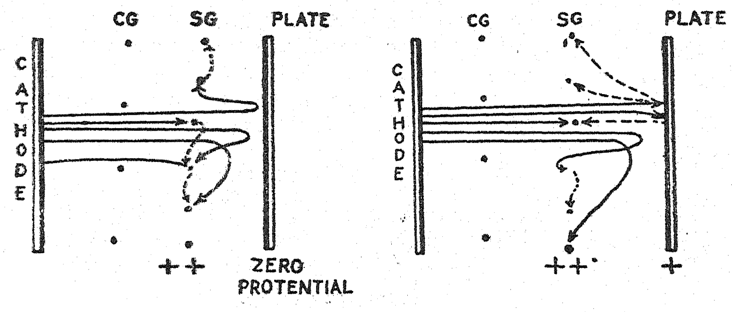
Observations

S. No.	Plate voltage	Screen grid voltage = V				Screen grid voltage = V			
		C. G. voltage = 0 volt		C. G. voltage = -3 volt		C. G. voltage = 0 volt		C. G. voltage = -3 volt	
	V_p	I_p	I_s	I_p	I_s	I_p	I_s	I_p	I_s
		mA	mA	mA	mA	mA	mA	mA	mA

Result. Fig. 34·13 depicts the plot of the required characteristics of the tetrode No.

Discussion of the Graph. (1) *Plate current curve.* Let us examine the curve of the plate currrent I_p. With the screen at 88 volts and plate at zero, virtually all the space current flows to the screen. As the plate voltage is increased, some of the electrons pass through the mesh of the screen grid and reach the plate resulting in a rise of the plate current. But as the plate voltage rises the electrons strike the plate with greater energy, knocking other electrons off the plate. This phenomenon is known as secondary emission. The number of secondary electrons per primary electron is usally greater than unity. Since the plate is still less positive than the screen, the secondary electrons

are attracted to the screen grid rather than back to the plate, causing a loss of plate current that is depicted by the dip in the curve. Fig. 34·14 shows the emission of secondary electrons by broken lines. It



(a) Anode at Zero Potential (b) Anode below Screen Potential

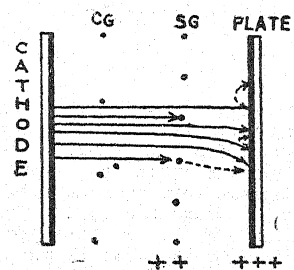


Fig. 34.14 (a, b & c)
(c) Anode above Screen Potential

shows secondary electrons going to the screen grid thus lowering the plate current.

As the plate voltage begins to approach that of the screen some of the secondary electrons are recaptured by the plate and hence the plate current begins to rise. When the plate voltage equals the screen grid voltage, the total space current is equally shared. As the plate voltage exceeds that of the screen grid, more and more of the space current flows to the plate. At a certain plate voltage, the maximum plate current is reached, after which the increase in the plate voltage has almost no effect on the plate current.

(ii) *The Screen Current Curve*. Let us now examine the screen current curve. As there is a steady voltage of 88 volts applied to the screen, practically all the space current flows to the screen when the anode is at zero potential. As the plate voltage increases, some of the

space current is drawn to the plate, and hence the screen current drops by the same amount. As a result of secondary emission, the plate current is reduced but because the secondary electrons are drawn to the screen, the screen current registers a rise in its value. This is so because the screen is still at a higher potential than the plate. As the plate voltage approaches the screen voltage, some of the secondary electrons are attracted by the plate, thus reducing the screen current. When the plate voltage exceeds the screen voltage, most of the space current flows to the plate and the screen current drops to a low steady value.

The total space current graph $I_p + I_s$ is a straight line parallel to the plate potential axis.

Oral questions

What are diode and triode valves ? Who invented them ? How many types of valves do you know ? Distinguish between high vacuum and gas-filled tubes. Of what material is the filament made ? Why are oxide coated filaments preferred to pure tungsten filaments ? What is the construction of the grid ? Of what material is it made ? Describe the internal action of a triode. What is meant by space charge ? On what factors does the current leaving the filament depend ? What is the function of the grid ? How does the electron-current leaving the filament divide between the grid and the plate ?

What are the three constants of a triode ? Define amplification factor, grid-plate transconductance and plate resistance of a triode. In what units are they expressed ? What is the relation between them ? What do you understand by characteristic curves of triode ? How will you draw grid voltage-plate current curves of a triode and determine from them the constants of the tube ? What precautions do you take in this experiment ? Why should the filament current and the plate potential be kept constant for each set of observation ? Why should the grid potential not exceed 20 volts ? Why do you use straight portions of the characteristic curves for the determination of the tube constants ? Are the tube constants really constant ? What are the uses of a triode ? How can a triode be used as (*i*) an amplifier, (*ii*) an oscillator and (*iii*) a rectifier ? What achievements have these uses of the triode made possible?

What is a tetrode ? Explain the meaning of secondary emission. Explain the function of the screen grid in a tetrode. What are the advantages of a tetrode over a triode ? Which of the two—the triode and the tetrode—will have a higher amplification factor and why ? Why do manufacturers not specify the amplification factor of a tetrode ? What does the kink in the characteristic curve of a tetrode indicate and to what use can it be put ?

CHAPTER XXXV

MISCELLANEOUS EXPERIMENTS IN ELECTRICITY

Experiment 35·1

Object. To determine the electronic charge *e* by Millikan's oil-drop method.

Apparatus. Millikan's oil-drop apparatus, atomiser, oil, power supply, voltmeter.

Description of apparatus. *Millikan's oil-drop Apparatus.* The laboratory form of the apparatus is depicted in figure 35·1. A and B

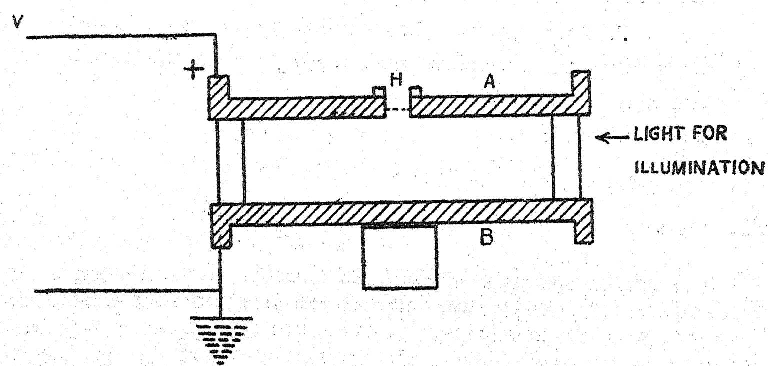


Fig. 35·1

are two optically worked metal plates which form the plates of a parallel plate condenser. These are separated by three optically plane parallel plates of glass about 1·5 cm.thick. A large constant electrostatic field may be maintained between A and B by supplying the necessary p. d. across A and B from a power supply unit. Drops of oil from an atomiser are sprayed above A and fall through the small holes at H into the condenser. Some of the drops are ionized carrying one or more electrons. The space between the plates is illuminated and the motion of the charged drops is observed through a telescope and timed through a given distance with the help of the graduated glass graticule placed in the position of the cross-wires of the eyepiece of the telescope. The speed of the charged drop is measured (a) under gravity alone, and (b) under gravity and a known electric field E between the plates A and B.

Theory. (i) *Motion under gravity alone.* When no electric field is applied, the charged drop between the condenser plates A and B moves down under the action of gravity alone. This motion in the beginning is accelerated and is opposed by the viscous forces of the air and as such the drop acquires a steady downward velocity v_1 when the upward viscous force equals the downward gravitational force.

Let, the radius of the drop be a

the density of the oil be ρ

the density of air be σ

The coefficient of viscosity of air be η

the steady velocity under gravity alone be v_1.

Hence,

wt. of the drop $= \frac{4}{3}\pi a^3 \rho g$

Loss in weight due to buoyancy of air $= \frac{4}{3}\pi a^3 \sigma g$

$\therefore$ Apparent weight of the drop $= \frac{4}{3}\pi a^3 (\rho - \sigma) g$

This gives the *downward gravitational force* on the drop.

From Stoke's law,

The Viscous force $= 6\pi\eta a v_1$

$$\text{Hence,}\quad \frac{4}{3}\pi a^3 (\rho-\sigma) g = 6\pi\eta a v_1 \tag{35.1}$$

$$\text{which gives}\quad a = \left[\frac{9\eta v_1}{2(\rho-\sigma) g}\right]^{\frac{1}{2}} \tag{35.2}$$

(ii) *Motion under electric field and Gravity.* Let an electric field E of sufficient intensity be applied between the condenser plates A and B so that it is sufficiently great to overcome the apparent weight of the drop. Let the charge on the drop be ne where e is the electronic charge. Let v_2 be the constant speed of the same drop in the upward direction. Then,

The net upward force acting on the drop $= ne \,.\, E - \frac{4}{3}\pi a^3 (\rho-\sigma) g$ while the viscous force opposing this $= 6\pi\eta a v_2$

Hence,

$$ne\,E - \frac{4}{3}\pi a^3 (\rho - \sigma) g = 6\pi\eta a v_2 \tag{35.3}$$

Adding equations (35.1) and (35.3) we have

$$ne \,.\, E = 6\pi\eta a (v_1 + v_2)$$

$$\text{or}\quad ne = \frac{6\pi\eta a (v_1 + v_2)}{E}$$

Substituting for a from equation (35.2) in the above,

$$ne = \frac{6\pi\eta}{E}(v_1 + v_2)\left[\frac{9\eta v_1}{2(\rho - \sigma) g}\right]^{\frac{1}{2}}$$

$$\text{or}\quad ne = \frac{9\sqrt{2}\,.\,\pi\eta^{3/2}}{\sqrt{g(\rho-\sigma)}} \,,\, \frac{(v_1+v_2)\sqrt{v_1}}{E} \tag{35.4}$$

If V be the potential difference between the condenser plates A and B and d the distance between them,

$$E = \frac{V}{300d} \text{ in e. s. units.}$$

Hence,

$$ne = \frac{9\sqrt{2\pi\eta^{3/2}} \times 300d}{\sqrt{g(\rho-\sigma)}} \times \frac{(v_1+v_2)\sqrt{v_1}}{V} \tag{35.5}$$

$$= K \times \frac{(v_1+v_2)\sqrt{v_1}}{V} \tag{35.6}$$

where K is a constant equal to

$$\frac{9\sqrt{2\pi\eta^{3/2}} \times 300d}{\sqrt{g(\rho-\sigma)}}$$

Method. Focus the light from a lamp by means of a lens L into the region between the condenser plates A and B. Insert a thin pin in one of the holes in the middle of the upper plate A, rotate the telescope and focus it so that a well defined image of the pin is formed in a dark background. This evidently implies that the telescope is not in line with the source of light. The darkness of the background should not be such that readings of the glass graticule in the telescope eyepiece may be difficult. When the adjustment has been made, take out the pin.

With the help of the atomiser, spray drops of oil in H. Some of these drops due to friction get charged and enter the region between the condenser plates through the holes in A at H.

Observe some of these drops through the telescope. To ascertain which of these are charged and which uncharged, switch on the electric field by applying a p. d. of say 450 volts across AB, with A at a positive potential. Drops which start moving up are charged. Examine one such drop. When it approaches the upper plate A, *switch off* the power supply, the drop then starts descending down under the action of gravity alone. When the drop approaches the plate B, *switch on* the power supply; it then starts moving up. This alternate process of switching on and off of the power supply can keep the drop under observation for any length of time.

Having thus chosen a drop, examine its motion with the telescope. When it is moving down under the action of gravity alone, find the time it takes to move, say through 40 divisions of the glass graticule in the telescope eyepiece. Knowing the value of one scale division of the glass graticule, calculate v_1 from its time of descent for 40 such divisions.

When the drop is reaching B, switch on the power supply and again find the time it takes to move up, say, 40 divisions of the scale.

From this timing calculate v_2, the velocity under the action of both the electrostatic field and gravity.

Repeat these observations a number of times and compute the values of v_1 and v_2 for the particular field E between the plates. To evaluate E, measure the p. d. V across the condenser plates with a voltmeter.

Repeat the experiment with different values of V and choosing different drops.

For each set of observation, calculate ne. It will be found to be different in each case for the simple reason that n may be different for drops examined. By rounding up the values find the highest common factor for these calculated values of ne, then assign the value to n in each case and find e for each set of observation. Finally compute the mean value of e. Compare this with the standard value and compute the percentage error.

Sources of error and precautions. (1) The telescope for observing the drops should be focussed in the manner described. Care should be taken to see that the dark background in which observations are to be made is not so dark as to make it difficult to read the glass graticule.

When the telescope has been focussed, the pin used in focussing should be taken out.

(2) A drop once chosen for observation should be kept moving up (by switching on the power supply) and down (by switching off the power supply) so that at least five sets are taken for v_1 and v_2, potential V being constant for the whole set.

(3) The timing of the drop for calculations of v_1 and v_2 should be done with an accurate stop watch reading to $\frac{1}{10}$ of a second.

(4) Many drops should be examined and under different electric fields E.

(5) If a lateral displacement of the drop occurs, it is due to collision and this drop should be then rejected. Whenever the speed of the drop suddenly changes, it may have acquired more charge or mass. When this happens, the drop cannot be used to repeat its earlier observations but may be used as a fresh drop for a fresh set of observations.

Observations. I. *Constants*

Viscosity of air η	=	$18{\cdot}1 \times 10^{-5}$ poise
Density of air, σ	=	0·00124 gm. per c.c.
Density of oil, ρ	=	(0·874) gm. per c.c.
Value of g at –	=	cm./sec.^{2}
Room temperature	=	°C

Value of one scale division of glass graticule $= 0 \cdot 01$ cm.
Distance between the plates, d = cm.
Least count of the stop watch = $0 \cdot 1$ Sec.

II. *Table for v_1 and v_2*

Set No.	P. D. across AB	Time for Travelling 40 Divisions				Calculated value of ne, in e. s. u.
		Under Gravity Alone		Under Gravitational & Electric Fields		
		Time	Mean	Time	Mean	
I	450 volts	6·8 sec.	6·6 sec.	3.0 sec.	3·2 sec.	$120 \cdot 4 \times 10^{-10}$
II						
... ... VII						

Calculations. I. Common to all sets

$$(\rho-\sigma) =$$

$$\therefore \text{ Constant} \quad K = \frac{9\sqrt{2\pi\eta^{3/2}}\, d \times 300}{\sqrt{g(\rho-\sigma)}}$$

$$=$$

$$=$$

II. For each set separately

$$v_1 =$$

$$v_2 =$$

$$V =$$

$$\therefore \frac{(v_1+v_2)\sqrt{v_1}}{V} =$$

$$=$$

$$\therefore \quad ne = K \cdot \frac{((v_1+v_2)\sqrt{v_1}}{V}$$

$$=$$

$$= \qquad \text{e. s. u.}$$

In this manner calculate ne for each set and then tabulate your results as follows by assigning the proper value of n in each case.

Set No.	ne in e. s. u.	Assigned value of n	Calculated value of e in e. s. u.
1	120×10^{-10}	25	$4{\cdot}800 \times 10^{-10}$
2	..	..	..
...			
...			
...			
7			
		Mean	

Result. The electronic charge as determined experimentally by Millikan's oil drop method = e. s. u.

Standard value = $4{\cdot}8036 \times 10^{-10}$ e. s. u.

∴ Percentage Error =

Experiment 35·2

Object. To determine the value of $\frac{e}{m}$ of an electron by Thomson's method.

Apparatus. Cathode ray tube with power supply unit, one pair of bar magnets, compass box and voltmeter (if not fixed in the power supply).

Description of apparatus. Thomson's cathode ray tube is shown simplified in figure 35·2. C and A are the cathode and the anode respectively. Narrow slits are cut in opposite plates at A so

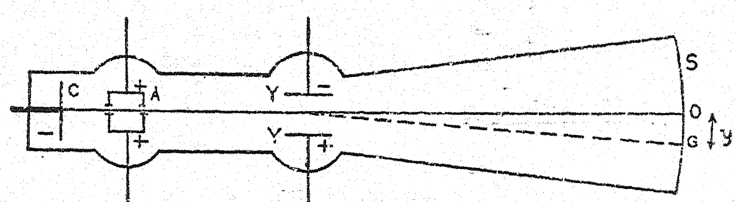


Fig. 35·2

hat the cathode rays passing through are limited to a narrow beam. The rays then strike the fluorescent screen S at O and produce a

glow there. The rays can be deflected electrostatically by connecting the horizontal plates YY to a suitable potential from the power supply unit. The cathode beam may also be magnetically deflected

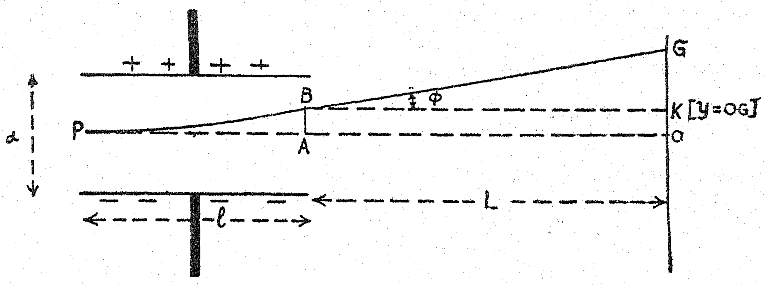


Fig. 35·3

by placing a magnet along the scale on the wooden stand which lies perpendicular to the longitudinal axis of the tube CAO. It is, however, desirable to use two bar magnets, one on each arm suitably disposed and symmetrically situated perpendicular to the axis CAO. The field so produced by the magnets at a point on the axis CAO can be measured by removing the cathode ray tube from the stand that carries the magnets and placing the compass box there and observing the deflection of the compass needle.

Theory. Let the cathode ray tube be placed in the direction of the horizontal component H_e of the earth's magnetic field. In such a case the path of the electron coincides with the direction of H_e. Consequently the electrons will suffer no deflection on account of H_e. The vertical component of the earth's magnetic field, however, will only cause a lateral displacement and does not interfere with our observations as we are measuring vertical displacements due to an applied vertical electrostatic field or an applied horizontal magnetic field perpendicular to CAO.

Let an electric field E be applied to the horizontal plates. If the distance between the plates is d cm. and the potential applied is V volts, then the field

$$E = \frac{V \times 10^8}{d} \text{ e. m. u.} \tag{35.7}$$

When the electrostatic field is E e. m. u., the force on the electron is Ee where e is the charge in e. m. u. on the electron. This force is parallel to the field E and not normal both to the path and the magnetic field as when the magnetic field is applied. Let m be the mass of the electron. Denoting PO as the X-axis and the normal to OP at P in the plane of the paper as Y-axis, the acceleration of the electron is $\frac{d^2y}{dt^2}$. Hence the force on the electron due to the electrostatic field is given by

A Text Book, of

$$m\frac{d^2y}{dt^2} = \mathrm{E}e, \tag{35.8}$$

Since the velocity of the electron v in the path is $\frac{dx}{dt}$, we have from the above equation on integration

$$\frac{dy}{dt} = \frac{\mathrm{E}e}{m} \cdot t + \mathrm{B}$$

where B is the constant of integration. We then have

$$\frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{\mathrm{E}e}{m} \cdot t + \mathrm{B}$$

or $$v \cdot \frac{dy}{dx} = \frac{\mathrm{E}e}{m} \cdot t + \mathrm{B}$$

From the conditions of the experiment, at the point P,

Instant $t = 0,\quad \frac{dy}{dx} = 0,$

hence the constant of integration B = 0.

$$\therefore \quad v\frac{dy}{dx} = \frac{\mathrm{E}e}{m}t \tag{35.9}$$

If the electron takes a time t_0 to traverse the length of the plates l,

$$v \cdot t_0 = l$$

or $$t_0 = \frac{l}{v}$$

At this instant,

$$v\left(\frac{dy}{dx}\right)_{t=t_0} = \mathrm{E}.\frac{e}{m} \cdot t_0 = \frac{\mathrm{E}e}{m} \cdot \frac{l}{v}$$

For the point B, $t = t_0$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{\text{at B}} = \frac{\mathrm{E}e\,l}{m\,v^2}$$

This gives the tangent to the path of the electron at the point B. On coming out of the electrostatic field the electron moves along this tangent at B, *i.e.*, along BG and strikes the screen of the cathode ray tube at G distant OG $= y$ from the point O where the beam was focussed in absence of the electrostatic filed. OG $= y$ measures the displacement of the spot on the screen due to the action of the electrostatic field on the electron. Putting the angle BGK as ϕ, we have

$$\tan\phi = \left(\frac{dy}{dx}\right)_{\text{at B}} = \frac{\mathrm{E}el}{mv^2}$$

On the screen the value of

$$KG = BK \tan \phi$$

If BK, the distance of the screen from the end of the plate is L,

$$KG = L \tan \phi = \frac{L.\ Eel}{mv^2} \qquad (35.10)$$

To obtain the value of y, which is given by

$$\begin{aligned} y &= OK + KG \\ &= AB + KG, \qquad (35.11) \end{aligned}$$

we should obtain the value of AB, *i.e.*, the displacement the electron undergoes in traversing the distance l, the length of the plates, under the action of the electrostatic field. At any point distant x from P in the field,

$$x = vt$$

and

$$\frac{dy}{dx} = \frac{Ee}{mv^2} .x$$

by substituting the value of $t = x\ v$ in equation (35.9). Hence

$$AB = \int_0^{AB} dy = \int_0^{l} \frac{Ee}{mv^2} . x\, dx$$

$$= \frac{1}{2} . \frac{Eel^2}{mv^2} \qquad (35.12)$$

Hence substituting the value of KG from equation (35.10) and of AB from equation (35.12) in equation (35.11) we have the displacement OG $= y$ on the screen given by

$$\begin{aligned} y &= \frac{1}{2}\frac{Eel^2}{mv^2} + \frac{EelL}{mv^2} \\ &= \frac{Eel}{mv^2}\left[\frac{l}{2} + L\right] \end{aligned}$$

or

$$\frac{e}{m} = \frac{v^2 y}{El\left(\frac{l}{2} + L\right)} \qquad (35.13)$$

e/m can be determined provided v is also known experimentally. This is conveniently done as follows.

Two bar magnets, one on each arm, are placed in the E-W direction on the wooden stand so that the magnetic field lies horizontally in the region of the electrostatic field and in such a direction as to displace the electron beam opposite to the direction in which it is

displaced by the electrostatic field. The two magnets are symmetrically situated and so moved along the scale that the displacement $OG=y$ due to the electrostatic field is completely neutralised and the spot lies at O. Under this condition the forces acting on the electron due to the electrostatic field and that due to the magnetic field are equal and opposite.

Force on the electron due to electrostatic field $=Ee$

Force on the electron due to magnetic field $=Hev$

$$\therefore \quad Hev=Ee$$

$$v=\frac{E}{H} \tag{35.14}$$

Substituting this value of v in (35.13), we have

$$\frac{e}{m}=\frac{(E^2/H^2)y}{El\left(\frac{l}{2}+L\right)}$$

$$=\frac{Ey}{H^2l\left(\frac{l}{2}+L\right)} \tag{35.15}$$

In this derivation it is assumed that H is uniform. H can be measured by removing the cathode ray tube and placing the compass bxo between the two magnets which lie with their axes in the E–W direction If the deflection of the needle is θ and H_e is the horizontal component of the earth's magnetic field,

$$H = H_e \tan \theta$$

whence

$$\frac{e}{m}=\frac{Ey}{H_e^2 \tan^2\theta\, l\left(\frac{l}{2}+L\right)}$$

$$=\frac{Vy\times 10^8}{H_e^2 \tan^2\theta\, l\left(\frac{l}{2}+L\right)d} \text{ e.m.u per gm.} \tag{35.16}$$

since $E = \dfrac{V\times 10^8}{d}$ in e.m.u.

All the factors on the right hand side of the equation (35.16) being known, e/m is easily calculated in e. m. u. per gm.

Method. Place the wooden stand for mounting the magnet in a direction perpendicular to the magnetic meridian with the help of a compass box. Now place the cathode ray tube perpendicular to the wooden stand and at its centre. The longitudinal axis of the C.R. tube is then in the direction of the horizontal component of the earth's magnetic field.

Carefully study the pamphl.t pertaining to the operation of the cathode ray tube. Switch on the power supply. Adjust the intensity and focusing knobs of the cathode ray tube until a well defined spot appears on the screen. Adjust it to lie in the centre of the screen, Note this position of the spot on the graduated vertical scale on the screen. This is the position of the spot when no potential has been applied to the deflecting plates.

Now apply a certain steady potential difference V volt across the deflecting plates and measure it with an accurate high resistance voltmeter. The spot on the screen is found to be displaced vertically upwards. Note this position of the spot. The difference between the two positions of the spot gives the displacement y, for the applied p. d. of V volt.

Place two equal large bar magnets in the E.W direction, one on each arm of the wooden stand along the graduated scale with their proper poles pointing towards the cathode ray tube so as to reduce the electrostatic deflection. The magnets should lie symmetrically, *i.e.*, equidistant from the axis of the C. R. tube and should be displaced along the scales symmetrically so as to reduce the displacement of the spot to zero. Note the positions of the poles of the magnets nearest the C. R. tube on the scale fixed to the wooden frame. Let these be r_1 and r_2 respectively. This adjustment implies that the field H produced by the magnets in their respective positions produces an equal and opposite displacement of the electron beam.

Remove the magnets. Reverse the applied voltage when the spot on the screen will be deflected vertically downwards. Note the position of the deflected spot and find the displacement y_2 for the same applied voltage V volts. Calculate the mean value of y_1 and y_2; this gives the mean displacement y for the applied potential V volts.

Place the magnets again on the wooden stand in the direction required to reduce the deflection and as before, arrange the poles of these magnets symmetrically and at such a distance apart that the field is of sufficient strength to reduce the deflection to zero. Again note the positions of the poles of the magnet nearest to the C. R. tube, on the scale of the wooden stand. Let these be r_1' and r_2'. Switch off the power supply.

The value of H to balance the electrostatic deflection may be determined in the following manner.

Remove the cathode ray tube from the wooden stand which carries the magnets and place a compass box such that its centre lies on the common axis of the magnets. Adjust the pointer at zero-zero mark. Place the bar magnets with the same opposite poles towards the C. R. tube at the same distances r_1 and r_2 respectively as in the first part of the experiment when the deflection was reduced

to zero. Observe the deflection of the pointer, reading both ends. Find the mean deflection θ_1. Reverse the magnets pole for pole and adjust their distances at r_1' and r_2', the distances of the poles when the deflection was reduced to zero. Again read both ends of the pointer and measure the mean deflection θ_2. Take the mean of θ_1 and θ_2. Let this be θ.

If H_e be the horizontal component of the earth's magnetic field, the field H due to magnets is given by

$$H = H_e \tan \theta$$

This gives H to balance the effect of the electrostatic field E.

Repeat the experiment by altering V and adjusting and measuring H to balance.

The constants of the apparatus l, L and d are known; H_e is taken known at the place of observation. From equation (35.16) we have

$$\frac{e}{m} = \frac{10^8\, Vy}{H_e^2\, l\left(\frac{l}{2} + L\right) d. \tan^2 \theta}$$

$$= K. \frac{Vy}{\tan^2 \theta} \tag{35.17}$$

where $$K = \frac{10^8}{H_e^2\, l\left(\frac{l}{2} + L\right) d} \tag{35.18}$$

Calculate the constant K.

Plot a graph between the product Vy and $\tan^2\theta$. Draw the best straight line through these plotted points and find the value of $Vy/\tan^2\theta$. Knowing K from equation (35.18) and $Vy/\tan^2\theta$ from the graph, calculate e/m from the relation

$$\frac{e}{m} = K. \frac{Vy}{\tan^2\theta} \quad e. m. u./gm. \tag{35.17}$$

Sources of Error and precautions. (1) The cathode ray tube should be placed with its longitudinal axis in the direction of the horizontal component of the earth's magnetic field.

(2) The spot on the screen must be adjusted to be sharp and intense. It should not be allowed to remain on the screen at a given position for a long time.

(3) When the potential difference V is applied to the deflecting plates, its value must be so adjusted as to give the displacement Y in exact mms.

(4) As the plates are not too far apart we can take the electric field E so produced as a uniform field and of an extent coinciding with the limit of the plates, *i.e.*, we can neglect the end corrections. We assume the field H as uniform in strength and extent, which is incorrect.

Observations. I. *Known Constants*

Length of the deflecting plates, l = cm.
Distance of the screen from the end of the plates, L = cm.
Distance between the plates, d = cm.
Value of H_e = oersted.

II. *Determination of y and V.*

Initial position of the spot = cm.

S. N.	Applied Potential Difference V volt	Deflected position of spot when V is				Mean deflection	When y is restored to zero	
		Direct	y_1	Reversed	y_2	$= \frac{1 \pm}{2}$	Position of magnet on eastarm Distance of the nearest pole to C. R. Tube	Position of the other magnet on west arm. Distance of the nearest pole to C. R. Tube
		Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.
1	Direct } Reverse }						N. pole r_1 S. pole r_2	S. pole r_1' N. pole r_2'
2								
3								
4								
5								
6								

III. *Determination of H*

N	For y restored to Zero		Defection of the Compass needle		Mean	Mean defection $\theta = (\theta_1+\theta_2)/2$	$H = H_e \tan\theta$
	Position of the magnet on E arm.	Position of the magnet on W arm.					
	Distance of the pole nearest to C. R. tube.		one end deg.	other end deg.			
1.	N-pole r_1 S-pole r_2	S pole r_1' N pole r_2'		}	θ_1 θ_2		
2.							
7							

Calculations. Calculate Vy for each set and the corresponding values of $\tan^2\theta$. Tabulate as below.

S. No.	V	y	$V.y$	$\tan^2\theta$	$\frac{Vy}{\tan^2\theta}$	Point on the graph
1						
2						
:						
:						
7						
				Mean		

Also find the value of $Vy/\tan^2\theta$ graphically.

$$K = \frac{10^9}{H_e^2\, l\left(\frac{l}{2} + L\right) d}$$

$$=$$

$$=$$

$$\therefore \quad \frac{e}{m} = K.\frac{Vy}{\tan^2\theta} =$$

$$= \quad \text{e. m. u. per gm.}$$

Result. The value of e/m of an electron = e. m. u. per gm.

Standard value = $1{\cdot}76 \times 10^7$ e. m. u. /gm.

Percentage error =

35·1. **Cathode Ray Oscillograph.** For the study of any transient phenomena, *i.e.*, either a potential or a current that varies with time, it is necessary to have an instrument that draws out the potential or current on, say,a vertical axis while the elapsed time is run out along the corresponding horizontal axis. This was initially achieved by mechanical devices. But with the discovery of the fact that beams of electrons can be deflected by transverse electric and magnetic fields and that electrons have very little inertia, it has been possible to so use

the electron beam in oscillographs. This led to the development of the cathode ray oscillograph.

The cathode of an oscillograph tube is usually an indirectly heated nickel cylinder *C* whose flat end is oxide coated. The cylindrical control electrode *G* lies coaxial with C and encloses a diaphragm through

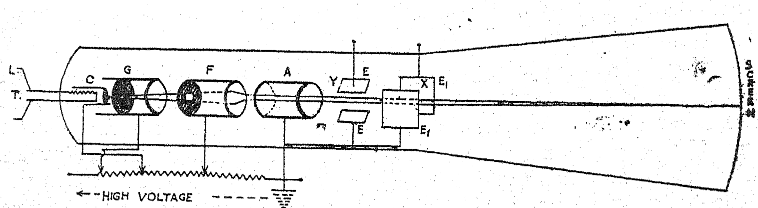


Fig. 35·4

which the electrons pass on their way towards the anode A. This control electrode is often called the grid and it has a negative potential with respect to the cathode. By varying the grid potential, the density of the electron beam and the brilliance of the spot on the screen can be controlled. As the electrons travel between the cathode and the anode A, they are accelerated by the positive anode potential, which may be a thousand volts or more with respect to the cathode. The anode A is an open cylinder or diaphragm and the electrons because of their high speeds pass through it to the screen S. The focussing may be achieved by magnetic means or by an electrostatic field if it has a radial component which causes the electrons to be deflected towards the axis. This component is obtained by an additional cylindrical electrode *F* called the focussing electrode at a potential lower than that of A. F and A are sometimes referred to as the first and the second anode respectively and because of their difference in potential, there is an electrostatic field inside each cylinder, near the gaps between them. The non-uniformity of this field gives it the radial component necessary for focussing.

Between the anode A and the screen S are a system of deflecting plates. These consist of two small pairs of metal plates EE and E_1E_1. In the figure they are shown separated along the beam axis, though in many oscillographs they are coterminous. EE are horizontal and represent a parallel plate condenser system with a field Y determined by the plate configuration and the potential V_y applied to it. This potential V_y causes the vertical deflection and comes from the amplifier connected to the transient that is desired to be studied. The vertical plates E_1E_1 give an electric field X between them when a potential V_x is provided across them and cause a horizontal deflection of the

electron beam. One pate of each pair is connected to the anode. If this is not done there would be an electrostatic field between them and the electron gun. Further, since it would be dangerous to have the deflector plates at a high potential, the anode is earthed, and the cathode and the other electrodes are given negative potential with respect to the earth. The free plate in each deflector system is usually referred to the X-and Y-plate of the oscillograph.

In the part of the tube between the anode A and the screen S, the space through which the electron beam passes must be free from all electrostatic and magnetic fields, except those required for deflection. If this is not so, the deflections will not measure the deflecting fields. Also the screen itself must not be below the anode potential or the beam will turn back before reaching it. Hence to make this space equipotential, the inner walls of the tube are coated with graphite which is connected to the second anode A. To exclude stray magnetic fields the tube is surrounded by a shield of high-permeability alloy. The screen is coated with a fluorescent material such as zinc orthosilicate and where the electron beam is focussed, a brilliant spot is visible on the screen due to fluorescence.

Time Base for C. R. O. Suppose an alternating p.d. is applied to the Y-plates EE. The spot on the screen traces out a vertical line proportional to the applied voltage. In order to convert this line into a wave, some other deflecting force, whose direction is perpendicular to that of the alternating one, is reqiured to provide a "time base". If a steady voltage is applied to the X-plates E_1E_1 (which are not connected to the alternating supply), the effect will be merely to displace the line traced out by the spot from its initial position by a certain definite amount. However, if this voltage can be made to increase in magnitude at a constant rate, the displacement of the spot in the direction perpendicular to that of the deflection due to the alternating field, will be at a uniform rate and a wave with a linear time base will be the result. In this manner the wave form of the alternating p.d. across Y-plates can be obtained on the screen of the oscillograph.

One time base circuit is described below. Refer to fig. 35·5. The capacitor C is charged through the high resistance R_1 at a nearly constan rate until the p.d. across C is sufficient to fire the thyratron V.

The capacitor C then discharges rapidly through V, the current being limited to a safe value by R_2. When the capacitor voltage is no longer high enough to maintain the discharge, the discharge is stopped and the capacitor commences to recharge. The action then repeats,

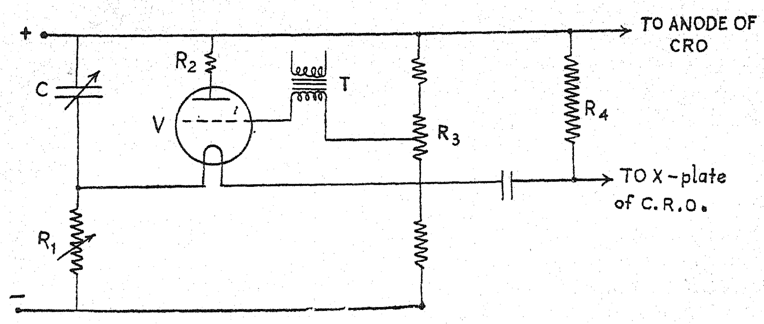


Fig. 35·5 Time base with thyratron discharge valve

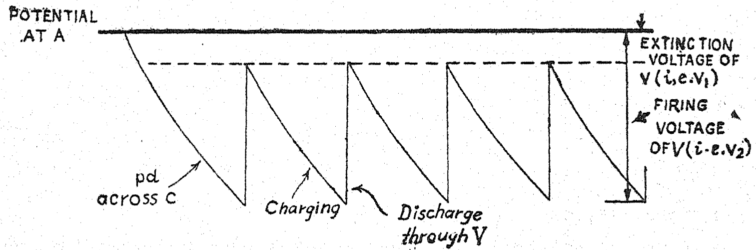


Fig. 35·6

the resulting potential variation across C having the form shown in fig. 35·6. The voltage is applied to X-plate via C_0.

During the charging period the spot is swept across the screen at a nearly uniform rate and is suddenly returned during the discharge period. The periodic time depends on the product CR_1 and the magnitude of $V_2—V_1$. It is usually adjusted in coarse steps by changing C. For fine setting R_1 is adjusted. The length of the sweep is proportional to $V_2—V_1$ and the deflection sensitivity of the cathode ray tube. It is controlled by means of R_3 which alters the grid potential of the thyratron and hence its firing voltage V_2. The capacitor charging curve is of course exponential but a sufficiently small section of it, which is used here, can be regarded as almost linear. Greater linearity can be obtained if R_1 is replaced by a pentode valve.

In order to synchronise the time base either the voltage under test or a voltage synchronous with it, and obtained from another part of the circuit, is applied to the transformer T. The secondary voltage of T triggers the thyratron and synchronises the time base if the latter has been adjusted very nearly to the correct frequency. When synchronisation has been achieved, the trace is then repeated again and again in the same position and appears stationary on the screen.

35.2. Applications of Cathode Ray Oscillograph. Periodic and transient phenomena can be observed with this instrument, provided, in the latter case, that the phenomenon exists for a period which is long enough to make an impression on the retina of the observer's eye.

Some applications to the study of periodic phenomena are given below.

Exercise 1. Study of A. C. supply wave form. The operation is relatively simple. The filament and then the high potential source are turned on and the position of the electron beam on the screen S is noted. The focussing control rheostats are turned until the beam gives a sharp fine image on the screen. Then the bias on the deflecting sets of plates X and Y are adjusted to place the point of impact on the left hand centre of the screen. The time base sweep circuit is tested by putting on a repeat sweep so that it sweeps across the screen from left to right in a rapid succession of sweeps. This gives one a chance to see that the time axis is clearly defined and straight.

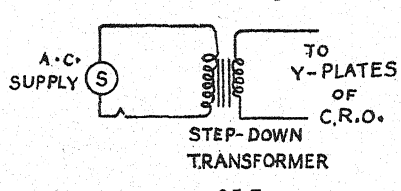


Fig. 35·7

Next with the help of a step-down transformer, the output of the secondary is fed on Y-plates of the C.R.O. and the time base switch synchronised to the supply frequency is switched on. What now appears on the screen is the wave form of the A.C. supply.

If the voltage senstivity of the oscillographs for the Y-axis is known, the peak voltage of the supply can be calculated from the measurement of the wave form.

Exercise 2. Study of Hysterisis Cycle. A circuit as depicted in figure 35·8 is connected up so that the p.d. across the ohmic resistance R can be fed on the X-plates of the C. R.O. and the p.d. across the condenser C across the Y-plates. Having adjusted the spot on

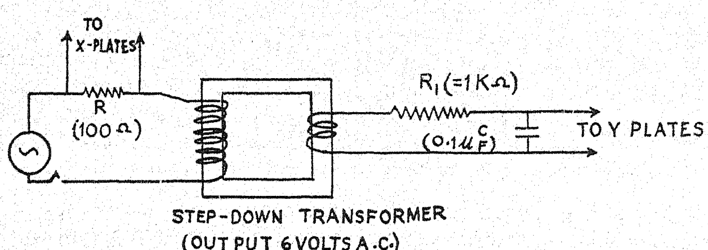


Fig. 35·8

the screen now to lie at its centre, A. C. supply in the transformer is switched on so that the requisite p. d. s. are applied across X-and Y-plates. It will be noticed that a stationary hysterisis loop appears on the screen. This may be traced on a tracing paper.

The calculation of hysterisis loss from the area of the traced loop is beyond the scop of the present book.

Exercise 3. Comparison of Frequencies. A simple and useful application of the C.R.O. is as a frequency comparator for an audio-frequency oscillator which may be calibrated in terms of a known frequency.

"When two simple harmonic motions are plotted against each other at right angles the resulting configuration is called a Lissajous figure. Since SHM plotted against time gives sinusoidal configurations, two sinusoidal electrical inputs to an oscilloscope will give a Lissajous pattern on the screen. The particular pattern depends upon the frequency, amplitude and phase relationships of the two inputs.

The frequency ratio of the two inputs may be determined from an analysis of the Lissajous figure produced. If a Lissajous figure is enclosed in a rectangle whose sides are parallel to the formation axes of the figure, the frequency ratio of the two inputs may be determined by counting the points of tangency to the sides of the rectangle enclosing the pattern. The ratio of the tangency points is in the inverse ratio of the input frequencies.

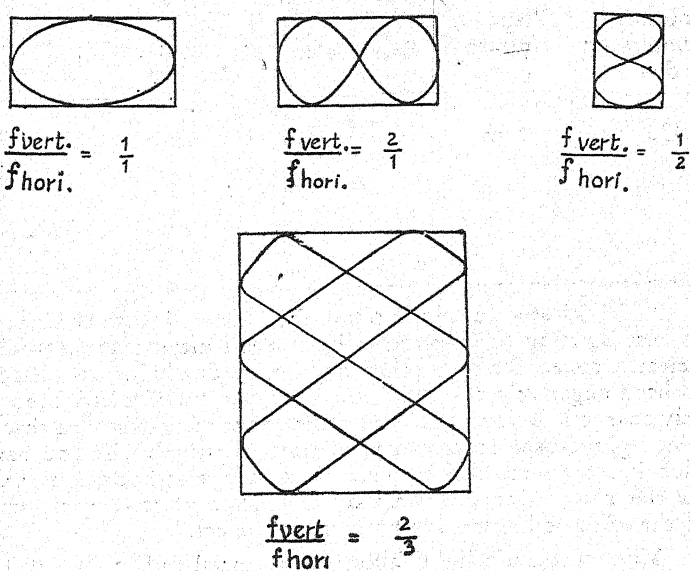


Fig. 35·9

The procedure is as follows:

(a) Connect one signal generator to the vertical input and the other to the horizontal input of the oscilloscope. Switch controls so that the oscilloscope accepts the output of the signal generator instead of the horizontal sweep. Set both generators for 1000 cycles, and *make gain adjustments* until an ellipse of satisfactory size is observed on the screen. Adjust controls as necessary to "stop" the ellipse. By switching one of the generators off and on, cause the ellipse to change phase, noting the various shapes it assumes. By phase changes and amplitude adjustments see if you can get a circular configuration.

(b) Leaving the vertical (Y) input at 1000 cycles and assuming it to be the standard, adjust the horizontal input generator (the variable) to approximately 500 c.p. s. to obtain the 1-2 Lissajous figure, a figure 8 on its side.

Next obtain the 2 : 1 pattern by varying the horizontal input frequency. This is an upright figure of 8.

In like manner, obtain Lissajous figures down to 1 : 5 and up-to 5 : 1. Sketch all the figures obtained and compare the frequency obtained from Lissajous ratios with the dial reading of the horizontal input signal generator."

Record the data in a neat table.

Vertical Input = 1000 c. p. s.

Horizontal Input frequency on dial	Shape of figure	No. of tangency points On X-axis	On Y-axis	$\frac{\text{vert.}}{\text{hori.}}$ =	hori.
490	∞	2	1	$\frac{2}{1}$	$\frac{1000}{2}=500$
1990	8	1	2	$\frac{1}{2}$	$1000\times2=2000$
...					
...					

35.3. Photo-electric Effect. In 1888, Hallwachs shone ultraviolet light on to an electrically charged metal plate, standing on an electroscope. He found that the plate gradually lost its charge, if it had been negatively charged at the start, but not if it had been positively charged. Many years later other experiments showed that ultraviolet light caused metal plates to emit negatively charged particles, which had the same charge to mass ratio as the cathode rays, *i.e.*, they were electrons. Hence this emission is called photo-electric emission and the effect is known as photo-electric effect.

Monochromatic light falling on a metal plate A will liberate photo-electrons; these can be detected as a current if they are attrated

to a metal electrode B by means of a potential difference V applied between A and B such that B is at positive potential. A galvanometer G could serve to measure this photo-electric current.

Fig. 35·10 is a graphical representation of the data between the photo-electric current i and the coresponding potential difference V.

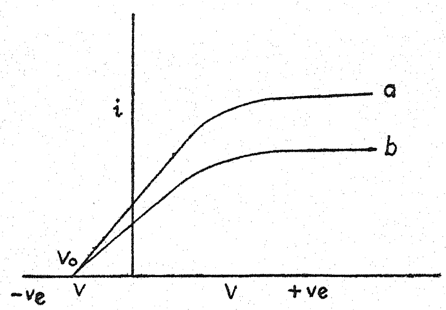


Fig. 35·10

If V is made large enough, the photo-electric current reaches a saturation value at which all photo-electrons emitted by the plate A are collected by the anode B. If V were reversed in sign, the photo-electric current does not immediately drop to zero; this proves that photo electrons are emitted with a finite velocity. Some do reach the anode B despite the fact that the electric field opposes them. However, when this reversed potential is made large enough a value V_o, the stopping potential, is reached at which the photo-electric current is zero. Evidently eV_o where e is the electronic charge measures the kinetic energy K_{max} for the fastest ejected photo-electrons, *i.e.*,

$$K_{max}=eV_o$$

From the graphs a and b corresponding to different intensities of the incident beam of light, it can be clearly inferred that K_{max} is independent of the intensity of light.

Fig. 35·11 shows the stopping potential V_0 as a function of the frequency of the incident light for sodium.

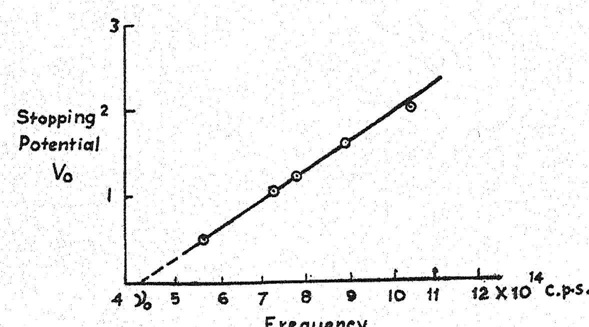


Fig. 35·11

Fig. 34·11. A plot of miltikan's measurement of stopping potentials at various frequencies. The cut off frequency (by extrapolation) is $4{\cdot}39 \times 10^{14}$ c.p.s.

It can evidently be noted from the graph that there is a definite cut off frequency below which no photo-electric effect occurs.

The three major features of photo-electric effect could not be explained on the basis of the wave theory.

1. K_{max} $(=eV_0)$ is independent of the frequency of light.

2. For each surface there exist a characteristics cut off frequency v_o. For frequencies less than this, the photo-electric effect disappears, no matter how intense the illumination.

3. No detectable time lag has ever been measured between the falling of light and the ejection of the photo-electrons.

It was Einstein who successfully explained the photo-electric effect by making an assumption that energy is a light beam travels in concentrated bundles called PHOTONS. The energy E of a single photon is given by

$$E=h\mu$$

where V is the frequency and h, the planck's constant.

Applying the pho on concept to the photo-electric effect, Einstein wrote

$$h\mu=E_o+K_{max} \tag{35.19}$$

where $h\mu$ is the energy of the photon. The equation above expresses the fact that the photon carries the energy $h\mu$ into the surface. Part of this energy E_0 is used in causing the electrons to pass through the metal surface. The excess energy $(h\mu-E_o)$ is given to the electron in the form of kinetic energy; in case the electron does not lose energy by internal collisions, as it escapes from the metal, it will exhibit it all as kinetic energy after it emerges. Thus K_{max} represents the maximum kinetic energy that the photo-electrons can have outside the surface, in nearly all cases it will have less energy than this because of internal losses.

Einstein's theory clearly explains the three features of photo-electric effect mentioned earlier. K_{max} is independent of the light intensity. Doubling the intensity of light merely doubles the number of photons and thus doubles the photo-electric current; it does not change the energy $h\mu$ of the individual photons or the nature of the individual photo-electric processes described by equation 35.19 above.

The existence of a characteristic frequency is explained in this manner. If K_{max} equals zero,

$$h\mu_o=E_o$$

which asserts that the photon has just enough energy to eject the photo-electrons but none extra to appear as kinetic energy. E_o is called the

work functions of the substance. If v is reduced below V_0, the individual photons,no matter how many of them there are, (*i.e.*, no matter how intense the beam of light) will not have enough energy to eject photo-electrons.

Further there is no question of a time lag between the falling of light and the ejection of a photo-electron because the requisite energy is supplied in a concentrated bundle.

35·4. **Photo-electric Cells.** There are three main type of photo-electric cells.

(i) Photo-conductive

(ii) Photo-voltaic

(iii) Photo-emissive.

The oldest type is the photo-conductive type which suffers a change of resistance when illuminated. This is now of limited industrial importance. The photo-voltaic type is a generating cell and will pass a current when illuminated. This has been described in the chapter on photometry—Experiment 16·2. The photo-emissive type emits eletrons from an illuminated cathode and requires a positive collecting anode like a thermionic valve.

35·5. **Photo-emissive Cells.** Fig. 35·12 represents a common arrangement of electrodes. The cathode has an emissive coating such as sodium, potassium or caesium, on the side facing the anode. Some of the light falling on either side of the V-cathode is reflected on to the other side and thus has two chances of being absorbed and causing emission. The rather sketchy anode in the form of a loop of platinum wire is satisfactory because most electrons shooting past the anode will slow up and will ultimatey be attracted back to the anode. The electrodes are supported in an evacuated glass bulb fitted with a base like a thermionic valve; the anode and cathode connections being brought out at opposite ends of the envelope to reduce the possibility of leakage.

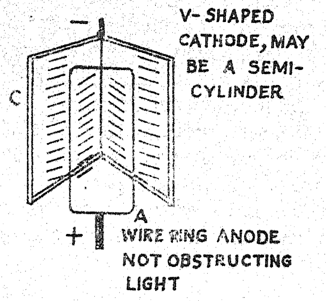


Fig. 35·12

35·6. **Spectral Response.** The emission varies with wavelength and the alkali metals show a selective effect, with a maximum in or near the visible part of the spectrum, as shown in figure 35·13. The emission is practically independent of temperature, suggesting that the effect is in reality more like ionisation than like thermionic emission, and that the electrons concerned are really those within the atom and not the free conductivity electrons.

Curves of these types are obtained experimentally in the following ways. Light from an incandescent lamp is passed through a prism of a

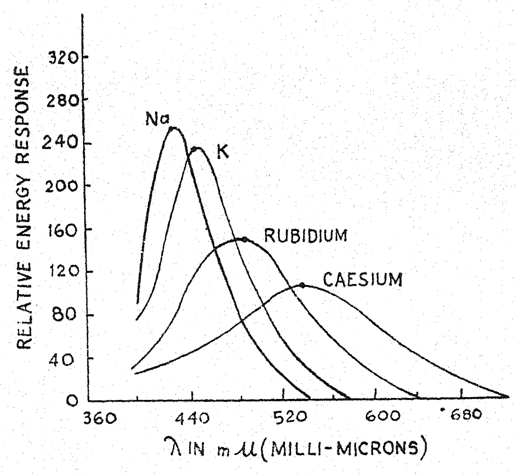


Fig. 35.13

monochromator for dispersion, a narrow band of wavelengths being selcted by means of an approximately placed system. The current given by the photo-electric surface when exposed to the light passing through the system of slits is noted. The current given by a blackened thermo-pile when exposed to the same light is also noted. The ratio of these two readings is plotted vrs. the mean wavelength of the incident light. Blackened thermopiles are used because they absorb all radiation incident upon them equally, regardless of the wavelength. This permits the measure of the energy contained in any part of the spectrum to be made. An automatic spectral sensitivity curve tracer has been designed for obtaining these curves quickly with the aid of a cathode ray tube.

Experiment 35·3

Object. To draw the characteristic curves of a photocell and to find the maximum velocity of the emitted electrons.

Apparatus. Photo-emissive all mounted inside a blackened wooden box with a wide slit, a sensitive suspended coil galvanometer, lamp and scale arrangement, an optical bench, power supply for D. C. anode potential or a dry battery 0-45 volts, a rheostat, a plug-key, a resistance box and connexion wires.

Theory. To draw the characteristic curves of a photocell, we shall study how the photo-electric current, for a given intensity of the beam of light, varies with the p. d. between the anode and the cathode. The current being proportional to the deflection of the galva-

nometer in the photocell circuit, it would suffice to plot a graph between p. d. and the deflection.

This study can be repeated for different light intensities by altering the distance of the photocell from the lamp.

If the applied p. d. is reversed in small steps of say 0·05 volts *i.e.*, the applied potential on the anode become negative in steps of 0·05 volts, the photo-electric current gradually decreases. From this part of the study a value of V_0 of the stopping potential (at which the photo-electric current is zero), can be computed graphically. Evidently eV_o where e is the electronic charge measures the kinetic energy K_{max} for the fastest ejected photo-electrons, whence

$$K_{max}=\tfrac{1}{2}mv^2_{maz}=eV_o$$

or
$$v_{max}=\sqrt{\frac{2eV_o}{m}}$$

If e/m is taken in e. m. u. as $1{\cdot}76\times10^7$ e. m. u. of charge per gm. V_0 should also be expressed in e. m. u. Thus, if V_0 is the p. d. in volts,

$$v_{max} = \sqrt{2\times1.76\times.0^7\times V_0\times10^8} \quad \text{cm./sec.} \qquad (35.20)$$

$$V_{max\cdot} = 2\times1{\cdot}76\times107\times V_0\times10^8 \quad \text{cm. sec.} \qquad (35.20)$$

Method. Make the connexions as depicted in Fig.35.14. Mount the photocell at one end of an optical bench. Arrange the source

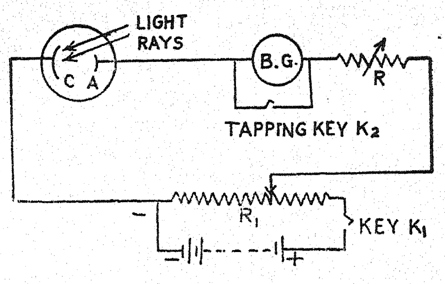


Fig. 35·14

of light from the cell, say at a distance of 60 cm. Place a suitable filter in the path of the rays coming from the lamp. Allow this light to fall on the cathode of the photocell, its anode being at a positive potential adjusted with the help of the potential divider R_1. Ajdust the p.d. as recorded with the voltmeter to be, say, 20 volts. When light falls on the cathode, note the deflection of the galvanometer. If the deflection is large, control the same with the variable resistance R, usually a dial resistance box, placed in series with the galvanometer. The apparatus is now ready for use; other voltages used will have to be less than 20 volts.

(a) **Anode voltage positive.** Keeping the intensity constant we wish to study the relationship between anode voltage and anode current. Hence in the first set of observations, keep the lamp, say, at a distance of 60 cm.

Allow the light to fall on the cathode, and note the deflection of the galvanometer. This corresponds to the anode voltage of 20 volts as recorded by the voltmeter. Repeat this by reducing the anode voltage first in steps of 2 volts and later in steps of one volt. Plot a graph between the anode potential and the corresponding deflection of the galvanometer.

This completes one set of observation.

Keep R fixed; increase the distance of the photocell from the source to, say, 80 cm. and repeat the whole set of observations and plot the graph.

Keeping R fixed, repeat the whole set of observations for distances between the source and the cell say, 90 cm., 100 cm.....and draw corresponding graphs.

(b) **Anode voltage negative.** For a distance of 60 cm. from the source to the cell, repeat the experiment by applying a negative anode potential in steps of 0·05 volt. Make the necessary changes in the connexions for this and use a low range voltmeter for reading the voltage. With the increase in negative potential the current and hence the deflection goes on rapidly falling. Take readings of the deflection for such negative potentials upto the stage when the deflection is reduced to 0·1 cm.

Repeat the experiment for a distance of, say, 100 cm. between the source and the cell.

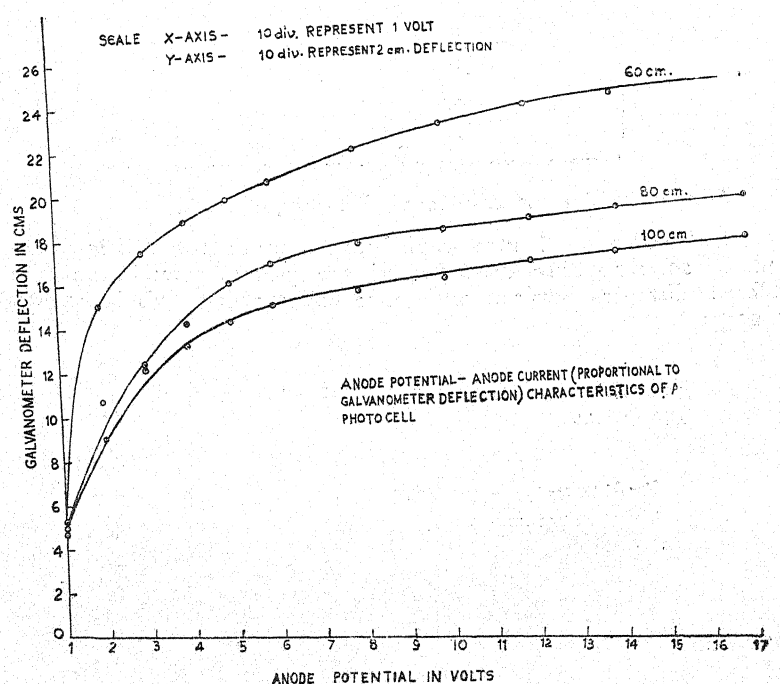
Plot both these graphs on a different sheet of graph paper with voltage, as usual, along negative X-axis and deflection along Y-axis. By extra-polation, find V_0 the negative potential at the anode needed to stop photo-emission. From a knowledge of V , calculate the velocity of the fastest ejected electrons.

Sources of error and precautions. (1) There should be no stray light in the room. It is desirable to perform the experiment in a separate small dark room.

(2) It is often found that different portions of the same emitting surface may possess different sensitivities. It is advisable therefore to illuminate a large part of the cathode uniformly rather than to focus the light source on only a portion of the photo-emissive surface.

(3) For positive anode potential observations, readings of deflection should be taken by altering the anode potential from a maximum to lower values first in steps of 2 volts and later in steps of one volt.

(4) Negative anode potential should be varied in steps of 0·05 volt and the corresponding graph should be plotted on a separate graph sheet.



Graph 1

Observations. **Table.** 1. *Anode Potential Positive*

Distance between source and photocell = 60 cm.

Initial position of the spot of the gal. on the scale = 0.0 cm.

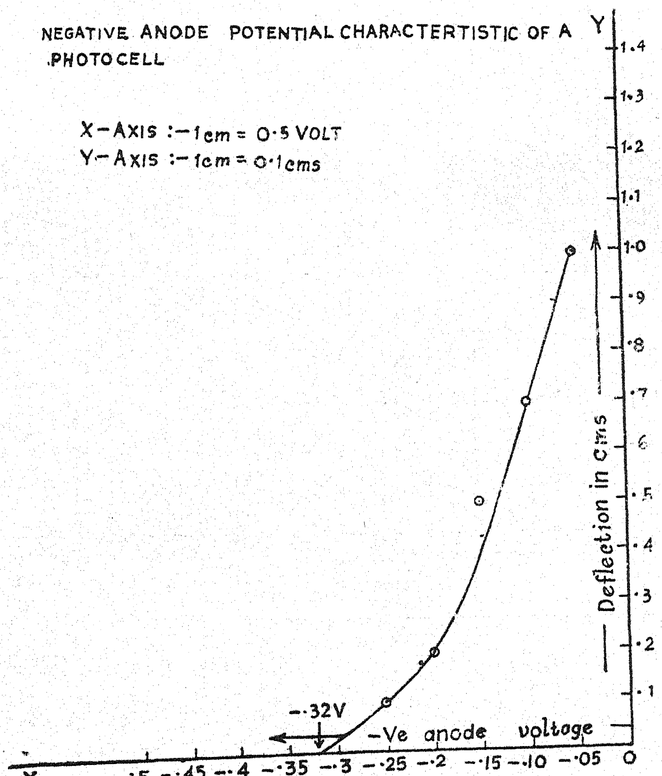
Anode Potential volts	Deflection of Galvanometer cm.	Anode Potential volts	Deflection of Galvanometer cm.
18	24.8	6	20.4
16		5	19.7
14	23.8	4	18.7
12	23.3	3	17.4
10	22.0	2	15.1
8	21.6	1	5.3

Table. 2. Anode Potential Negative

Anode Potential Volts	Deflection of Galvanometre cm.
—0.05	
—0.10	
—0.15	
—0.20	
—0.25	

Similar Tables for distances of 80, 90,....cms.

Calculations. 1. Plot graph between anode potential in volts and the corresponding deflection of the galvanometer in cms. for different distances between the source and the cell. This is shown in graph 1.



Graph 2

From graph 2, the negative stopping potential.

$$V_0 = \quad \text{Volt}$$

$$\therefore \quad V_{max} = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{2 \times 1{\cdot}76 \times 10^7 \times V_0 \times 10^8}$$

This gives the velocity of the fastest ejected electrons.

Result. 1. Characteristics curves (anode voltage vrs. galvanometer deflection—proportional to anode current) are shown in graph 1.

2. The stopping potential at which the photo-electric current is zero is $V_o =$ volt.

3. The velocity V_{max}. of the fastest ejected electrons is given by $V_{max} =$ cm./sec.

Exercise. To study the frequency response of a photo-electric cell.

Apparatus. As in the last experiment and a set of filters.

Method. Make the connexions as in the previous experiment. Adjust the distance of the photocell from the source to be, say, 60 cm. Ajdust R to such a value that with an anode potential of 20 volts the deflection is not beyond the scale when light is made to fall direct from the white light lamp on the cathode of the cell. We shall now keep the distance of 60 cm. and the value of R fixed for the set of observations.

With the anode potential of 20 volts, allow the light from the lamp to pass through the blue filter (mean transmission wavelength 4300 A) and observe the deflection of the galvanometer.

Replace the blue filter in succession by green, greenish yellow and yellow filters and observe the corresponding deflections of the galvanometer.

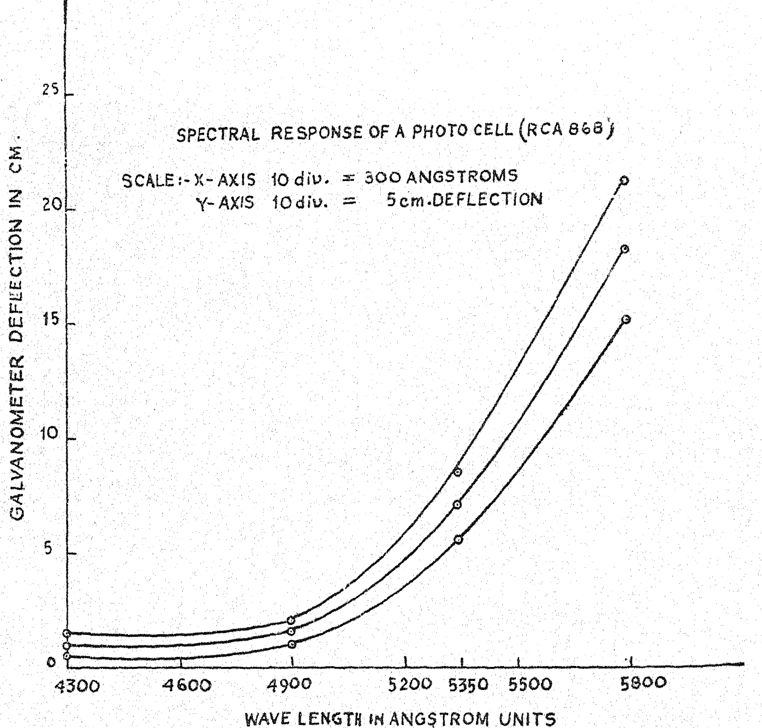
Reduce the anode potential to 12 volts and repeat the observations. Take another set of oservations with the anode potential set at 8 volts.

Tabulate your observations as follows :

Distance between lamp and cell = 60 cm.

Set No.	Anode Potential volts	Galvanometer Deflection in cms with			
		Blue filter $\lambda =$ A	Green filter $\lambda =$ A	Greenish yel-low filter $\lambda =$ A	Yellow filter $\lambda =$ A
1					
2					
3					

Plot a graph between mean wavelength transmitted by the filter along the X-axis and the corresponding galvanometer deflection along the Y-axis for a fixed anode potential. On the same sheet draw the similar graphs for other fixed anode potentials. This is shown in graph 35·3.



Graph 35·3

The graph reveals how the response of a photo-emissive cell varies with wavelength of light.

QUESTIONS

'e' BY MILLIKAN'S OIL DROP METHOD.

What is the construction of Millikan's Oil drop apparatus ? How do you introduce liquid drops in between the two plates ? How do these drops get charged ? What type of liquid are you using and why ? Why are the plates made of metal ? What is the function of the water cell in front of the light source ? How do you select a particular drop for working ? When you establish an electric field, some of the drops move against gravity, why is it so? What is the condition that a drop may remain stationary? How can you determine the velocity of the drop? What is Stoke's Law ? What are the limitations of Stoke's law ? When the drop is falling under gravity why does it move downwards with a uniform velocity ? How does the determination of the charge on the various drops enable you to determine the electronic charge. What are other methods for determining 'e'. What precautions do you take in the experiment ?

e/m BY THOMSON METHOD

What is a cathode ray tube? Give the construction of the tube. Is the ratio of the electronic charge to mass dependent on the material of the cathode ? How do you produce cathode rays in your experiment ? What is the principle of the experiment? How is the electron beam deflected from its path when the magnetic field is applied ? Does the application of the magnetic field alter the Kinetic energy of the moving electron ? How do you produce a magnetic field and how do you measure it ? How do you apply the electrostatic field and how do you calculate it ? What precautions do you take in your experiment ? How can you proceed to get a well defined spot on the screen ? What is meant by accelerating voltage? What value of accelerating voltage are you using here ? How do you get such a large voltage ? Why are the deflecting plates short and curved at the end towards the screen ? Why do you have the plates o close to each other ? What may be the order of the capacity of the deflecting plates ? Is the value of e/m of an electron constant ?

CATHODE RAY OSCILLOSCOPE

What do you mean by cathode rays ? Who discovered them ? What is the construction of the cathode ray oscilloscope ? Why are there more than one anode ? What type of emission is there ? How can you concentrate the beam of electrons ? What do you mean by a fluorescent material ? What do you mean by horizontal and vertical gain ? What is time base ? Give the diagram of a time base circuit. How can you examine the wave form of the a c supply ? What is sweep selection ? What happens when you apply a D. C. voltage on the YY plates of the apparatus ? Can you measure an unknown A.C. or D.C. Voltage with C.R.O. ? How can you compare two unknown frequencies ? Can you measure an unknown frequency ? What are Lissajous figures ? How are they used to compare two frequencies ? What are the other uses of C.R.O. ?

PHOTO-CELL

What is photo-electric effect ? Who discovered this effect ? What are the laws of photo-electric emission ? What are the factors on which photo-electric current depends ? Is the photo-electric emission possible for all wavelengths of light ? What is photo-electric work function ? What is the difference between photo-electric emission and thermionic emission. Explain Einstein's photo-electric equation ? What is threshold frequency? Is the photo-electric emission possible when light rays are replaced by X-rays ? What is a photo-cell ? How many types of

photo-cell do you know ? What is the difference between gas-filled and vacuum photocell ? Of what material is the cathode of a photo-voltaic cell made ? What type of cell are you using ? What is the shape of the cathode and why ? What is the shape of the anode and why ? What do you mean by spectral response of a photocell ? What do you mean by characteristics of a photocell and how do you study them ? How do you calculate the velocity of the fastest ejected electrons ? Why is the cut off voltage negative ? Is the cut off voltage dependent on the intensity of light ? To what uses is a photocell put ?

APPENDIX I

CONVERSION FACTORS

To convert		Multiply by	Logarithm
Length :			
In. into cm.	... ...	2·540	0·4048
Ft. into metres	... ...	·3048	$\bar{1}$·4840
Yds. into metres	... ...	·9144	$\bar{1}$·9611
Miles into km.	... ...	2·6093	0·2066
Area :			
Sq. in. into sq. cm.	...	6·451	0·8096
Sq. ft. into sq. metres	...	·0929	·9680
Sq. yds. into sq. metres	...	·8361	·9223
Acres into sq. metres	...	4047	3·6071
Volume :			
Cub. in. into c. c.	...	16·39	1·2143
Cub. in. into litres	...	·01639	$\bar{2}$·2145
Cub. ft. into litres	...	28·32	1·4521
Gallons into litres	...	4·546	0·6677
Gallons into cub. ft.	..	·1606	1·2057
Mass :			
Grain into m. gm.	...	64·8	1·8116
Oz. (*av.*) into gm.	...	28·35	1 4525
lb. into gm.	...	453·6	2 6567
Ton into kgm.	...	1016	3·0069
Force, work, *etc.* :			
Gm.-wt. into dynes	...	981	2.9917
lb.-wt. into poundals	...	32·2	1·5079
Poundals into dynes	.	1·3825 × 10^4	14 407
Cm.-gm. into ergs	..	981	2·9917

CONVERSION FACTORS—(*concluded*)

To convert	Multiply by	Logarithm
Ft. lbs. into Joules ...	1·356	0·1324
Horse Power into ft. lbs./sec. ...	550	2·7404
Horse Power into Watts ...	745·7	2·8726
K. W.H. into Joules ...	$3{\cdot}600 \times 10^6$	6·5563
Miscellaneous :		
Radians into degree ...	57·296	1·7582
$\log_{10} N$ into $\log_e N$ ...	2·303	·3623
lbs. of water at 62^0F into c.c. ...	454·6	2·6577
lbs. of water at 62^0F into gallons ...	·100	$\bar{1}{\cdot}0000$
Atmospheres into dynes per sq. cm.	$1{\cdot}014 \times 10^6$	6·0059
Atmospheres into lbs. per sq. in. ...	14·70	1·1673
m. p. h. into cm. per sec. ...	44·70	1·6503

USEFUL DATA AND FORMULAE

$\pi = 3{\cdot}14159$

dyne = force which, acting on a mass of 1 gm. produces an acceleration of 1 cm. per sec. per sec.

1 Bar = 10^6 dynes per sq. cm.

1 Atmosphere = 760 mm. of Hg. = 1·0136 bar.

1 erg = work done by a force of 1 dyne acting through 1 cm.

1 Joule = 10^7 ergs

1 Watt = 1 Joule per sec.

R, the gas constant = 8·305 Joules per gm. molecule = 1·988 cal. per gm. molecule

1 gallon of water weighs 10 lbs. approx.

1 cub. ft. of water weighs 62·43 lbs.

Area of a circle, radius r ... $= \pi r^2$

Area of an ellipse of semi-axes a and b ... ... $= \pi ab$

Area of the curved surface of a cylinder of length l, radius r $= 2\pi rl$.

Area of the curved surface of a cone of height $h = \pi r\sqrt{r^2 + h^2}$

Area of the curved surface of a sphere of radius r ... $= 4\pi r^2$

Volume of a cylinder ... $= \pi r^2 l$

Volume of a cone ... $= \frac{1}{3}\pi r^2 l$

Volume of a sphere ... $= \frac{4}{3}\pi r^3$

Volume of an ellipsoid-axes $2a$, $2b$, $2c$ ... ... $= \frac{4}{3}\pi abc$

APPENDIX II

Physical Constants

BOILING POINT OF WATER UNDER VARIOUS BAROMETRIC PRESSURES

+mm. →	0	1	2	3	4	5	6	7	8	9
mm.										
700	97·71	·75	·79	·83	·87	·91	·95	·99	98·03	·07
710	98·11	·14	·18	·22	·26	·30	·34	.38	·42	·45
720	98·49	·53	·57	·61	·65	·69	·72	·76	·80	·84
730	98·88	·91	·95	·99	99·03	·07	·10	·14	·18	·22
740	99·25	·29	·33	·37	·41	·44	·48	·52	·56	·59
750	99·63	·67	·70	·74	·78	·81	.85	·89	·93	·96
760	100·00	·03	·07	·11	·15	·18	·22	·26	·29	·33

ELECTRO-CHEMICAL-EQUIVALENT

Element	E. C. E. (gm./coulomb)	Chemical equivalent
Silver ...	·0011180	107·88
Copper ...	·0003295	31·88
Lead ...	·0010731	103·61
Hydrogen ...	·0000105	1·0081
Oxygen ...	·0000829	8·0000

RESISTANCE OF WIRES OF VARIOUS GAUGES AND MATERIALS

S. W. G No.	Diameter in mm.	Area of Cross-section Sq. cm.	Resistance per metre (ohm/metre) Copper	Eureka	Manga-nin	German Silver
10	3·25	·08302	·0021	·057	·051	·049
12	2·64	·05480	·0032	·086	·077	·041
14	2·03	·03243	·0054	·146	·131	·070
16	1·63	·02076	·0083	·228	·204	·109
18	1·22	·01168	·0148	·405	·361	·193
20	·914	·006567	·0260	·722	·645	·345
22	·711	·00397	·0435	1·20	1·07	·57
24	·559	·002452	·070	1·93	1·73	·92
26	·457	·001642	·105	2·89	2·58	1·38
28	·374	·001110	·155	4·27	3·82	2·02
30	·315	·000779	·222	6·08	5·45	2·90
32	·274	·000591	·293	8·02	7·18	3·83
34	·234	·000429	·404	11·1	9·90	5·27
36	·193	·000293	·590	16·2	14·50	7·74
38	·152	·0001824	·950	26·0	23·2	12·4
40	·122	·0001170	1·48	40·6	36·3	19·4

MAGNETIC ELEMENTS AND g

Station	Declination	Angle of Dip	Horizontal component H	Vertical component V	g
Agra	0° 10′ E	40° 40′	·3484	·2985	979·06
Ajmer	1° 10′ E	39° 20′	·3483	·2861	978·98
Aligarh	0° 20′ E	41° 50′	·3455	·3091	978·08
Allahabad	0° 20′ W	37° 10′	·3629	·2758	978·94
Bareilly	0° 20′ E	42° 20′	·3436	·3136	...
Banaras	0° 30′ W	37° 10′	·3635	·2764	...
Bombay	0° 20′ W	25° 30′	·3761	·1792	978·63
Calcutta	0° 00′ W	31° 30′	·3819	·2342	978·78
Kanpur	0° 00′	38° 39′	·3628	·2901	978·97
Chandausi	0° 30′ E	42° 40′	·3429	·3167	...
Dehradun	0° 50′ E	45° 50′	·3315	·3405	979·06
Delhi	0° 40′ E	42° 52′	·3453	·3204	979·15
Gorakhpur	0° 20′ W	43° 40′	·3576	·2972	978·94
Gwalior	0° 20′ E	39° 00′	·3531	·2857	978·96
Jaipur	0° 30′ E	40° 30′	·3470	·2961	978·52
Jodhpur	0° 00′	39° 10′	·3482	·2835	978·92
Khurja	0° 30′ E	42° 10′	·3426	·3109	978·08
Lucknow	0° 10′ W	40° 00′	·3536	·2965	979·00
Meerut	0° 40′ E	43° 30′	·3389	·3212	979·15
Udaipur	0° 00′	35° 50′	·3620	·2613	978·9

MATHEMATICAL TABLES

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

Natural Sines

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences. 1′	2′	3′	4′	5′
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	14
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10°	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	11	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20°	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30°	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	9	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40°	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences 1′	2′	3′	4′	5′
45°	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50°	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60°	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70°	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80°	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	·9998	9999	9999	9999	9999	1·000	1·000	1·000	1·000	1·000	0	0	0	0	0

NATURAL COSINES

N.B.—Subtract **Mean Differences**

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences 1′	2′	3′	4′	5′
0°	1·000	1·000	1·000	1·000	1·000	1·000	·9999	9999	9999	9999	0	0	0	0	0
1	·9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	·9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	·9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	·9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	·9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	·9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	·9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	·9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	·9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10°	·9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	·9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	·9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	·9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	·9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	·9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	·9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	·9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	·9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	·9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20°	·9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	·9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	·9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	·9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	·9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	·9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	·8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	·8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	·8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30°	·8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	·8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	·8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	·8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	·8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	·8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	·8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	·7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	·7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	·7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40°	·7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	·7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	·7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	·7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

NATURAL COSINES

***N.B.—Subtract* Mean Differences**

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences 1′	2′	3′	4′	5′
45°	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	·6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	·6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	·6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50°	·6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	·6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60°	·5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70°	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80°	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	14
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15

NATURAL TANGENTS

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
											1′	2′	3′	4′	5′
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10°	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20°	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30°	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40°	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

	0′	6′	12′	18	24′	30	36′	42′	48′	54′	Mean Differences 1′	2′	3′	4′	5′
45°	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50°	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60°	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	0̄057	0̄145	0̄233	0̄323	0̄413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70°	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	0̄061	0̄237	0̄415	0̄595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972					
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	0̄045	0̄504	0̄970					
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80°	5·6713	7297	7894	8502	9124	9758	0̄405	1̄066	1̄742	2̄432					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	0̄264					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	0̄285					
83	8·1443	2636	3863	5126	6427	7769	9152	0̄579	2̄052	3̄572					
84	9·514	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					

Mean differences no longer sufficiently accurate.

POWERS, ROOTS & RECIPROCALS

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	$\sqrt{10n}$	$\frac{1}{\sqrt{n}}$	$\frac{1}{\sqrt{10n}}$
1	1	1	1·0000	1·0000	1·00000	3·1623	1·00000	0·31623
2	4	8	1·4142	1·2599	0·50000	4·4721	0·70711	0·22361
3	9	27	1·7321	1·4422	0·33333	5·4772	0·57735	0·18257
4	16	64	2·0000	1·5874	0·25000	6·3245	0·50000	0·15811
5	25	125	2·2361	1·7100	0·20000	7·0711	0·44721	0·14142
6	36	216	2·4495	1·8171	0·16667	7·7460	0·40825	0·12910
7	49	343	2·6458	1·9129	0·14286	8·3666	0·37796	0·11952
8	64	512	2·8284	2·0000	0·12500	8·9443	0·35355	0·11180
9	81	729	3·0000	2·0801	0·11111	9·4868	0·33333	0·10541
10	100	1000	3·1623	2·1544	0·10000	10·0000	0·31623	0·10000
11	121	1331	3·3166	2·2240	0·09091	10·4881	0·30151	0·09535
12	144	1728	3·4641	2·2894	0·08333	10·9545	0·28868	0·09129
13	169	2197	3·6056	2·3513	0·07692	11·4018	0·27735	0·08771
14	196	2744	3·7417	2·4101	0·07143	11·8322	0·26726	0·08452
15	225	3375	3·8730	2·4662	0·06667	12·2474	0·25820	0·08165
16	256	4096	4·0000	2·5198	0·06250	12·6491	0·25000	0·07906
17	289	4913	4·1231	2·5713	0·05882	13·0384	0·24253	0·07670
18	324	5832	4·2426	2·6207	0·05556	13·4164	0·23570	0·07454
19	361	6859	4·3589	2·6684	0·05263	13·7840	0·22942	0·07255
20	400	8000	4·4721	2·7144	0·05000	14·1421	0·22361	0·07071
21	441	9261	4·5826	2·7589	0·04762	14·4914	0·21822	0·06901
22	484	10648	4·6904	2·8020	0·04545	14·8324	0·21320	0·06742
23	529	12167	4·7958	2·8439	0·04348	15·1658	0·20851	0·06594
24	576	13824	4·8990	2·8845	0·04167	15·4919	0·20412	0·06455
25	625	15625	5·0000	2·9240	0·04000	15·8114	0·20000	0·06325
26	676	17576	5·0990	2·9625	0·03846	16·1245	0·19612	0·06202
27	729	19683	5·1962	3·0000	0·03704	16·4317	0·19245	0·06086
28	784	21952	5·2915	3·0366	0·03571	16·7332	0·18898	0·05976
29	841	24389	5·3852	3·0723	0·03448	17·0294	0·18570	0·05872
30	900	27000	5·4772	3·1072	0·03333	17·3205	0·18257	0·05774
31	961	29791	5·5678	3·1414	0·03226	17·6068	0·17961	0·05680
32	1024	32768	5·6569	3·1748	0·03125	17·8885	0·17678	0·05590
33	1089	35937	5·7446	3·2075	0·03030	18·1659	0·17408	0·05505
34	1156	39304	5·8310	3·2396	0·02941	18·4391	0·17150	0·05423
35	1225	42875	5·9161	3·2711	0·02857	18·7083	0·16903	0·05345
36	1296	46656	6·0000	3·3019	0·02778	18·9737	0·16667	0·05270
37	1369	50653	6·0828	3·3322	0·02703	19·2354	0·16440	0·05199
38	1444	54872	6·1644	3·3620	0·02632	19·4936	0·16222	0·05130
39	1521	59319	6·2450	2·3912	0·02564	19·7484	0·16013	0·05064
40	1600	64000	6·3245	3·4200	0·02500	20·0000	0·15811	0·05000
41	1681	68921	6·4031	3·4482	0·02439	20·2485	0·15617	0·04939
42	1764	74088	6·4807	3·4760	0·02381	20·4939	0·15430	0·04880
43	1849	79507	6·5574	3·5034	0·02326	20·7364	0·15250	0·04822
44	1936	85184	6·6332	3·5303	0·02273	20·9762	0·15076	0·04767
45	2025	91125	6·7082	3·5569	0·02222	21·2132	0·14907	0·04714
46	2116	97336	6·7823	3·5830	0·02174	21·4476	0·14744	0·04663
47	2209	103823	6·8557	3·6088	0·02128	21·6795	0·14587	0·04613
48	2304	110592	6·9282	3·6342	0·02083	21·9089	0·14434	0·04564
49	2401	117649	7·0000	3·6593	0·02041	22·1359	0·14286	0·04518
50	2500	125000	7·0711	3·6840	0·02000	22·3607	0·14142	0·04472

POWERS, ROOTS & RECIPROCALS

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	$\sqrt{10n}$	$\frac{1}{\sqrt{n}}$	$\frac{1}{\sqrt{10n}}$
51	2601	132651	7·1414	3·7084	0·01961	22·5832	0·14003	0·04428
52	2704	140608	7·2111	3·7325	0·01923	22·8035	0·13868	0·04385
53	2809	148877	7·2802	3·7563	0·01887	23·0217	0·13736	0·04344
54	2916	157464	7·3485	3·7798	0·01852	23·2379	0·13608	0·04303
55	3025	166375	7·4162	3·8030	0·01818	23·4521	0·13484	0·04264
56	3136	175616	7·4833	3·8259	0·01786	23·6643	0·13363	0·04226
57	3249	185193	7·5498	3·8485	0·01754	23·8747	0·13245	0·04189
58	3364	195112	7·6158	3·8709	0·01724	24·0832	0·13131	0·04152
59	3481	205379	7·6811	3·8930	0·01695	24·2899	0·13019	0·04117
60	3600	216000	7·7460	3·9149	0·01667	24·4949	0·12910	0·04082
61	3721	226981	7·8102	3·9365	0·01639	24·6982	0·12804	0·04049
62	3844	238328	7·8740	3·9579	0·01613	24·8998	0·12700	0·04016
63	3969	250047	7·9373	3·9791	0·01587	25·0998	0·12599	0·03984
64	4096	262144	8·0000	4·0000	0·01563	25·2982	0·12500	0·03953
65	4225	274625	8·0623	4·0207	0·01538	25·4951	0·12403	0·03922
66	4356	287496	8·1240	4·0412	0·01515	25·6905	0·12309	0·03892
67	4489	300763	8·1854	4·0615	0·01493	25·8844	0·12217	0·03863
68	4624	314432	8·2462	4·0817	0·01471	26·0768	0·12127	0·03835
69	4761	328509	8·3066	4·1016	0·01449	26·2679	0·12039	0·03807
70	4900	343000	8·3666	4·1213	0·01429	26·4575	0·11952	0·03780
71	5041	357911	8·4261	4·1408	0·01408	26·6458	0·11868	0·03753
72	5184	373248	8·4853	4·1602	0·01389	26·8328	0·11785	0·03727
73	5329	389017	8·5440	4·1793	0·01370	27·0185	0·11704	0·03701
74	5476	405224	8·6023	4·1983	0·01351	27·2029	0·11625	0·03676
75	5625	421875	8·6603	4·2172	0·01333	27·3861	0·11547	0·03651
76	5776	438976	8·7178	4·2358	0·01316	27·5681	0·11471	0·03627
77	5929	456533	8·7750	4·2543	0·01299	27·7489	0·11396	0·03604
78	6084	474552	8·8318	4·2727	0·01282	27·9285	0·11323	0·03581
79	6241	493039	8·8882	4·2908	0·01266	28·1069	0·11251	0·03558
80	6400	512000	8·9443	4·3089	0·01250	28·2843	0·11180	0·03536
81	6561	531441	9·0000	4·3267	0·01235	28·4604	0·11111	0·03514
82	6724	551368	9·0554	4·3445	0·01220	28·6356	0·11043	0·03492
83	6889	571787	9·1104	4·3621	0·01205	28·8097	0·10976	0·03471
84	7056	592704	9·1652	4·3795	0·01190	28·9828	0·10911	0·03450
85	7225	614125	9·2195	4·3968	0·01176	29·1548	0·10847	0·03430
86	7396	636056	9·2736	4·4140	0·01163	29·3258	0·10783	0·03410
87	7569	658503	9·3274	4·4310	0·01149	29·4958	0·10721	0·03390
88	7744	681472	9·3808	4·4480	0·01136	29·6648	0·10660	0·03371
89	7921	704969	9·4340	4·4647	0·01124	29·8329	0·10600	0·03352
90	8100	729000	9·4868	4·4814	0·01111	30·0000	0·10541	0·03333
91	8281	753571	9·5394	4·4979	0·01099	30·1662	0·10483	0·03315
92	8464	778688	9·5917	4·5144	0·01087	30·3315	0·10426	0·03297
93	8649	804357	9·6437	4·5307	0·01075	30·4959	0·10370	0·03279
94	8836	830584	9·6954	4·5468	0·01064	30·6594	0·10314	0·03262
95	9025	857375	9·7468	4·5629	0·01053	30·8221	0·10260	0·03244
96	9216	884736	9·7980	4·5789	0·01042	30·9839	0·10206	0·03227
97	9409	912673	9·8489	4·5947	0·01031	31·1448	0·10153	0·03211
98	9604	941192	9·8995	4·6104	0·01020	31·3050	0·10102	0·03194
99	9801	970299	9·9499	4·6261	0·01010	31·4643	0·10050	0·03178
100	10000	1000000	10·0000	4·6416	0·01000	31·6228	0·10000	0·03162

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